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Week 11: Verifying Correctness of Recursive Programs

1. Write down a proof that the following recursive factorial function is correct using proof by induction. Put your inductive proof into a pdf file (text answers.pdf). Hint: review the lecture slides for the two components of a proof by induction, i.e. (a) the base case and (b) the inductive step.

In this case we a base cases:

The base case: If (n = 1)

return 1

We have a recursive step: If(n > 1)

return n * fact (n - 1)

We need to prove if n = 1 we can return the number 1. If we take the factorial number of 1 the output would be 1. Indeed, we can confirm that the fact of integer 1 is 1. The base case does what we want it to.

We now need to prove the recursive step, if (n > 1) we can return n * fact (n -1) for all n.

Suppose fact() has given some integer k > 1 as input. In this case the function will return:

fact(k) = k * fact(k - 1)

If fact(k) is correct the function compute the fact of integers from 1 to k. This will only be correct if fact(k - 1) is correct.

If we suppose that fact(k - 1) is correct then we know that fact(k) = k * (fact of integers 1 to (k-1) for k > 1)

So far we have proven that:

- Fact(1) is correct
- Fact(k) is correct if fact(k - 1) is correct for k > 1

We want to now combine these two facts to prove that fact() is correct for all positive integers, n.

Suppose we pick some specific integer e.g. k= 3:

- Fact(3) is correct if fact(2) is correct
- Fact(2) is correct if fact(1) is correct
- We have already proven that fact(1) is correct
- Therefore fact(2) is correct
- Therefore fact(3) is correct

In general, no matter which integer we choose for k, we will create a sequence of if-then propositions of the form:

- If fact(i - 1) is correct then fact(i) is correct

Because each if-then proposition uses (i-1) to prove the case for (i), eventually we will arrive at the proposition:

- If fact(1) is correct then fact(2) is correct

The base case fact(1) proves the case for fact(2), which in turn proves the case for fact(3), etc. all the way up to fact(k)

This proves that our function is correct for all positive integers, n.