數理統計學自主學習週作業

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以下程式碼皆使用 R 語言來撰寫數理統計學教科書中有關機率分配的三題習題，除了 4.58(f)較為複雜在 Workspac 上執行之外，其他都是在 console 中直接執行，執行結果即在程式碼下方

4.58 題使用 x 所對應的累積機率：pnorm(x, mean=0, sd=1)程式指令來求常態分配某個區間的機率值，而其指令的括號內皆只寫出 x 的值，沒有寫出 mean 和 sd，因為此題在問的是標準常態分配，可以忽略 mean 和 sd 值的標示，另外在 4.58(f)中有使用到程式指令 seq(from=, to=, by=)來成立 x 軸以及 polygon()函數來繪出形狀

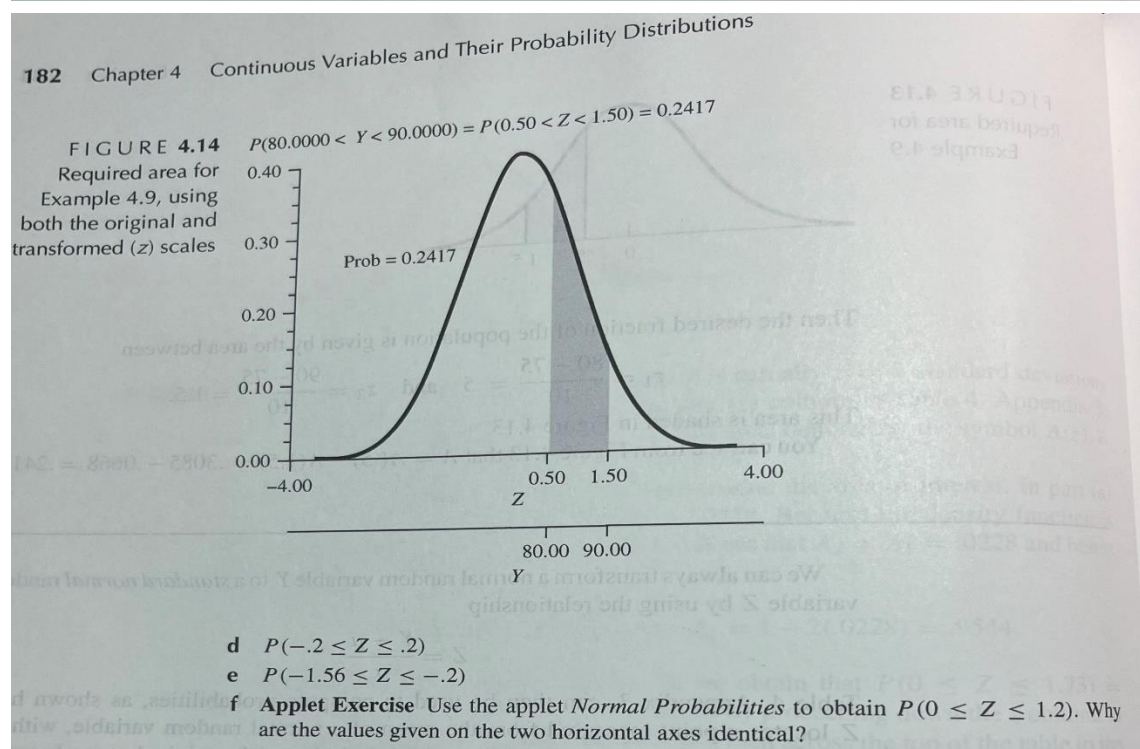## 4.58 題目

4.58 Use Table 4, Appendix 3, to find the following probabilities for a standard normal random variable Z:

  a $P(0 \le Z \le 1.2)$
  b $P(-.9 \le Z \le 0)$
  c $P(.3 \le Z \le 1.56)$

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FIGURE 4.14
Required area for Example 4.9, using both the original and transformed (z) scales

$P(80.0000 < Y < 90.0000) = P(0.50 < Z < 1.50) = 0.2417$

Prob = 0.2417

  d $P(-.2 \le Z \le .2)$
  e $P(-1.56 \le Z \le -.2)$
  f **Applet Exercise** Use the applet *Normal Probabilities* to obtain $P(0 \le Z \le 1.2)$. Why are the values given on the two horizontal axes identical?

以下是 4.58 題的解題程式碼

4.58(a)

```
> a = pnorm(1.2) - pnorm(0)
> print(a)
[1] 0.3849303
```

4.58(b)

```
> b = pnorm(0) - pnorm(-0.9)
> print(b)
[1] 0.3159399
```

4.58(c)

```
> c = pnorm(1.56) - pnorm(0.3)
> print(c)
[1] 0.3227086
```

4.58(d)

```
> d = pnorm(0.2) - pnorm(-0.2)
> print(d)
[1] 0.1585194
```
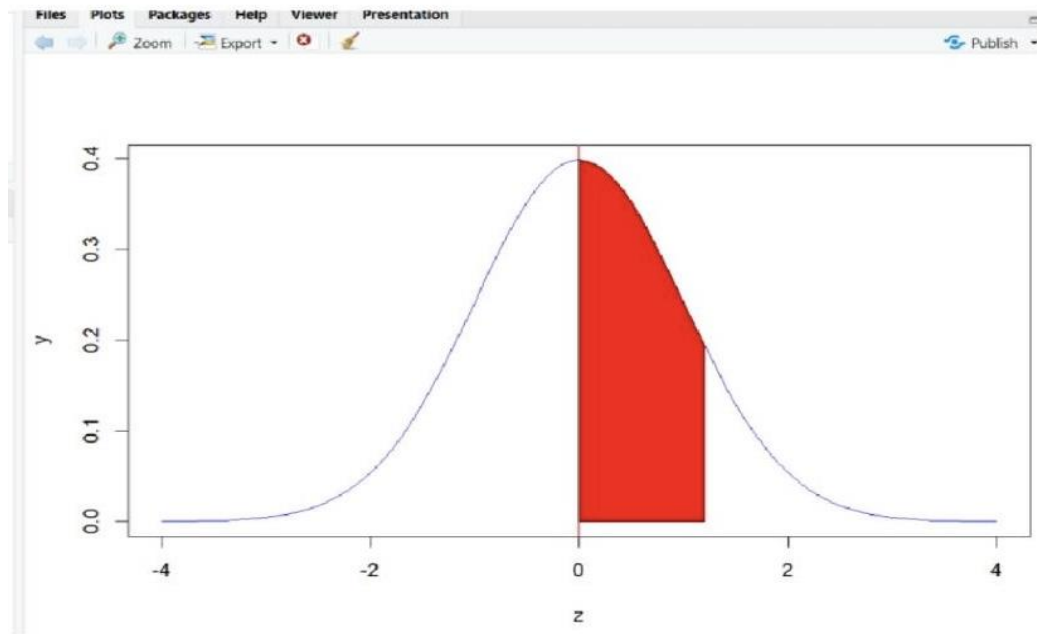
4.58(e)

```
> e = pnorm(-0.2) - pnorm(-1.56)
> print(e)
[1] 0.3613603
```

4.58(f)

```
z = seq(from=0,to=1.2,by=0.1)
y=dnorm(z)
polygon(c(0,z,1.2),c(0,y,0),col("red"))
pnorm(1.2)-pnorm(0)
```

執行結果如下

4.86 題　使用 pgamma() 程式指令來求 Gamma 分配 0 到某個數的機率值

4.86 題目

4.86 Applet Exercise When we discussed the χ^2 distribution in this section, we presented (with justification to follow in Chapter 6) the fact that if Y is gamma distributed with $\alpha = n/2$ for some integer n, then $2Y/\beta$ has a χ^2 distribution. In particular, it was stated that when $\alpha = 1.5$ and $\beta = 4$, $W = Y/2$ has a χ^2 distribution with 3 degrees of freedom.

a　Use the applet Gamma Probabilities and Quantiles to find $P(Y < 3.5)$.

b　Use the applet Gamma Probabilities and Quantiles to find $P(W < 1.75)$. [Hint: Recall that the χ^2 distribution with ν degrees of freedom is just a gamma distribution with $\alpha = \nu/2$ and $\beta = 2$.]

c　Compare your answers to parts (a) and (b).

以下是 4.86 解題程式碼

4.86(a)

```
# a = p(Y<3.5)=pgamma(3.5,1.5,1/4)
> a = pgamma(3.5,1.5,1/4)
> print(a)
[1] 0.3741245
```

4.86(b)

#P(Y<3.5)=P(W=2Y/4<3.5*2/4)=P(W<1.75)，而又因為 2Y/β 服從卡方分配，所以套用其分配 α=ν/2 和 β=2 的性質

```
# b = p(W<1.75)
> b = pgamma(1.75,1.5,1/2)
> print(b)
[1] 0.3741245
```

4.86(c)

The answer of a and b are the same

4.123 題套用了 R 語言中的 Beta 函數程式指令 beta(α, β) 以及機率 p 所對應的分位數：qbeta(p, shape1, shape2)

4.123 題目

4.123 The relative humidity Y, when measured at a location, has a probability density function given by

$$f(y) = \begin{cases} ky^3(1-y)^2, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a　Find the value of k that makes $f(y)$ a density function.

b　**Applet Exercise** Use the applet *Beta Probabilities and Quantiles* to find a humidity value that is exceeded only 5% of the time.

以下是 4.123 的解題程式碼
4.123(a)

#$\int_0^1 f(y)\, dy = k \int_0^1 y^3(1-y)^2\, dy = k\, B(4,3) = 1$

#k=$\dfrac{1}{B(4,3)}$

```
> k=1/beta(4,3)
> print(k)
[1] 60
```

4.123(b)

```
> b=qbeta(0.95,4,3)
> print(b)
[1] 0.8468389
```