FC715E
Engineering

Theme: 5 Hour: 1   Kinematics 1

The lecture notes included in this presentation have been adapted from the resources accompanying the textbook in the recommended reading list:
“Physics” by James S. Walker, Pearson 2010.

Theme 5: Kinematics 1

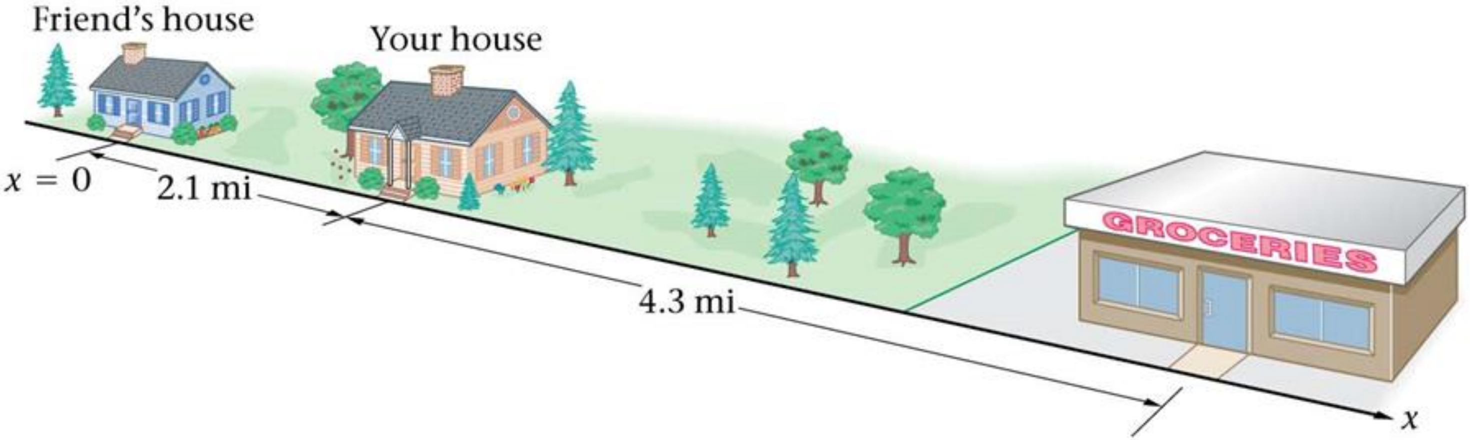
5.1 Distance and displacement

5.2 Average speed and velocity

5.3 Motion with constant acceleration

5.4 Use of graphs in kinematics

Distance vs displacement



Must define coordinate system:

•Origin is friend's house

•Positive direction is left to right

Worked example

QUICK EXAMPLE 2-1 DISPLACEMENT AND DISTANCE

Predict/Calculate

Referring to Figure 2-2, suppose you take a trip from your friend's house to the grocery store and then to your house. **(a)** Is the displacement for this trip positive, negative, or zero? Explain. **(b)** Find the displacement for this trip. **(c)** What is the distance covered in this trip?

REASONING AND SOLUTION

Displacement is the final position minus the initial position; it can be positive or negative. Distance is the length of travel, which is always positive.

1. **Part (a)** The displacement is positive because the final position is to the right (positive direction) of the initial position.
2. **Part (b)** Determine the initial position for the trip, using Figure 2-2: $x_i = 0$
3. Determine the final position for the trip, using Figure 2-2: $x_f = 2.1\ \text{mi}$
4. Subtract x_i from x_f to find the displacement. Notice that the result is positive, as expected: $\Delta x = x_f - x_i = 2.1\ \text{mi} - 0 = 2.1\ \text{mi}$
5. **Part (c)** Add the distances for the various parts of the trip: $2.1\ \text{mi} + 4.3\ \text{mi} + 4.3\ \text{mi} = 10.7\ \text{mi}$

How to calculate speed

General formula:

$$V = \frac{S}{t}$$

Where *S* is the distance covered and *t* is the time interval.

But there is more to it than that!

Average vs instantaneous speed

- Average speed
  - The *total* distance travelled divided by the *total* time taken
  - At any moment in time the actual speed will *not* necessary be equal to the average speed
- Instantaneous speed
  - Actual speed occurring at one instant in time
  - The speed shown on a speedometer (in a car, for example)

  Mathematically:

$$V_{inst} = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t}$$

Units of speed

•SI unit is m/s

•We sometimes use km/hr (kilometres per hour) or mph (miles per hour), depending on where we live

•To convert from m/s to km/hr just multiply by a factor of 3.6 (for the opposite, divide by 3.6)

•To (roughly) convert from m/s to mph just multiply by a factor of 2.24 (for the opposite, divide by 2.24)

Usain Bolt world record

In 2009 in Berlin he ran the 100 metre dash in 9.58 seconds!

Image credit: "100 m final Berlin 2009" by , attribution: Erik van Leeuwen (bron: Wikipedia). - http://www.erki.nl/. Licensed under GFDL via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:100_m_final_Berlin_2009.JPG#mediaviewer/File:100_m_final_Berlin_2009.JPG

Usain Bolt world record (cont.)

How fast did he actually run?

•His *reaction time* (time between hearing the pistol and starting) was 0.146 s

•By subtracting this from 9.58 we get a run time of 9.434 s

•So his *average* speed was:

total distance/total run time, i.e. 100/9.434 = 10.6 m/s (= 38.15 km/hr)

•This does not tell us anything about the variation of his performance throughout the race

Usain Bolt’s instantaneous speed

- The *split time* is the time taken to complete each 10 metre segment of the race

- This gives a better approximation of the *instantaneous* speed (although still an approximation)

Usain Bolt split times

Position (m)	Time (s)	Speed (m/s)
10	1.89	5.73
20	2.88	10.10
30	3.78	11.11
40	4.64	11.63
50	5.47	12.05
60	6.29	12.20
70	7.10	12.35
80	7.92	12.20
90	8.75	12.05
100	9.58	12.05

These are the split times of Usain Bolt's world record obtained from http://berlin.iaaf.org

Usain Bolt instantaneous speed

How do we derive the speed for each split?

•We divide 10 m by the time taken

•For example for last segment we have,
10/(9.58-8.75) = 10/0.83 = 12.05 m/s

•We can deduce that the maximum split speed is 12.35 m/s, just over 44.46 km/hr

•In actual fact these are the average speeds for each split but they do provide useful performance indications

Scalar and vector quantities

•Speed does not tell us anything about Bolt's direction

•We can draw an arrow to point in the direction of motion, also, the larger the speed, the larger the arrow: we then have the *velocity* of Bolt

•The velocity arrow gives us information both on the value of the speed (i.e. how fast he is going) *and* on his direction

•Velocity needs both the magnitude (numerical value) and the direction, it is therefore a *vector* quantity

•Speed only needs the magnitude, it is therefore a *scalar* quantity

Velocity vs speed

$$V = \frac{\Delta x}{\Delta t}$$

Where, Δx is the displacement and Δt is the elapsed time
Note that velocity can have a positive or negative sign denoting direction!

Changing direction

Assume that Usain Bolt is running the 200 m event at a constant speed. Is his velocity constant or not ?

- His speed may be constant but his direction is changing
- The direction of the velocity arrow is therefore changing
- So, his velocity is NOT constant!

Acceleration

•Whenever we have a change in velocity (magnitude or direction) we will have an *acceleration*

•Acceleration can be defined as how fast the velocity is changing over time

How do we calculate acceleration ?

We use the formula,

$$a = \frac{v - v_0}{t}$$

Where v and v_0 are the final and initial speeds and t is the time interval.

Worked example

What is the acceleration of Usain Bolt if it takes him 2 seconds to reach a speed of 10 m/s (assume his initial speed is zero) ?

Solution

$$a = \frac{v - v_0}{t} = \frac{10 - 0}{2} = 5 \text{m/s}^2$$

- SI units are m/s^2
- Sometimes it can be measured in units of acceleration of gravity (*g*)
- The acceleration of gravity is (roughly) 9.81 m/s^2
- This is how fast a body would accelerate in free fall (ignoring air resistance)

Motion with constant acceleration

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Can use above equations only when acceleration is constant!
Usually simplified, as $x_0 = 0$.

Worked example

A plane must attain a takeoff speed before reaching the end of a runway. Find the minimum runway length if the acceleration of the plane is 2.2 m/s^2 and the takeoff speed is 95 m/s. Find also the time taken for takeoff.

Solution

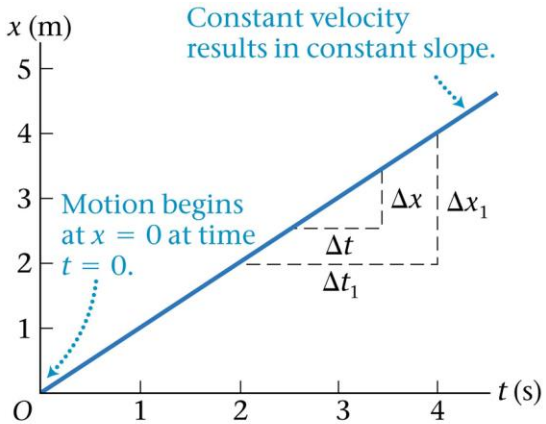
Choose positive direction from left to right!

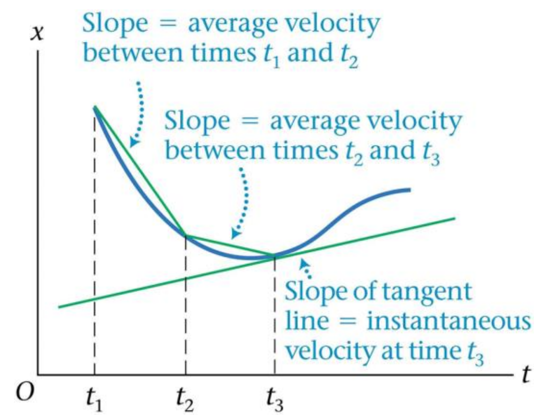
$$\Delta x = \frac{v^2 - v_0^2}{2a} \Rightarrow \Delta x = \frac{95^2 - 0^2}{2 \cdot (2.2)} \Rightarrow \Delta x = 2050m$$

$$\Delta x = v_0 t + \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2\Delta x}{a}} \Rightarrow t = \sqrt{\frac{2 \cdot 2050}{2.2}} \Rightarrow t = 43.17s$$

Use of graphs in kinematics

Position vs time graphs:





For constant velocity (linear change in distance), the slope of the graph gives velocity.

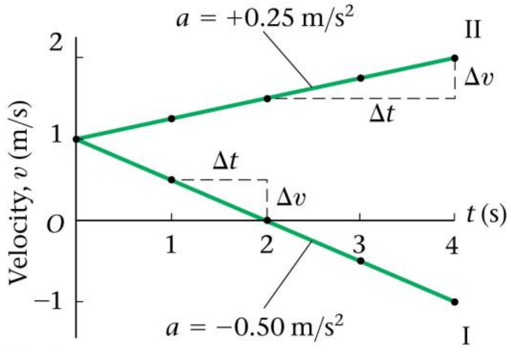
For varying velocity, slope of tangent gives the instantaneous velocity, slope of line between two points gives average velocity.

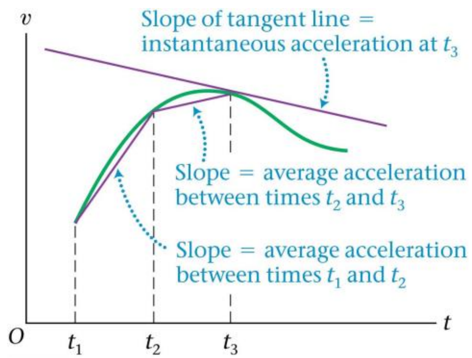
Check out a relevant animation here!

Images in this and next two slides, from: Walker J. S., Physics Technology Update, 4^{th} edition, Pearson 2014

Use of graphs in kinematics (cont.)

Velocity vs time graphs:





For constant acceleration (linear change in velocity), the slope of the graph gives acceleration.

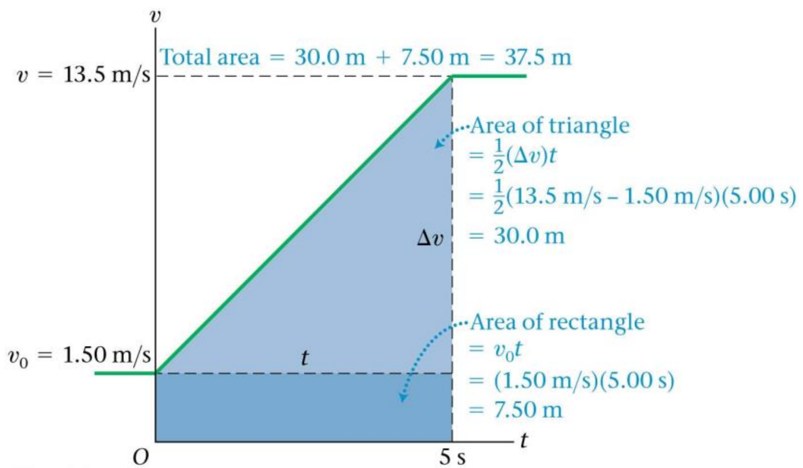
For varying acceleration, slope of tangent gives the instantaneous acceleration, slope of line between two time points gives the average one.

Solving problems with varying acceleration requires more advanced calculus!

Images in this and next two slides, from: Walker J. S., Physics Technology Update, 4th edition, Pearson 2014

Use of graphs in kinematics (cont.)

Velocity vs time graphs:



Area under the velocity curve gives distance travelled!
In this example, velocity increases linearly from 1.5 m/s to 13.5 m/s.
Total distance travelled is calculated at 37.5 m.

Images in this and next two slides, from: Walker J. S., Physics Technology Update, 4th edition, Pearson 2014

Use of graphs in kinematics (cont.)

Important points:

- If it is *distance* you are calculating any area below the x-axis in the Velocity vs Time graph is ADDED (i.e. everything is taken as positive)
- If it is *displacement* you are calculating then any area below the x-axis is SUBTRACTED.

Also,

If you are calculating average *speed*, then you use Distance/Time.
If you are calculating average *velocity* then you use Displacement/Time.

Images in this and next two slides, from: Walker J. S., Physics Technology Update, 4th edition, Pearson 2014