Exercise 1

We can identify g as a function $g : \mathbb{R}^{M+M} \to \mathbb{R}$ and write $h : \mathbb{R}^N \to \mathbb{R}$ equivalently as $h = g \circ \tilde{f}$ where $\tilde{f} : \mathbb{R}^N \to \mathbb{R}^{M+M}$ is defined as $\tilde{f}(x) = [f(x), f(x)]^T$. Every column $n \in \{1, \ldots, N\}$ in jacobian of $\tilde{f}$ equals

$$\begin{aligned} D_i\tilde{f}(x) = \tfrac{\partial \tilde{f}}{\partial x_1} = \left[\tfrac{\partial \tilde{f}_1}{\partial x_i}(x), \ldots, \tfrac{\partial \tilde{f}_M}{\partial x_i}(x), \tfrac{\partial \tilde{f}_{M+1}}{\partial x_i}(x), \ldots, \tfrac{\partial \tilde{f}_{M+M}}{\partial x_i}(x)\right]^T \\ = \left[\tfrac{\partial f_1}{\partial x_i}(x), \ldots, \tfrac{\partial f_M}{\partial x_i}(x), \tfrac{\partial f_1}{\partial x_i}(x), \ldots, \tfrac{\partial f_M}{\partial x_i}(x)\right]^T = [D_i f(x), D_i f(x)]^T \end{aligned}$$

and hence $D\tilde{f}(x) = [Df(x), Df(x)]^T \in \mathbb{R}^{(M+M)\times N}$. This is sufficient for

$$Dh(x) = Dg(\tilde{f}(x)) \cdot D\tilde{f}(x) = Dg([f(x), f(x)]^T) \cdot [Df(x), Df(x)]^T$$

Exercise 2

It is

$$\frac{\partial E}{\partial w}(\mathbf{w}) = \frac{1}{L}\sum_{l=1}^{L} 2 \cdot (d_l - f(\mathbf{x}_l; \mathbf{w})) \cdot \frac{\partial f}{\partial w}(\mathbf{w})$$

and

$$\frac{\partial f}{\partial w^{2,0}_{1,1}}(\mathbf{x}; \mathbf{w}) = \sum_{j=0}^{2} w^{1,1}_{0,j} \cdot x_j, \qquad \frac{\partial f}{\partial w^{1,0}_{0,1}}(\mathbf{x}; \mathbf{w}) = w^{2,0}_{1,0} \cdot x_1$$

for $\mathbf{x} = [x_0, x_1, x_2]^T$.