Algebraic Methods in Combinatorics

Instructor: Benny Sudakov

Assignment 6

To be completed by October 30th

The solution of each problem should be no longer than one page!

Problem 1. Let $\mathcal{F}$ be a family of distinct proper subsets of $\{1, 2, \ldots, n\}$. Suppose that for every $1 \leq i \neq j \leq n$ there is a unique member of $\mathcal{F}$ that contains both i and j. Prove that $|\mathcal{F}| \geq n$.

Problem 2. Let G be a complete graph with vertex set V, where $|V| = n$.

(a) Let $B_1, \ldots, B_m$ be subgraphs of G that are complete bipartite graphs, and suppose that every edge of G belongs to an odd number of B_i's. Prove that $m \geq (n-1)/2$.

(b)* Let $B_1, \ldots, B_m$ be subgraphs of G that are complete bipartite graphs, and suppose that every edge of G belongs to exactly one of the B_i's. Prove that $m \geq n - 1$.

[Hint: .A ot gnidnopserroc mrof citardauq eht redisnoC .xirtam 1-lla eht si J erehw
,$I - J = T^A + A$ htiw A xirtam gnidnopserroc eht redisnoC]

Problem 3. Let $A_1, \ldots, A_m$ be subsets of $[n]$ such that $|A_i \cap A_j|$ is divisible by 6 for every distinct $1 \leq i, j \leq m$, and $|A_i|$ is not divisible by 6 for every $1 \leq i \leq m$. Show that $m \leq 2n$.

Problem 4. Let $\mathcal{A}$ be a finite family of at least $r+1$ sets of size r, such that any $r+1$ sets in $\mathcal{A}$ intersect. Show that all the sets in $\mathcal{A}$ intersect.