

Exercises

- (1) Write down a proof that the following recursive factorial function is correct using *proof by induction*. Put your inductive proof into a pdf file (text_answers.pdf).
Hint: review the lecture slides for the two components of a proof by induction, i.e. (a) the base case and (b) the inductive step.

```
/* Factorial function definition */
int fact(int n)
{
    /* pre-condition */
    assert (n >= 1);

    /* post-condition */
    if(n > 1)
        return n * fact(n - 1);
    else
        return 1;
}
```

The base case

If $n = 1$ then the function returns 1, so $fact(1) = 1$

And $fact(2) = 2$

The inductive step

Inductive hypothesis: assume $fact(k - 1)$ and $fact(k - 2)$ correctly compute the $(k - 1)^{th}$ and $(k - 2)^{th}$ factorial number, denoted F_{k-1} and F_{k-2} , for integer $k > 2$

- Then: $fact(k) = fact(k - 1) \cdot fact(k - 2) = F_{k-1} \cdot F_{k-2}$

which is the definition of the k^{th} Factorial number, F_k

- Therefore, fact calculates the n^{th} Factorial number, for all integers $n \geq 1$