Fractal dimensions in Riemann's zeta function

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We tend to think that infinity must be the realm of philosophy. Our body can’ t stand eternity. It is just the ideal. Riemann’ s zeta function must be almost correct, but nobody can prove this difficulty because of infinity. You can see the goal, but it moves forward with you. You never reach it. I have found the pattern of prime numbers which are fractals based on sieve of Eratosthenes. There is nothing new. You just expand integers or you divide them such as 1–100, 1–10000, and 1–1000000. I can’ t see infinity either, but a part includes everything because of fractal dimensions. Therefore, I think that we don’ t need to count prime numbers infinitely.

At first, Riemann’ s zeta function is based on sieve of Eratosthenes, and we need to count prime numbers to know zeros of Riemann’ s zeta function.

This is zeta function.

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s} + \frac{1}{7^s} + \frac{1}{8^s} + \frac{1}{9^s} + \cdot\cdot\cdot\cdot\cdot\cdot \qquad ①$$

Then, you put sieve of Eratosthenes in it.

$$\frac{1}{2^s}\zeta(s) = \frac{1}{2^s} + \frac{1}{4^s} + \frac{1}{6^s} + \frac{1}{8^s} + \frac{1}{10^s} + \cdot\cdot\cdot\cdot\cdot\cdot \qquad ②$$

① − ②

$$\left(1-\frac{1}{2^s}\right)\zeta(s) = 1 + \frac{1}{3^s} + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{9^s} + \frac{1}{11^s} + \cdot\cdot\cdot\cdot \qquad ③$$

$$\frac{1}{3^s}\left(1-\frac{1}{2^s}\right)\zeta(s) = \frac{1}{3^s} + \frac{1}{9^s} + \frac{1}{15^s} + \frac{1}{21^s} + \cdot\cdot\cdot\cdot \qquad ④$$

③ − ④

$$\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{2^s}\right)\zeta(s) = 1 + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{11^s} + \cdot\cdot\cdot \qquad ⑤$$

When you put prime numbers in zeta function infinitely, you can erase all numbers except for 1.

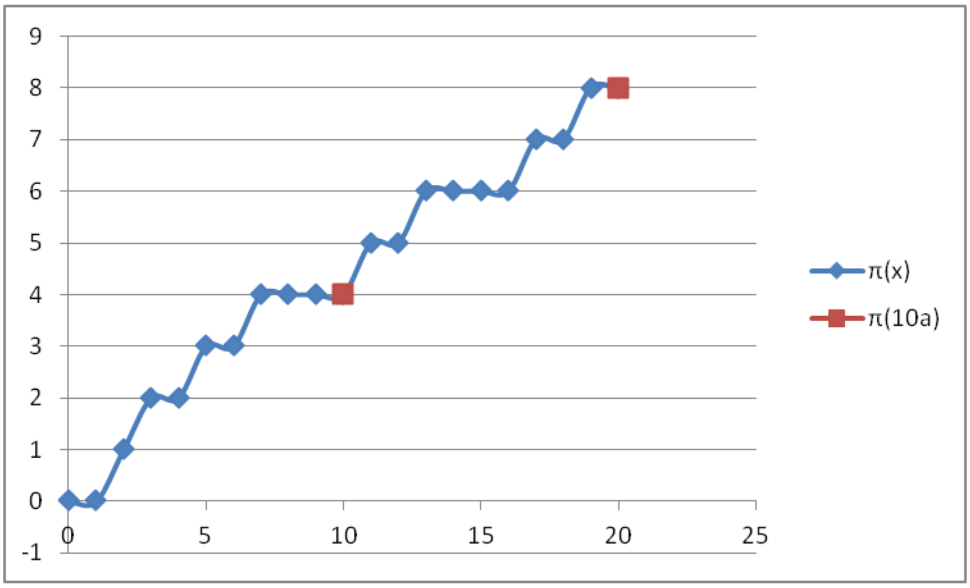
$$\cdots\left(1-\frac{1}{13^s}\right)\left(1-\frac{1}{11^s}\right)\left(1-\frac{1}{7^s}\right)\left(1-\frac{1}{5^s}\right)\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{2^s}\right)\times\zeta(s)=1$$

$$\zeta(s)=(1-2^{-s})^{-1}(1-3^{-s})^{-1}(1-5^{-s})^{-1}(1-7^{-s})^{-1}(1-11^{-s})^{-1}\cdots$$

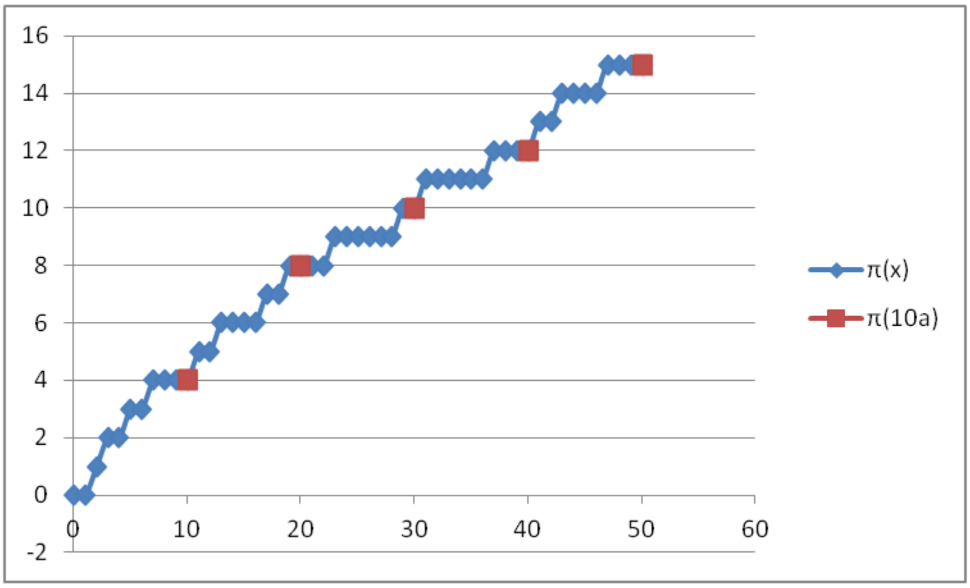
$$\zeta(s)=\prod_p(1-p^{-s})^{-1}$$

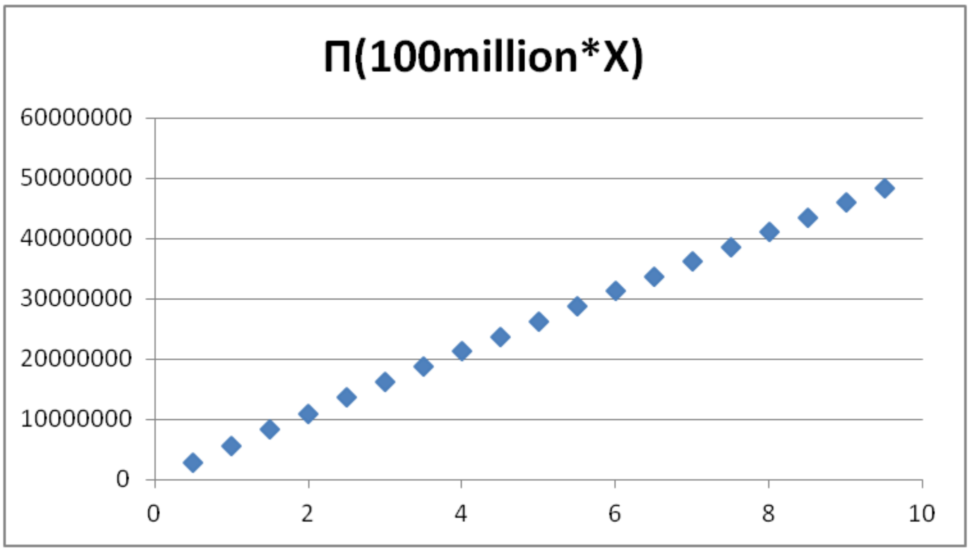
Zeros of Riemann' s zeta function mean $\zeta(s)=0$,and s must be always 0.5+zi which are complex numbers. i is an imaginary number. As you know, Riemann' s zeta function includes infinity. Therefore, you need to find all prime numbers to prove $\zeta(s)=0$ and 0.5+zi. It is impossible physically. Riemann' s zeta function expands integers to know the sum of prime numbers in a certain integer.

Then, you need to understand how to count prime numbers. Basically, there is no pattern to find prime numbers, but when you decide the realm of integer such as 100 or 10000, you can pick up prime numbers and count them. Visual basic is useful. The program is based on sieve of Eratosthenes. $\pi(x)$ means the sum of prime numbers until x. When x is 5, $\pi(x)=3$. The graph is like this.



$\pi(10a)$ means that x are 10,20,30,40・・・・. You count prime numbers infinitely with the same pattern.





J function is based on $\Pi(x)$, and when you calculate J function, you would know that it is almost the same with 0.5+zi.

$$J(x) = \Pi(x) + \frac{1}{2}\Pi\left(\sqrt{x}\right) + \frac{1}{3}\Pi\left(\sqrt[3]{x}\right) + \frac{1}{4}\Pi\left(\sqrt[4]{x}\right) + \frac{1}{5}\Pi\left(\sqrt[5]{x}\right) + \cdot\ \cdot\ \cdot$$

x→J(x)→0.5+zi

40→14.61667→14.1435657

65→21.5333→21.0279853

80→25.28333→25.01585333

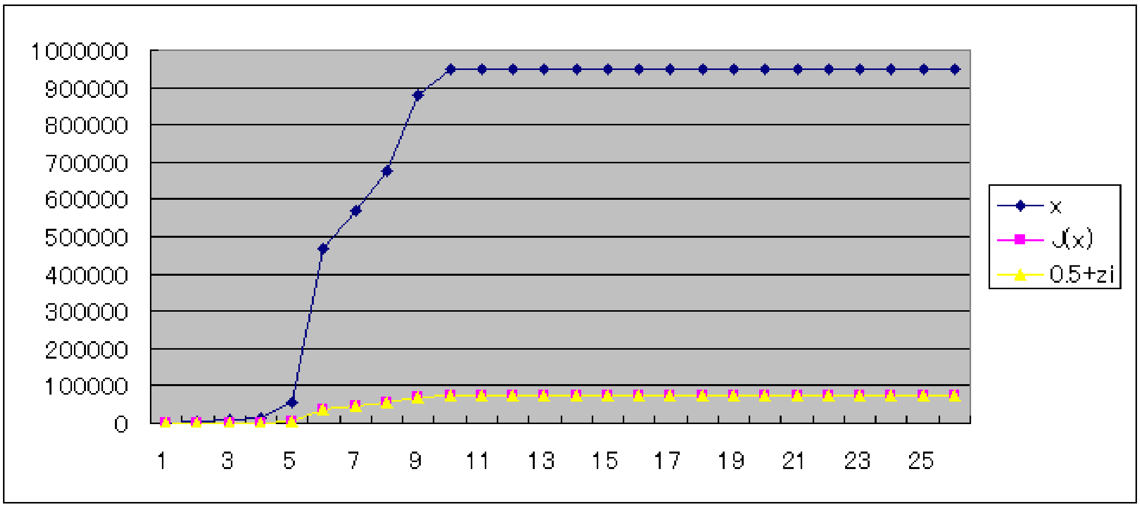
105→30.5333→30.42810832

110→32.5333→32.93879514

137→37.5095→37.58932556

153→40.5095→40.92105478

948881→74920.11→74920.83



Therefore, as long as you continue to count prime numbers, you can understand J(x) and Zeros of Riemann's zeta function. Of course, these are approximate values.

Secondly, I found the pattern of prime numbers.

ρ must be optional odd numbers.Ш is apparent prime numbers such as 2,3,5. In this case, you can ignore 2 because only odd numbers must be the target to expand prime numbers.

Ш＝ {3,5,7,11,13,17,‥・,X,‥∞・・}

Ш1=3, Ш2=5, Ш3=7, Ш4=11, ・・Шn=X・・

If ρ is divided by Ш and you can get the integral answer, it is Шn^2.

Шn^2+2Шn,

Шn^2+2Шn+2Шn,

Шn^2+2Шn+2Шn+2Шn,

Шn^2+2Шn+2Шn+2Шn+2Шn,

Шn^2+2Шn+2Шn+2Шn+2Шn+2Шn,

Шn^2+2Шn+2Шn+2Шn+2Шn+2Шn+2Шn,・・・is not prime numbers.

Therefore," Шn^2+2(д-1)Шn {д＞1} " must not be prime numbers.

Moreover, " $Шn^2 + \sum 2Шn$ " isn't also prime numbers.

For example, you can find prime numbers up to 100 by following this theory.

《３、５、７、９、１１、１３，１５、１７、１９、２１、２３、２５、２７、２９、３１、３３、３５、３７、３９、４１、４３、４５、４７、４９、５１、５３、５５、５７、５９、６１、６３、６５、６７、６９、７１、７３、７５、７７、７９、８１、８３、８５、８７、８９、９１、９３、９５、９７、９９》are the odd numbers.

3,5,7 must be the apparent prime numbers because of $\sqrt{100}$=10.

Ш1=3,Ш2=5,Ш3=7

9 is divided by Ш1=3, so ρ=9 and 9+2(д−1)Ш1 is working.

д＞1

Therefore

９＋２×３＝１５、９＋４×３＝２１、９＋６×３＝２７、９＋８×３＝３３、

９＋１０×３＝３９、９＋１２×３＝４５、９＋１４×３＝５１、９＋１６×３＝５７、

９＋１８×３＝６３、９＋２０×３＝６９、９＋２２×３＝７５、９＋２４×３＝８１、

９＋２６×３＝８７、９＋２８×３＝９３、９＋３０×３＝９９

１５、２１、２７、３３、３９、４５、５１、５７、６３、６９、７５、８１、８７、９３、９９ are not prime numbers.

The next is Ш2=5 and 25 is divided by this number.

ρ=25 and ２５＋２（д－１）Ш2 is working.

Therefore

２５＋２×５＝３５、２５＋４×５＝４５、２５＋６×５＝５５、２５＋８×５＝６５、２５＋１０×５＝７５、２５＋１２×５＝８５、２５＋１４×５＝９５

３５、４５、５５、６５、７５、８５、９５ are not prime numbers.

The next is Ш3=7 and 49 is divided by this number.

ρ=49 and ４９＋２（д－１）Ш3 is working.

Therefore

４９＋２×７＝６３、４９＋４×７＝７７、４９＋６×７＝９１

６３、７７、９１ are not prime numbers.

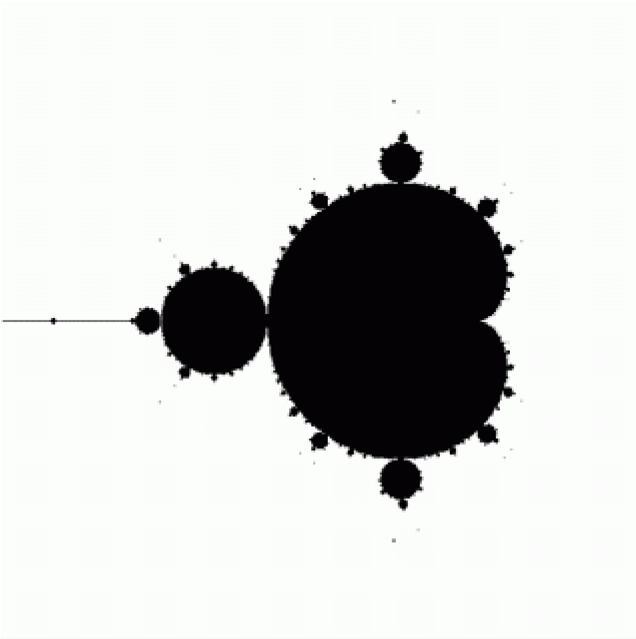
Then you can find prime numbers by 100.

３、５、７、１１、１３、１７、１９、２３、２９、３１、３７、４１、４３、４７、５３、５９、６１、６７、７１、７３、７９、８３、８９、９７

Finally, $Ш\mathrm{n}+1=Ш\mathrm{n}^2+\mathrm{C}$ is fractal dimensions.

I have found the pattern which is $Ш\mathbf{n}^2+\sum 2Ш\mathbf{n}$. You can see Π(x) through this pattern. This is also same with sieve of Eratosthenes. For example, x=(3,5,7,9,11) y=(3,5,7,9,11). z=x*y and z=x+Σ2x or z=y+Σ2y. 9=3*3 and 9=3+2*3,15=3*5 and 15=3+4*3=3^2+2*3 or 15=5*3 and 15=5+2*5, 21=3*7 and 21=3^2+4*3 or 21=7*3 and 21=7+2*7・・・・. z is not prime numbers, so x'=(3,5,7,11) y'=(3,5,7,11). x' and y' are prime numbers. Therefore, you can see Π(x) through $Ш\mathrm{n}^2+\sum 2Ш\mathrm{n}$ which is related to J(x) and zeros of Riemann's zeta function. It includes imaginary numbers.

Then, you can see $Ш\mathrm{n}+1=Ш\mathrm{n}^2+\mathrm{C}$.



This figure is well known as fractal dimensions.

$Z_{n+1} = Z_n^2 + C$

$Z_0=0$

$Z_1 = Z_0^2 + C = C$

$Z_2 = Z_1^2 + C = C^2 + C$

$Z_3 = Z_2^2 + C = (C^2 + C)^2 + C = C^4 + 2C^3 + C^2 + C$

$Z_4 = Z_3^2 + C = ((C^2 + C)^2 + C)^2 + C = C^8 + 4C^7 + 6C^6 + 6C^5 + 5C^4 + 2C^3 + C^2 + C$

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When C is 0, Zn=0.

When C is 1, Zn=0,1,2,5,26,・・・・∞.

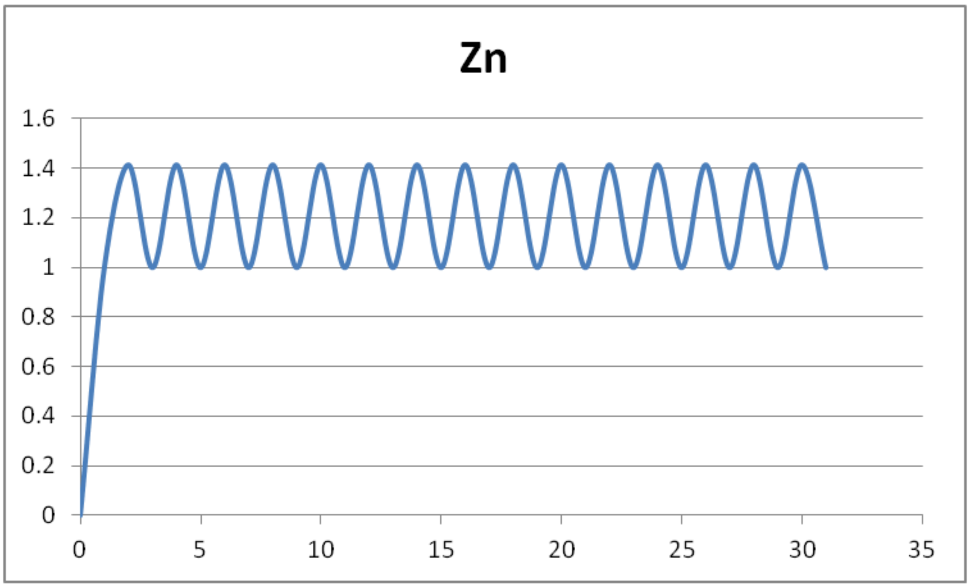
When C is i (i is an imaginary number, $i^2=-1$), Zn=0, i, −1+i, −i, −1+i, −i, −1+i, −i, ・・・.

You can ignore ∞ which is expanding the integer such as C=1,2,3・・.

You just care Zn=0, i, −1+i, −i, −1+i, −i, −1+i, −i, ・・・.

It is repeating of −1+i and –i, because $Z_3=(-1+i)^2+i=-i$, $Z_4=(-i)^2+i=-1+i$,

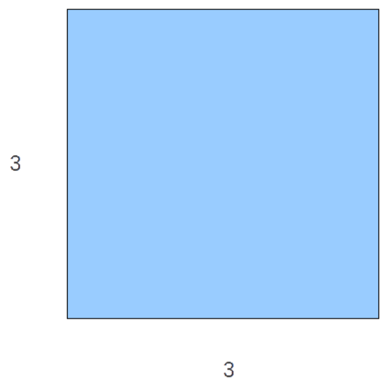
$Z_5=(-1+i)^2+i=-i$, $Z_6=(-i)^2+i=-1+i$, $Z_7=(-1+i)^2+i=-i$



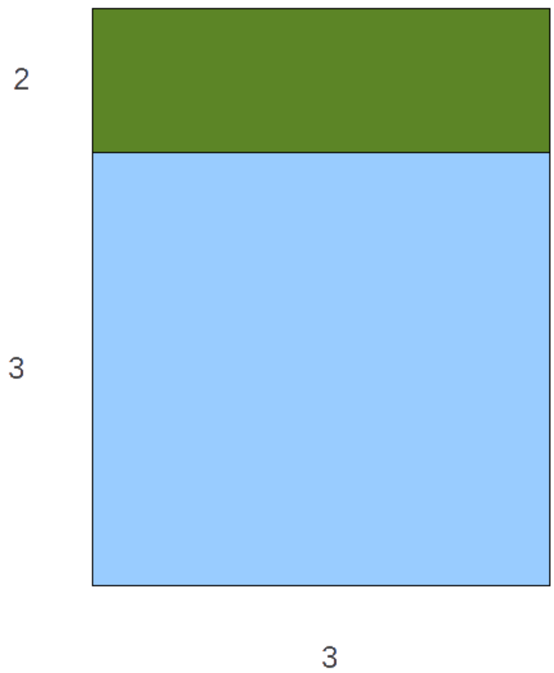
It would be the beginning and the ending.

As long as we see J(x), $Шn+1 = Шn^2 + C$ must be fractal dimensions. There is no ending. It is just beginning.

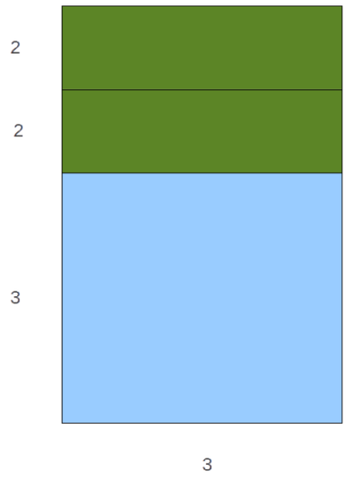
Prime numbers are also expressed as squares like ancient Babylonian. I found the pattern of prime numbers which were based on fractal. Any prime numbers are squared to find none prime numbers.

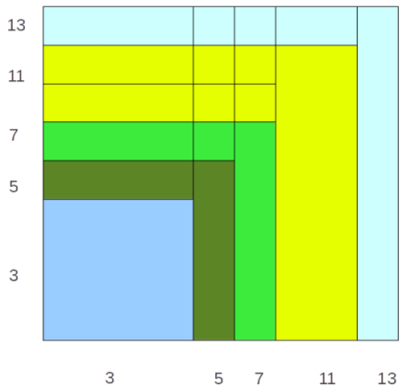


This is Ш$\mathbf{n}^2$.



Then you pile up 2Шn.





You can expand this square infinitely like J(x) and Zeros of Riemann’ s zeta function.

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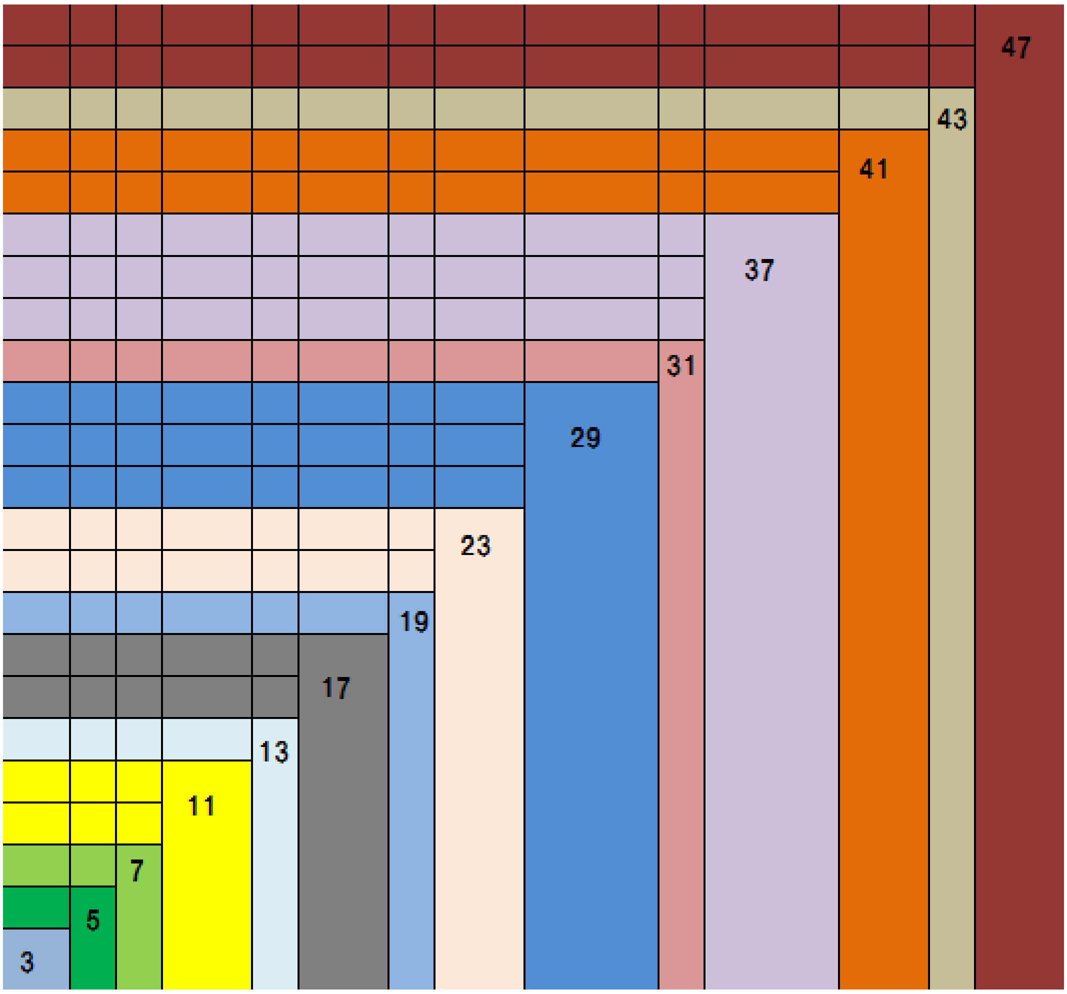

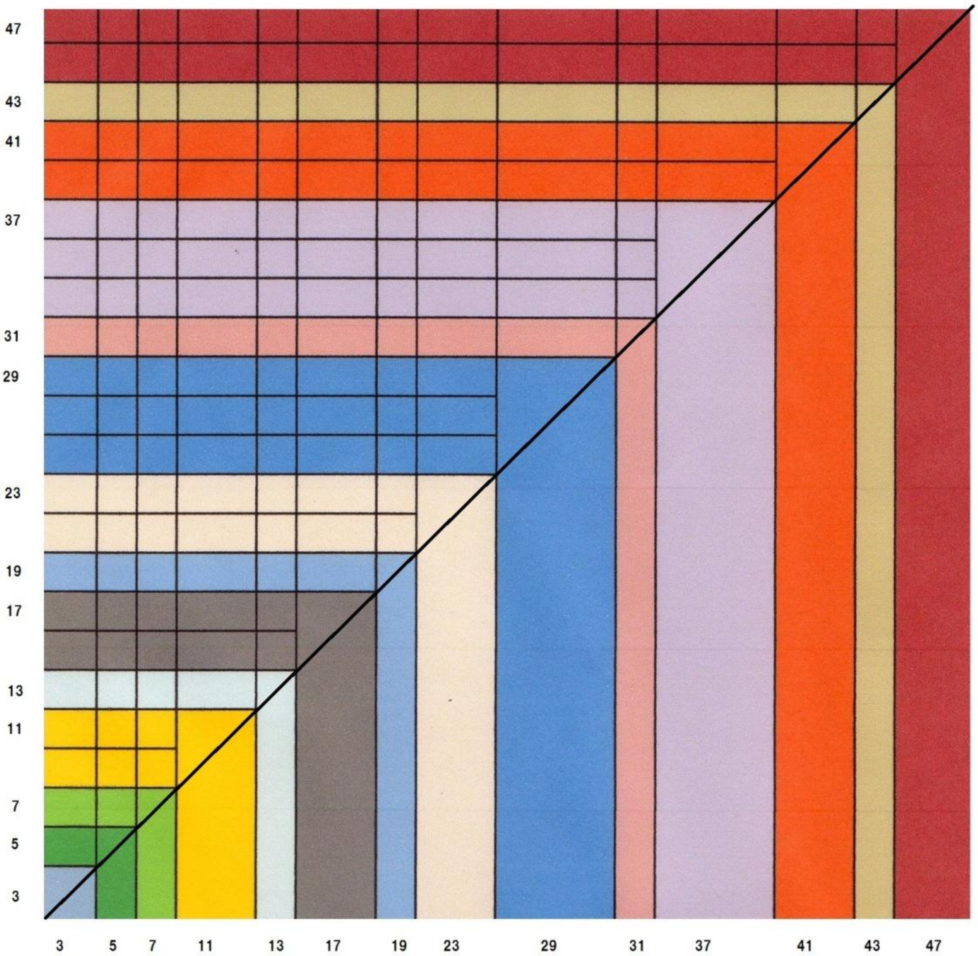

You can divide non prime numbers into prime numbers in 50%.

In conclusion, it is said that Zeros of Riemann's zeta function is almost correct but it is too hard to prove it. I don't have any new ideas to solve it. However, according to fractal dimensions, it is very difficult to change the pattern. I think that it is impossible. Therefore, as long as we can't deny Zeros of Riemann's zeta function, 90% would be almost 100%. Unfortunately, it is just my expectation.

References

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