Medical imaging is about taking pictures and seeing inside of the human body without incisions or having to cut it to see what is inside. What is an image? or more precisely what is a black and white digital image? A digital image or a picture is a collection of points, called pixels and is usually denoted by two coordinates (x,y), and each pixel has light intensity, called gray level, ranging from white to black.

Mathematically speaking, a black and white picture is a function f(x,y) that assigns to each pixel some number corresponding to its gray level. In the 1920s, pictures were coded using five distinct levels of gray resulting in low quality pictures. Nowadays, the number of gray levels is an integer power of 2, that is 2k for some positive integer k. The standard now is 8-bit images, that is 28=256 levels of gray, with 0 for white and 255 levels or shades of gray. An image with many variations in the gray levels tends to be sharper than an image with small variations in the gray scale. The latter tends to be dull and washed out [[3]](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6526197/#b0015).

One of the oldest techniques used in medical imaging is X-rays where the patient is placed between an X-ray source and a film sensitive to X-ray energy, but in digital radiography the film is digitized or the X-rays after passing through the patient are captured by a digital devise. The intensity of the X-rays changes as they pass through the patient and fall on the film or the devise. Another medical application of X-ray technology is in Angiography where an X-ray contrast medium is injected into the patient through a catheter which enhances the image of the blood vessels and enables the radiologist to see any blockage. X-rays are also used in industry and in screening passengers and luggage at airports.

But more modern and sophisticated machines than X-ray machines are the CT scanners which produce 3-dimensional images of organs inside the human body.

The first CT scanner invented by Allen Cormack and GGodfrey Hounsfield in 1963 had a single X-ray source and a detector which moved in parallel and rotated during the scanning process. This technique has been replaced by what is called a fan-beam scanner in which the source runs on a circle around the body firing a fan (or a cone) of X-rays which are received after they pass through the body by an array of detectors in the form of a ring encircling the patient and concentric with the source ring. The process is repeated and the data is collected and processed by a computer to construct an image that represents a slice of the object. The object is slowly moved in a direction perpendicular to the ring of detectors producing a set of slices of the object which, when put together, constitute a three-dimensional image of the object.

In 1956, Allen Cormack, a young South African physicist, was appointed at the Radiology Department at the Groote Schuur hospital, the teaching hospital for the University of Cape Town’s medical school. This hospital later became the site of the world’s first heart transplant. Cormack took on himself, as one of his first duties at the new job, the task of finding a set of maps of absorption coefficients for different sections of the human body.

The results of the task would make X-ray radiotherapy treatments more efficient. He soon realized that what he needed to complete his task was measurements of the absorption of X-rays along lines in thin sections of the body. Since the logarithm of the ratio of incident to emergent X-ray intensities along a given line is just the line integral of the absorption coefficient along that line, the problem mathematically was equivalent to finding a function f(x,y) from the values of its integrals along all or some lines in the plane [[4]](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6526197/#b0020).

A few years later, Cormack immigrated to the United States and became a naturalized citizen. Because of the demands of his new position, he had to pursue his problem part time as a hobby. But by 1963 he had already found three alternative forms of solutions to the problem and published his results. He contacted some research hospitals and groups, like NASA, to see if his work would be useful to them but received little or no response.

Cormack continued working on some generalizations of his problem, such as recovering a function from its line integrals along circles through the origin. Because there was almost no response to his publications, or at least that was what he thought, Cormack felt somewhat disappointed and forgot about the problem for a while.

By a mere accident, Cormack discovered that his mathematical results were a special case of a more general result by Johann Radon, in which Radon introduced an integral transform and its inverse and showed how one could construct a two-dimensional function f(x,y) from its line integrals. Even more, he showed how one can reconstruct an n-dimensional function from its integrals over hyper-planes of dimension n-1. That integral transform is now called the Radon transform. The transform and its inverse are the essence of the mathematical theory behind CT scans.

As is often the case with many beautiful and significant mathematical discoveries, the Radon transform was discovered and went unnoticed for very many years. And when it was rediscovered, it was rediscovered independently by several people in different fields. The Radon transformation without doubt is one of the most versatile function transformations. Its applications are numerous and its scope is immense. Chief among its applications are computed tomography (CT) and nuclear magnetic resonance (NMR).

Johann Radon was born in December 16th, 1887 and died on May 25th, 1956. He was an Austrian professor of mathematics who worked at different universities in Austria and Germany but his final destination was at the Institute of Mathematics at the University of Vienna where he was appointed professor in 1946. He later became a dean and the rector at the University of Vienna.

The saga of the Radon transform began in 1917 with the publication of Johann Radon’s seminal paper “On the determination of functions from their integrals along certain manifolds.” At that time, Radon was an assistant to Professor Emanuel Czuber at the University of Technology of Vienna.

In that paper Radon demonstrated how one could reconstruct a function of two variables from its integrals over all straight lines in the plane. He also discussed other generalizations of this problem, for example, reconstructing a function from its integrals over other smooth curves, as well as, reconstructing a function of n variables from its integrals over all hyper-planes.

One of the beauties and strength of mathematics may be gleaned from the following examples. We cannot visualize objects in dimensions higher than three, nevertheless, Radon’s result shows that we can theoretically construct images of n dimensional objects, which are functions of n variables, if we know their integrals over hyper-planes of dimension n-1.

Although his paper had some direct ramifications on solutions of hyperbolic partial differential equations with constant coefficients, it did not receive much attention even from Radon’s colleagues at the University of Vienna. This may be attributed to World War I and the turmoil that permeated the political atmosphere in Europe during that period. It should be emphasized that Radon did not have any applications in mind and probably never imagined that his work would be used in saving lives 50 years later.

In the late 1960s, at the Central Research Laboratories of a company called Electrical and Musical Industries (EMI), best known as publisher of the Beatles records, Godfrey Hounsfield, a British engineer, used some of Cormack’s ideas to develop a new X-ray machine that revolutionized the field of medical imaging. Soon after that Cormack and Hounsfield joined forces and collaborated in refining the invention and developing the CT-scanning technique. Although the first image obtained by CT scan took hours to process, it was the beginning of a new and remarkable invention.