# gmp

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with extensive quoting from the GMP Manual

This manual describes how to use the gmp Ruby gem, which provides bindings to the GNU multiple precision arithmetic library, version 4.3.x or 5.0.x.

# Contents

# 1  Introduction to GNU MP

This entire page is copied verbatim from the GMP Manual.

GNU MP is a portable library written in C for arbitrary precision arithmetic on integers, rational numbers, and floating-point numbers. It aims to provide the fastest possible arithmetic for all applications that need higher precision than is directly supported by the basic C types.

Many applications use just a few hundred bits of precision; but some applications may need thousands or even millions of bits. GMP is designed to give good performance for both, by choosing algorithms based on the sizes of the operands, and by carefully keeping the overhead at a minimum.

The speed of GMP is achieved by using fullwords as the basic arithmetic type, by using sophisticated algorithms, by including carefully optimized assembly code for the most common inner loops for many different CPUs, and by a general emphasis on speed (as opposed to simplicity or elegance).

There is assembly code for these CPUs: ARM, DEC Alpha 21064, 21164, and 21264, AMD 29000, AMD K6, K6-2, Athlon, and Athlon64, Hitachi SuperH and SH-2, HPPA 1.0, 1.1, and 2.0, Intel Pentium, Pentium Pro/II/III, Pentium 4, generic x86, Intel IA-64, i960, Motorola MC68000, MC68020, MC88100, and MC88110, Motorola/IBM PowerPC 32 and 64, National NS32000, IBM POWER, MIPS R3000, R4000, SPARCv7, SuperSPARC, generic SPARCv8, UltraSPARC, DEC VAX, and Zilog Z8000. Some optimizations also for Cray vector systems, Clipper, IBM ROMP (RT), and Pyramid AP/XP.

For up-to-date information on GMP, please see the GMP web pages at
`http://gmplib.org/`

The latest version of the library is available at
`ftp://ftp.gnu.org/gnu/gmp/`

Many sites around the world mirror 'ftp.gnu.org', please use a mirror near you, see `http://www.gnu.org/ord`
for a full list.

There are three public mailing lists of interest. One for release announcements, one for general questions and discussions about usage of the GMP library, and one for bug reports. For more information, see

`http://gmplib.org/mailman/listinfo/`.

The proper place for bug reports is gmp-bugs@gmplib.org. See Chapter 4 [Reporting Bugs], page 28 for information about reporting bugs.

## 2   Introduction to the gmp gem

The gmp Ruby gem is a Ruby library that provides bindings to GMP. The gem is incomplete, and will likely only include a subset of the GMP functions. It is built as a C extension for ruby, interacting with gmp.h. The gmp gem is not endorsed or supported by GNU or the GMP team. The gmp gem also does not ship with GMP, so GMP must be compiled separately.

## 3   Installing the gmp gem

### 3.1   Prerequisites

OK. First, we've got a few requirements. To install the gmp gem, you need one of the following versions of Ruby:

- (MRI) Ruby 1.8.6 - tested lightly.

- (MRI) Ruby 1.8.7 - tested seriously.

- (MRI) Ruby 1.9.1 - tested seriously.

As you can see only Matz's Ruby Interpreter (MRI) is supported. I haven't even put a thought into trying other interpreters/VMs. I intend to look into FFI, which supposedly will allow me to load this extension into JRuby and Rubinius, not sure about others...

Next is the platform, the combination of the architecture (processor) and OS. As far as I can tell, if you can compile GMP and Ruby on a given platform, you can use the gmp gem there too. Please report problems with that hypothesis.

Lastly, GMP. GMP must be compiled and working. "and working" means you ran "make check" while installing GMP. The following versions of GMP have been tested:

- GMP 4.3.1

- GMP 4.3.2

- GMP 5.0.0

- GMP 5.0.1

That's all. I don't intend to test any older versions, maybe 4.3.0 for completeness.

Here is a table of the exact environments on which I have tested the gmp gem:

| Platform | Ruby | GMP |
|---|---|---|
| Cygwin on x86 | (MRI) Ruby 1.8.7 | GMP 4.3.1 |
| Linux (LinuxMint 7) on x86 | (MRI) Ruby 1.8.7 | GMP 4.3.1 |
| Mac OS X 10.5.7 on x86 (32-bit) | (MRI) Ruby 1.8.6 | GMP 4.3.1 |
| Mac OS X 10.5.7 on x86 (32-bit) | (MRI) Ruby 1.9.1 | GMP 4.3.1 |
| Windows XP on x86 (32-bit) | (MRI) Ruby 1.9.1 | GMP 5.0.1 |

### 3.2 Installing

You may clone the gmp gem's git repository with:

```
git clone git://github.com/srawlins/gmp.git
```

Or you may install the gem from gemcutter:

```
gem install gmp
```

Or you may install the gem from github (old):

```
gem install srawlins-gmp
```

At this time, the gem does not self-compile (how does that work?). To compile the C extensions, do the following:

```
cd <gmp gem directory>/ext
ruby extconf.rb
make
```

There shouldn't be any errors, or warnings.

## 4 Testing the gmp gem

Testing the gmp gem is quite simple. The test/unit_tests.rb suite uses Unit::Test. You can run this test suite with:

```
cd <gmp gem directory>/test
ruby unit_tests.rb
```

All tests should pass. If you don't have the test-unit gem installed, then you may run into one error. I'm not sure why this is... I suspect a bug in Ruby's Test::Unit that the test-unit gem monkey patches.

## 5 GMP and gmp gem basics

### 5.1 Classes

The gmp gem includes the namespace `GMP` and four classes within `GMP`:

- `GMP::Z` - Methods for signed integer arithmetic. There are at least 45 methods here (still accounting).

- `GMP::Q` - Methods for rational number arithmetic. There are at least 11 methods here (still accounting).

- `GMP::F` - Methods for floating-point arithmetic. There are at least 6 methods here (still accounting).

- `GMP::RandState` - Methods for random number generation. There are 3 methods here.

In addition to the above four classes, there is also one constant within `GMP`:

- `GMP::GMP_VERSION` - The version of GMP compiled into the gmp gem.

# 6 Integer Functions

## 6.1 Initializing, Assigning Integers

**new**

$$GMP::Z.new \rightarrow integer$$
$$GMP::Z.new(numeric = 0) \rightarrow integer$$
$$GMP::Z.new(str) \rightarrow integer$$

This method creates a new *GMP::Z* integer. It takes one optional argument for the value of the integer. This argument can be one of several classes. Here are some examples:

```
GMP::Z.new              #=> 0 (default)
GMP::Z.new(1)           #=> 1 (Ruby Fixnum)
GMP::Z.new("127")       #=> 127 (Ruby String)
GMP::Z.new(4294967296)  #=> 4294967296 (Ruby Bignum)
GMP::Z.new(GMP::Z.new(31)) #=> 31 (GMP Integer)
```

There is also a convenience method available, `GMP::Z()`.

## 6.2 Converting Integers

**to_d**

$$integer.to\_d \rightarrow float$$

Returns *integer* as an Float if *integer* fits in a Float.
Otherwise returns the least significant part of `integer`, with the same sign as *integer*.
If *integer* is too big to fit in a Float, the returned result is probably not very useful. To find out if the value will fit, use the function *mpz_fits_slong_p* (**Unimplemented**).

**to_i**

$$integer.to\_i \rightarrow fixnum$$

Returns *integer* as a Fixnum if *integer* fits in a Fixnum.

Otherwise returns the least significant part of *integer*, with the same sign as *integer*.

If *integer* is too big to fit in a Fixnum, the returned result is probably not very useful. To find out if the value will fit, use the function *mpz_fits_slong_p* (**Unimplemented**).

**to_s**                                                    *integer*.to_s(*base = 10*) → *str*

> Converts *integer* to a string of digits in base *base*. The *base* argument may vary
> from 2 to 62 or from -2 to -36, or be a symbol, one of *:bin, :oct, :dec*, or *:hex*.
>
> For *base* in the range 2..36, digits and lower-case letters are used; for -2..-36
> (and *:bin, :oct, :dec*, and *:hex*), digits and upper-case letters are used; for 37..62,
> digits, upper-case letters, and lower-case letters (in that significance order) are
> used. Here are some examples:
>
> ```
> GMP::Z(1).to_s         #=> "1"
> GMP::Z(32).to_s(2)     #=> "100000"
> GMP::Z(32).to_s(4)     #=> "200"
> GMP::Z(10).to_s(16)    #=> "a"
> GMP::Z(10).to_s(-16)   #=> "A"
> GMP::Z(255).to_s(:bin) #=> "11111111"
> GMP::Z(255).to_s(:oct) #=> "377"
> GMP::Z(255).to_s(:dec) #=> "255"
> GMP::Z(255).to_s(:hex) #=> "ff"
> ```

## 6.3   Integer Arithmetic

**+**                                                 *integer* + *numeric* → *numeric*

> Returns the sum of *integer* and *numeric*. *numeric* can be an instance of *GMP::Z*,
> *Fixnum, GMP::Q, GMP::F*, or *Bignum*.

**add!**                                            *integer*.add!(*numeric*) → *numeric*

> Sums *integer* and *numeric*, in place. *numeric* can be an instance of *GMP::Z*,
> *Fixnum, GMP::Q, GMP::F*, or *Bignum*.

| - | $integer$ - $numeric$ → $numeric$ |
|---|---|
|   | $integer$.sub!($numeric$) → $numeric$ |

Returns the difference of *integer* and *numeric*. The destructive method calculates the difference in place. *numeric* can be an instance of *GMP::Z, Fixnum, GMP::Q, GMP::F,* or *Bignum.* Here are some examples:

```
seven = GMP::Z(7)
nine  = GMP::Z(9)
half  = GMP::Q(1,2)
pi    = GMP::F("3.14")
nine - 5       #=> 4 (GMP Integer)
nine - seven   #=> 2 (GMP Integer)
nine - (2**32) #=> -4294967287 (GMP Integer)
nine - nine    #=> 0 (GMP Integer)
nine - half    #=> 8.5 (GMP Rational)
nine - pi      #=> 5.86 (GMP Float)
```

| * | $integer$ * $numeric$ → $numeric$ |
|---|---|
|   | $integer$.mul($numeric$) → $numeric$ |
|   | $integer$.mul!($numeric$) → $numeric$ |

Returns the product of *integer* and *numeric*. The destructive method calculates the product in place. *numeric* can be an instance of *GMP::Z, Fixnum, GMP::Q, GMP::F,* or *Bignum.*

| << | $integer$ << $numeric$ → $integer$ |
|---|---|

Returns *integer* times 2 to the *numeric* power. This can also be defined as a left shift by *numeric* bits.

| -@ | -$integer$ |
|---|---|
|   | $integer$.neg |
|   | $integer$.neg! |

Returns the negation, the additive inverse, of *integer*. The destructive method negates in place.

| abs | $integer$.abs |
|---|---|
|   | $integer$.abs! |

Returns the absolute value of *integer*. The destructive method calculates the absolute value in place.

## 6.4   Integer Division

---

**tdiv** | $integer.\mathrm{tdiv}(numeric) \rightarrow integer$
---

Returns the division of *integer* by *numeric*, truncated. *numeric* can be an instance of $GMP :: Z$, *Fixnum*, *Bignum*. The return object's class is always $GMP :: Z$.

---

**fdiv** | $integer.\mathrm{fdiv}(numeric) \rightarrow integer$
---

Returns the division of *integer* by *numeric*, floored. *numeric* can be an instance of $GMP :: Z$, *Fixnum*, *Bignum*. The return object's class is always $GMP :: Z$.

---

**cdiv** | $integer.\mathrm{cdiv}(numeric) \rightarrow integer$
---

Returns the ceiling division of *integer* by *numeric*. *numeric* can be an instance of $GMP :: Z$, *Fixnum*, *Bignum*. The return object's class is always $GMP :: Z$.

---

**tmod** | $integer.\mathrm{tmod}(numeric) \rightarrow integer$
---

Returns the remainder after truncated division of *integer* by *numeric*. *numeric* can be an instance of $GMP :: Z$, *Fixnum*, or *Bignum*. The return object's class is always $GMP :: Z$.

---

**fmod** | $integer.\mathrm{fmod}(numeric) \rightarrow integer$
---

Returns the remainder after floored division of *integer* by *numeric*. *numeric* can be an instance of $GMP :: Z$, *Fixnum*, or *Bignum*. The return object's class is always $GMP :: Z$.

---

**cmod** | $integer.\mathrm{cmod}(numeric) \rightarrow integer$
---

Returns the remainder after ceilinged division of *integer* by *numeric*. *numeric* can be an instance of $GMP :: Z$, *Fixnum*, or *Bignum*. The return object's class is always $GMP :: Z$.

---

**%** | $integer \mathbin{\%} numeric \rightarrow integer$
---

Returns *integer* modulo *numeric*. *numeric* can be an instance of *GMP::Z*, *Fixnum*, or *Bignum*. The return object's class is always *GMP::Z*.

## 6.5   Integer Roots

---

**root** | $integer.\mathrm{root}(numeric) \rightarrow numeric$
---

Returns the integer part of the *numeric*'th root of *integer*.

| **sqrt** | $integer.\text{sqrt} \rightarrow numeric$ |
| --- | --- |
| | $integer.\text{sqrt!}(numeric) \rightarrow numeric$ |

Returns the truncated integer part of the square root of *integer*.

| **sqrtrem** | $integer.\text{sqrtrem} \rightarrow sqrt,\ rem$ |
| --- | --- |

Returns the truncated integer part of the square root of *integer* as *sqrt* and the remainder, $integer - sqrt * sqrt$, as *rem*, which will be zero if *integer* is a perfect square.

| **power?** | $integer.\text{power?} \rightarrow true \mid false$ |
| --- | --- |

Returns true if *integer* is a perfect power, i.e., if there exist integers $a$ and $b$, with $b > 1$, such that *integer* equals $a$ raised to the power $b$.

Under this definition both 0 and 1 are considered to be perfect powers. Negative values of integers are accepted, but of course can only be odd perfect powers.

| **square?** | $integer.\text{square?} \rightarrow true \mid false$ |
| --- | --- |

Returns true if *integer* is a perfect square, i.e., if the square root of *integer* is an integer. Under this definition both 0 and 1 are considered to be perfect squares.

## 6.6   Integer Exponentiation

| **\*\*** | $integer\ \text{**}\ numeric \rightarrow numeric$ |
| --- | --- |
| | $integer.\text{pow}(numeric) \rightarrow numeric$ |

Returns *integer* raised to the *numeric* power.

| **powmod** | $integer.\text{powmod}(exp,\ mod) \rightarrow integer$ |
| --- | --- |

Returns *integer* raised to the *exp* power, modulo *mod*. Negative *exp* is supported if an inverse, $integer^{-1}$ modulo *mod*, exists. If an inverse doesn't exist then a divide by zero exception is raised.

## 6.7   Number Theoretic Functions

| **is_probab_prime?** | $integer.\text{is\_probab\_prime?}(reps = 5) \rightarrow 0, 1, \text{or } 2$ |
| --- | --- |

Determine whether *integer* is prime. Returns 2 if *integer* is definitely prime, returns 1 if *integer* is probably prime (without being certain), or returns 0 if *integer* is definitely composite.

This function does some trial divisions, then some Miller-Rabin probabilistic primality tests. *reps* controls how many such tests are done, 5 to 10 is a reasonable number, more will reduce the chances of a composite being returned as probably prime.

Miller-Rabin and similar tests can be more properly called compositeness tests. Numbers which fail are known to be composite but those which pass might be prime or might be composite. Only a few composites pass, hence those which pass are considered probably prime.

| **next_prime** | $integer.\text{next\_prime} \rightarrow prime$ |
| --- | --- |
| | $integer.\text{nextprime} \rightarrow prime$ |
| | $integer.\text{next\_prime!} \rightarrow prime$ |
| | $integer.\text{nextprime!} \rightarrow prime$ |

Returns the next prime greater than *integer*. The destructive method sets *integer* to the next prime greater than *integer*.

This function uses a probabilistic algorithm to identify primes. For practical purposes it's adequate, the chance of a composite passing will be extremely small.

| **gcd** | $a.\text{gcd}(b) \rightarrow g$ |
| --- | --- |

Computes the greatest common divisor of *a* and *b*. *g* will always be positive, even if *a* or *b* is negative. *b* can be an instance of *GMP::Z*, *Fixnum*, or *Bignum*.

```
GMP::Z(24).gcd(GMP::Z(8))   #=> GMP::Z(8)
GMP::Z(24).gcd(8)    #=> GMP::Z(8)
GMP::Z(24).gcd(2**32)   #=> GMP::Z(8)
```

| **invert** | $a.\text{invert}(m) \rightarrow integer$ |
| --- | --- |

Computes the inverse of *a* mod *m*. *m* can be an instance of *GMP::Z*, *Fixnum*, or *Bignum*.

```
GMP::Z(2).invert(GMP::Z(11))   #=> GMP::Z(6)
GMP::Z(3).invert(11)    #=> GMP::Z(4)
GMP::Z(5).invert(11)    #=> GMP::Z(9)
```

| **jacobi** | $a$.jacobi($b$) $\rightarrow$ *integer* |
| --- | --- |
| | GMP::Z.jacobi($a$, $b$) $\rightarrow$ *integer* |

Returns the Jacobi symbol $(a/b)$. This is defined only for $b$ odd. If $b$ is even, a range exception will be raised.

*GMP::Z.jacobi* (the instance method) requires $b$ to be an instance of *GMP::Z*. *GMP::Z#jacobi* (the class method) requires $a$ and $b$ each to be an instance of *GMP::Z*, *Fixnum*, or *Bignum*.

| **legendre** | $a$.legendre($b$) $\rightarrow$ *integer* |
| --- | --- |

Returns the Legendre symbol $(a/b)$. This is defined only for $p$ an odd positive prime. If $p$ is even, negative, or composite, a range exception will be raised.

| **remove** | $n$.remove($factor$) $\rightarrow$ (*integer*, *times*) |
| --- | --- |

Remove all occurrences of the factor $factor$ from $n$. $factor$ can be an instance of *GMP::Z*, *Fixnum*, or *Bignum*. *integer* is the resulting integer, an instance of *GMP::Z*. *times* is how many times $factor$ was removed, a *Fixnum*.

| **fac** | GMP::Z.fac($n$) $\rightarrow$ *integer* |
| --- | --- |

Returns $n!$, or, $n$ factorial.

| **fib** | GMP::Z.fib($n$) $\rightarrow$ *integer* |
| --- | --- |

Returns $F[n]$, or, the $n$th Fibonacci number.

## 6.8   Integer Logic and Bit Fiddling

| **and** | $a$ & $b$ $\rightarrow$ *integer* |
| --- | --- |

Returns *integer*, the bitwise and of $a$ and $b$.

| **ior** | $a$ | $b$ $\rightarrow$ *integer* |
| --- | --- |

Returns *integer*, the bitwise inclusive or of $a$ and $b$.

| **xor** | $a$ ^ $b$ $\rightarrow$ *integer* |
| --- | --- |

Returns *integer*, the bitwise exclusive or of $a$ and $b$.

**scan0**                                           $n.\text{scan0}(i) \rightarrow integer$

Scans $n$, starting from bit $i$, towards more significant bits, until the first 0 bit is found. Return the index of the found bit.

If the bit at $i$ is already what's sought, then $i$ is returned.

If there's no bit found, then $INT2FIX(ULONG\_MAX)$ is returned. This will happen in scan0 past the end of a negative number.

# 7 Random Number Functions

## 7.1 Random State Initialization

**new**

$$GMP::RandState.new \rightarrow mersenne\ twister\ state$$
$$GMP::RandState.new(:default) \rightarrow mersenne\ twister\ state$$
$$GMP::RandState(:mt) \rightarrow mersenne\ twister\ random\ state$$
$$GMP::RandState.new(:lc\_2exp, a, c, m2exp) \rightarrow linear\ congruential\ state$$
$$GMP::RandState.new(:lc\_2exp\_size, size) \rightarrow linear\ congruential\ state$$

This method creates a new *GMP::RandState* instance. The first argument defaults to *:default* (also *:mt*), which initializes the *GMP::RandState* for a Mersenne Twister algorithm. No other arguments should be given if *:default* or *:mt* is specified.

If the first argument given is *:lc_2exp*, then the *GMP::RandState* is initialized for a linear congruential algorithm. *:lc_2exp* must be followed with $a$, $c$, and $m2exp$. The algorithm can then proceed as $(X = (a * X + c) \mod 2^{m2exp})$.

*GMP::RandState* can also be initialized for a linear congruential algorithm with *:lc_2exp_size*. This initializer instead takes just one argument, *size*. $a$, $c$, and $m2exp$ are then chosen from a table, with $m2exp/2 > size$. The maximum size currently supported is 128.

```
GMP::RandState.new
GMP::RandState.new(:mt)
GMP::RandState.new(:lc_2exp, 1103515245, 12345, 15)    #=> Perl's
old rand()
GMP::RandState.new(:lc_2exp, 25_214_903_917, 11, 48)    #=> drand48
```

## 7.2 Random State Seeding

**seed**
$$state.seed(integer) \rightarrow integer$$

Set an initial seed value into *state*. *integer* can be an instance of *GMP::Z*, *Fixnum*, or *Bignum*.

## 7.3 Integer Random Numbers

**urandomb**
$$state.urandomb(n) \rightarrow integer$$

Generates a uniformly distributed random integer in the range 0 to $2^n - 1$, inclusive.

**urandomm**
$$state.urandomm(n) \rightarrow integer$$

Generates a uniformly distributed random integer in the range 0 to $n-1$, inclusive. $n$ can be an instance of *GMP::Z*, *Fixnum*, or *Bignum*.