(a) $\mathbb{E}(\bar{x}) = \mathbb{E}\Big(\frac{1}{L}\sum_{l=1}^{L} x_l\Big) = \frac{1}{L}\sum_{l=1}^{L}\mathbb{E}(x_l) = \frac{1}{L}\sum_{l=1}^{L}\frac{\theta}{2} = \frac{\theta}{2}$

(b) With $\hat{x} := 2\bar{x}$ it is $\mathbb{E}(\hat{x}) = \mathbb{E}(2\bar{x}) = 2\mathbb{E}(\bar{x}) = \theta$.

(c) The standard error

$$\mathbb{E}((\hat{x}-\theta)^2) = \mathbb{E}((\hat{x}-\mathbb{E}(\hat{x}))^2) = \mathrm{Var}(\hat{x}) = \mathrm{Var}\Big(\frac{2}{n}\sum_{l=1}^{L} x_l\Big) = \frac{4}{L^2}\sum_{l=1}^{L}\mathrm{Var}(x_l) = \frac{4}{L}\mathrm{Var}(x_1)$$

clearly goes to zero for $L \to \infty$.

(d) Let $\tilde{x} := \hat{x} + \frac{1}{L}$. It is

$$\mathbb{E}(\tilde{x}) = \mathbb{E}(\hat{x} + \frac{1}{L}) = \theta + \frac{1}{L}$$

and

$$\mathbb{E}((\tilde{x}-\theta)^2) = \mathbb{E}((\hat{x} + \frac{1}{L} - \theta)^2) = \mathbb{E}((\hat{x}-\theta)^2 + \frac{1}{L^2} + \frac{2}{L}(\hat{x}-\theta)) = \frac{4}{L}\mathrm{Var}(x_1) + \frac{1}{L^2}$$

which goes to zero for $L \to \infty$.