Assignment week 11 - text answers

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(1) $fact(n)$ proof by induction

The base case is that, if n is equal to 1, then the function should return 1. It can be confirmed that, that is correct because the factorial of 1 is equal to 1.

To complete the proof, it is required to proof that the recursive step is correct.

This is our statement:

$$fact(n) = n \cdot fact(n-1)$$

$fact(n)$ depends on $fact(n-1)$

If $fact(n-1)$ is correct, then the following should be true:

$$fact(n) = n \cdot [product\ of\ all\ integers\ from\ 1\ to\ n-1]$$

To verify that $fact(n-1)$ is correct, we will look at some cases:

Suppose $n = 2$

Then

$$fact(2) = 2 \cdot fact(1)$$

And since $fact(1)$ is our base case and returns 1, $fact(2)$ returns $2 \cdot 1$, which is in fact correct.

Suppose $n = 3$

Then

$$fact(3) = 3 \cdot fact(2)$$

Again, we have already proven that $fact(2)$ is correct, so $fact(3)$ is also correct.

So, in general the base case $fact(1)$ proves $fact(2)$, and $fact(2)$ proves $fact(3)$, and therefore for all n, as $n \to \infty$, $fact(n-1)$ proves $fact(n)$. As a result of that, the program is correct for all positive integers.