# Graph Theory

## Instructor: Benny Sudakov

# Assignment 1

This exercise sheet contains problems relating to Section 1 of the lecture notes

**To be completed by February 29th 23:59**

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

**Problem 1:** Given a graph $G$ with vertex set $V = \{v_1, \ldots, v_n\}$ we define the *degree sequence* of $G$ to be the list $d(v_1), \ldots, d(v_n)$ of degrees in decreasing order. For each of the following lists, give an example of a graph with such a degree sequence or prove that no such graph exists:

(a) $3, 3, 2, 2, 2, 1$

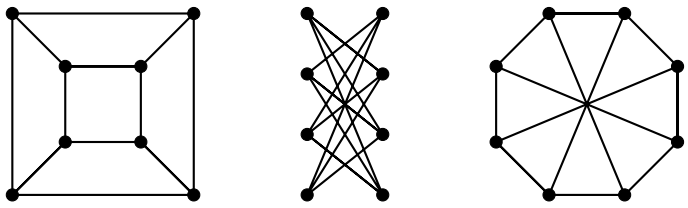
(b) $6, 6, 6, 4, 4, 2, 2$

(c) $6, 6, 6, 6, 5, 4, 2, 1$

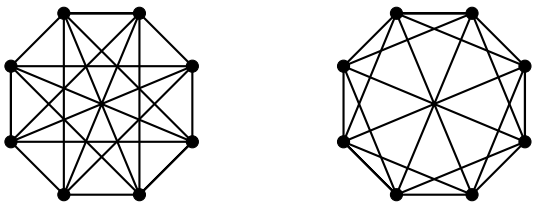
(d) $6, 6, 6, 4, 4, 3, 3$

**Problem 2:**

(a) Which of the following graphs are isomorphic? Why?



(b) Are the following graphs isomorphic?



**Problem 3:** Prove that if a graph $G$ is not connected then its complement $\overline{G}$ is connected. Is the converse also true?

**Problem 4:** Show that every graph on at least two vertices contains two vertices of equal degree.

**Problem 5:** Prove that every graph with $n \geq 7$ vertices and at least $5n - 14$ edges contains a subgraph with minimum degree at least 6.

**Problem 6:** Show that in a connected graph any two paths of maximum length share at least one vertex.

**Problem 7:** Prove that a graph is bipartite iff (if and only if) it contains no cycle of odd length.