Algebraic Methods in Combinatorics

Instructor: Benny Sudakov

Assignment 10

To be completed by November 27th, 20:00

Problem 1. Let $L(G)$ denote the line graph of G.

(a) Show that every eigenvalue λ of $L(G)$ satisfies $\lambda \geq -2$.

(b) Show that if G has more edges than vertices, then -2 is an eigenvalue of $L(G)$.

(c) For each odd $n \geq 3$, give an example of a graph with n vertices and n edges for which all eigenvalues of the line graph are greater than -2.

Problem 2. Let A be the adjacency matrix of a graph G.

(a) Suppose that the eigenvectors of A are also eigenvectors of J (the all 1 matrix). Show that there is a polynomial f such that $f(A) = J$.

(b) Deduce that there is a polynomial f such that $f(A) = J$ if and only if G is regular and connected.

Problem 3. Let $\Gamma = (\mathbb{Z}/2\mathbb{Z})^n$ and let $S = \{e_1, \ldots, e_n\} \subseteq \Gamma$ be the set of standard basis vectors. Recall that $G = G(\Gamma, S)$ is the hypercube graph. Find the eigenvalues of G.