

Exercises

- (1) Write down a proof that the following recursive factorial function is correct using *proof by induction*. Put your inductive proof into a pdf file (text_answers.pdf).
Hint: review the lecture slides for the two components of a proof by induction, i.e.
(a) the base case and (b) the inductive step.

```
/* Factorial function definition */
int fact(int n)
{
    /* pre-condition */
    assert (n >= 1);

    /* post-condition */
    if(n > 1)
        return n * fact(n - 1);
    else
        return 1;
}
```

We first need a **base case**.

We start by writing a bit of the sequence that is given by the factorial function, to give us an idea of what it might look like.

Lets say $n = 6$

Then the sequence is

$6*5*4*3*2*1$

In the case of factorial sequence the integer range is from $n \geq 1$

so n is the smallest integer we can input and therefore make a good base case.

Then we give the base case of

$\text{fact}(1) = 1$.

Next we need the **induction step**

Suppose we give k an input of $k > 1$.

Then : return $n * \text{fact}(n-1)$

We assume this is correct. This is called the induction hypothesis

Hereby we see that the k 'th number of the fact series is:

$\text{Fact}_k = k * \text{fact}(k-1)$

Let's take an example.

Lets say $k = 4$ which $k > 1$

The program would compute: $4*3*2*1 = 24$

We assume this is correct which then $\text{fact}(4)$ is correct.

Hereby I have proven that $\text{fact}(4)$ is correct, which then means that $\text{fact}(3)$ would be correct and $\text{fact}(2)$... and so on.