Algebraic Methods in Combinatorics

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Assignment 2

To be completed by October 2

The solution of each problem should be no longer than one page!

Problem 1. Let p be prime. A set $A \subseteq \mathbb{F}_p^n$ is called a *Nikodym* set if for each point $x \in \mathbb{F}_p^n$, there is a line $L(x)$ that contains x and satisfies $L(x) \setminus \{x\} \subseteq A$. Show that any Nikodym set in $\mathbb{F}_p^n$ has size at least $\binom{p-2+n}{n}$.

Problem 2. Let S be an s-distance set in $\mathbb{R}^n$, i.e. there are at most s possible distances between distinct points in S.

(a) Show that $|S| \leq \binom{n+s+1}{s}$.

(b) Show that there exists an s-distance set in $\mathbb{R}^n$ of size at least $\binom{n+1}{s}$.

Problem 3. Let $\mathcal{A}$ be a family of subsets of $[n]$, such that the *symmetric differences* $A \triangle B$ (where $A \triangle B = (A \setminus B) \cup (B \setminus A)$) for distinct $A, B \in \mathcal{A}$, have only two possible sizes.

(a) Show that $|\mathcal{A}| \leq 1 + \frac{n(n+1)}{2}$.

(b) Find such a family $\mathcal{A}$ of size at least $1 + \frac{n(n-1)}{2}$.