

A Z-test is being carried out, so we need to find the values of 'z' below which and above which we would reject the null hypothesis. Since no particular direction of difference has been specified, we're doing a two-tailed test.

Given that the significance level α is 0.05 (in a two-tailed test), this means that the upper 2.5% (0.025) and the lower 2.5% (0.025) of Z-scores under the normal distribution would constitute our rejection region. The critical Z-scores corresponding to these probabilities from a standard normal distribution table are approximately -1.96 and +1.96.

So if your calculated Z-score is less than -1.96 or greater than 1.96, you would reject the null hypothesis. This constitutes your rejection region.

(ii) Find the z-value; add it to your picture:

The Z-value or test statistic for a z-score test is calculated as follows:

$$z = (x - \mu_0) / (\sigma / \sqrt{n})$$

where x is the sample mean, μ_0 is the population mean under the null hypothesis, σ is the population standard deviation, and n is the sample size.

In this case, $x = 6.25$, $\mu_0 = 5$, $\sigma = 10$ (since, $\sigma^2 = 100$ or variance is given as 100, so standard deviation $\sigma = \sqrt{100} = 10$), and $n = 64$.

So, $z = (6.25 - 5) / (10 / \sqrt{64}) = 1.25 * \sqrt{64} / 10 = 2.5$.

Our z-value is 2.5.

(iii) Decide whether or not to reject H_0 in favor of H_A :

Since the calculated Z-value (2.5) falls in the rejection region (anything greater than 1.96 or less than -1.96), we reject the null hypothesis in favor of the alternative hypothesis. Our sample mean is sufficiently different from the null hypothesis mean to believe that the difference is not due to random chance alone.

(iv) Find the p-value for this data; add to your picture:

The P-value is the probability of observing a more extreme test statistic in the direction of the alternative hypothesis from what was actually observed, assuming the null hypothesis is true.

For a Z-value of 2.5, the p-value is given by the area that is more extreme than this. You can find this by looking it up in the Z-table or using a software or a calculator. This value is approximately 0.0062. Recall that since this is a two-tailed test, we may have to multiply that value by 2, but even without doing so, our p-value (0.0062) is less than our significance level 0.05.

(v) What's the connection between the answers to (ii), (iii), and (iv)?

The Z-value in (ii) can be directly used to make a decision in (iii), as if it falls into the rejection region, we can reject the null hypothesis. Also, the Z-value can be used to calculate the p-value. If the p-value is less than the significance level, it means that the observed data would be very unlikely if the null

hypothesis were true, so you reject the null hypothesis, which is consistent with the decision in (iii). So (ii), (iii), and (iv) are all connected and are different ways of making the same decision about the null hypothesis.