

Vraag 1/ Question 1 (25)

1.1 Gebruik die τ - notasie om die looptyd van die volgende programlyne te bepaal. (15)	1.1 Use τ - notation to determine <i>the running time</i> of the following program lines (15)
<pre> 1. for (int i=0; i<=(n+1); i++) { 2. b=arr[i]+1; }</pre>	
<p>1a. $t_{\text{fetch}} + t_{\text{store}}$ ✓✓</p> <p>1b. $(3t_{\text{fetch}} + t_{<} + t_{+}) \checkmark\checkmark (n+3) \checkmark\checkmark = 3t_{\text{fetch}} n + t_{<} n + t_{+} n + 9t_{\text{fetch}} + 3t_{<} + 3t_{+}$</p> <p>1c. $(2t_{\text{fetch}} + t_{+} + t_{\text{store}}) \checkmark\checkmark (n+2) \checkmark\checkmark = 2t_{\text{fetch}} n + t_{+} n + t_{\text{store}} n + 4t_{\text{fetch}} + 2t_{+} + 2t_{\text{store}}$</p> <p>2. $(4t_{\text{fetch}} + t_{+} + t_{\text{store}} + t_{[.]}) \checkmark\checkmark (n+2) \checkmark = 4t_{\text{fetch}} n + t_{+} n + t_{\text{store}} n + t_{[.]} n + 8t_{\text{fetch}} + 2t_{+} + 2t_{\text{store}} + 2t_{[.]}$</p> <p>Total: $9t_{\text{fetch}} n + 3t_{+} n + 2t_{\text{store}} n + t_{[.]} n + t_{<} n + 22t_{\text{fetch}} + 7t_{+} + 5t_{\text{store}} + 2t_{[.]} + 3t_{<} \checkmark\checkmark$</p>	
1.2 Bepaal die looptyd van al drie dele van lyn 8 in konteks van hierdie programdeel. Jy hoef nie die uitdrukkings te vereenvoudig nie. Maak gebruik van die vereenvoudigde model. (8)	1.2 . Determine the running time of <i>all three parts of line 8</i> in context of this program segment. You need <i>not</i> simplify the expressions. Use the simplified model. (8)
<pre> 1 public class Question1.2 2 { 3 public static int numbers (int n) 4 { 5 int prod = 1; 6 for (int i=1; i<n; i++) 7 { 8 for (int j=1; j<i; ++j) 9 prod *= (j+2)j; 10 } 11 return prod; 12 } 13 }</pre>	
<p>8a $2(n-1)$ ✓✓</p> <p>8b $3 \sum_{i=1}^{n-1} (i)$ ✓✓✓</p> <p>8c $4 \sum_{i=1}^{n-1} (i-1)$ ✓✓✓</p>	
1.3 Gee die bewys van die vergelyking (2)	1.3. Give proof for the following equation (2)
$\sum_{i=1}^n (i) = \frac{n(n+1)}{2}$ <p>$\sum_{i=1}^n (i) = 1 + 2 + 3 + \dots + (n-2) + (n-1) + n$ but the following is also true ✓</p> <p>$\sum_{i=1}^n (i) = n + (n-1) + (n-2) + \dots + 1 + 2 + 3$ if you add these lines together – you get=✓</p>	

$$\sum_{i=1}^n (i) = (n+1) + (n+1) + (n+1) + \dots + (n+1) + (n+1) + (n+1) \quad \checkmark$$

there are n pairs of (n+1) \checkmark

$$2 \sum_{i=1}^n (i) = n(n+1) \quad \text{- after dividing each side by 2: } \checkmark$$

$$\sum_{i=1}^n (i) = \frac{n(n+1)}{2} \quad \text{which proves the equation}$$

4 Gebruik die vergelyking van vraag 3 om die volgende uitdrukking te vereenvoudig: (7)

4. Use the equation in Question 3 to simplify the following expression: (7)

$$2 \sum_{i=0}^{n-2} (i+2)$$

$$2 \sum_{i=0}^{n-2} (i+2)$$

$$= 2 \left[\sum_{i=0}^{n-2} i + \sum_{i=0}^{n-2} 2 \right] \checkmark$$

$$= 2 \left[0 + \sum_{i=1}^n i - (n-1) - (n) + 2 \sum_{i=0}^{n-2} 1 \right] \checkmark \checkmark$$

$$= 2 \left[0 + \frac{n(n+1)}{2} - (n-1) - (n) + 2 \sum_{i=0}^{n-2} 1 \right] \checkmark \checkmark$$

$$= 2 \left[\frac{n(n+1)}{2} - 2n + 1 + 2(n-1) \right] \checkmark$$

$$= 2 \left[\frac{n(n+1)}{2} - 2n + 1 + 2n - 2 \right]$$

$$= 2 \left[\frac{n(n+1)}{2} - 1 \right]$$

$$= 2 \left[\frac{n(n+1) - 2}{2} \right]$$

$$= 2 \left[\frac{n^2 + n - 2}{2} \right]$$

$$= n^2 + n - 2 \checkmark$$