Algebraic Methods in Combinatorics

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Assignment 8

To be completed by November 13, 14:00

Problem 1. Let G be a connected graph with eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$ and let $v = (v_1, \ldots, v_n)^T$ be an eigenvector of G with eigenvalue λ_1. Prove the following statements.

(a) v_i is non-zero for every i.

(b) All the v_i's have the same sign.

(c) $\lambda_1 > \lambda_2$.

(d) Let H be a subgraph of G (which is not G itself). Then $\lambda_1(H) < \lambda_1(G)$.

(e) If G is d-regular, then $v_1 = \cdots = v_n$.

Problem 2. Let G be a graph with maximum degree d. Show that:

(a) $\lambda_1(G) \leq d$.

(b) If G is d-regular then $\lambda_1 = d$.

(c) If G is connected and not d-regular, then $\lambda_1 < d$.

Problem 3. Let G be a bipartite graph that does not have a *perfect matching*, i.e. a set of vertex-disjoint edges that cover all the vertices. Show that 0 is an eigenvalue of G.