Problem 1 Use a reduction to show that the language ALL_{TM} is undecidable

$$ALL_{TM} = \{\langle M \rangle \mid \text{where } M \text{ is a TM and } L(M) = \Sigma*\}$$

Problem 2 A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing Machine has any useless states. Formulate this problem as a language and show that it is undecidable.

Problem 3 If $A \leq_m B$ and B is a regular language, does this imply that A is a regular language? Why or why not?

Problem 4 Prove that the language

$$LOOP_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ loops on all inputs}\}$$

is not recognizable.

Problem 5 Prove that the 3-SAT problem discussed in class is an element of NP by giving a verifier and a NTM decider that run in poly-time.