R 语言期末大作业

1 基本模型

1.1 AR(1)model

Consider the following AR(1) model

$$X_t = \mu + \phi X_{t-1} + \varepsilon_t, \tag{1}$$

where $\varepsilon_t \sim N(0,1)$.

1.2 AR(1) model with GARCH(p,q) errors

Consider the following AR(1) model with GARCH(p,q) errors:

$$\begin{cases} X_t = \mu + \phi X_{t-1} + \varepsilon_t, \\ \varepsilon_t = \sigma_t \eta_t, \quad \sigma_t^2 = \omega_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2, \end{cases} \tag{2}$$

for $t = 1, \cdots, n$, where $\{\eta_t\}$ is a sequence of independent and identically distributed random variables with zero mean and variance one. $\omega_0 > 0, a_i \geq 0$, for $i = 1, \cdots, r-1, a_r > 0, b_j \geq 0$, for $j = 1, \cdots, s-1, b_s > 0$. The intercept μ may be zero or nonzero, and the regression coefficient ϕ may be (i) $\phi < 1$, (ii) $\phi = 1 + c/n$ and (iii) $\phi = 1$, which represents stationary, near unit root and unit root respectively.

Set $\mu = 0$ or $\mu = 0.01$, $\omega_0 = 0.3$, $a = 0.3, b = 0.2$.

2 检验方法

Here we are interested in testing whether the $\{X_t\}_{t=1}^n$ is a unit root process, that is,

$$\mathcal{H}_0 : \phi = 1 \quad \text{versus} \quad \mathcal{H}_1 : \phi \neq 1.$$

Obtain the estimators $\hat{\mu}, \hat{\phi}$ by solving

$$\begin{cases} 0 &= \sum\limits_{t=1}^{m}(X_t - \mu - \phi X_{t-1})\frac{1}{\sqrt{1+\sum\limits_{k=1}^{d_0}(X_{t-k}-X_{t-k-1})^2}} \\ 0 &= \sum\limits_{t=m+1}^{2m}(X_t - \mu - \phi X_{t-1})\frac{(X_{t-1}-\bar{X}_m)}{\sqrt{(1+(X_{t-1}-\bar{X}_m)^2)\left(1+\sum\limits_{k=1}^{d_0}(X_{t-k}-X_{t-k-1})^2\right)}} \end{cases}$$

where $m = [n/2]$ with $[\cdot]$ being the floor function, $d_0 \geq \max\{p+1, q+1\}$, and $\bar{X}_m = \frac{1}{m}\sum_{t=1}^{m} X_t$. Consider using the following score equations using the random weighting:

• *Step A1* Draw a random sample with sample size $2m$ from a distribution function with mean one and variance one, say the standard exponential distribution. Denote them by $\delta_1^b, \cdots \delta_{2m}^b$.

• *Step A2* Solve the following score equations to get estimators $\hat{\mu}^b$ and $\hat{\phi}^b$ for μ and ϕ:

$$\begin{cases} 0 &= \sum\limits_{t=1}^{m}\delta_t^b(X_t - \mu - \phi X_{t-1})\frac{1}{\sqrt{1+\sum\limits_{k=1}^{d_0}(X_{t-k}-X_{t-k-1})^2}} \\ 0 &= \sum\limits_{t=m+1}^{2m}\delta_t^b(X_t - \mu - \phi X_{t-1})\frac{(X_{t-1}-\bar{X}_m)}{\sqrt{(1+(X_{t-1}-\bar{X}_m)^2)\left(1+\sum\limits_{k=1}^{d_0}(X_{t-k}-X_{t-k-1})^2\right)}} \end{cases}$$

• *Step A3* Repeat the above two steps B times to get $\{\hat{\mu}^b, \hat{\phi}^b\}_{b=1}^B$. $B = 1000$.

Obtain the Statistic:

$$\frac{(\hat{\phi} - \phi_0)^2}{\frac{1}{B}\sum_{b=1}^{B}(\hat{\phi}^b - \hat{\phi})^2} \xrightarrow{d} \chi_1^2,$$

as $B \to \infty$, and $n \to \infty$, where χ_1^2 denotes a chi-squared distributed variable with one degree of freedom. Note that we denote ϕ_0 as the true value of ϕ.

3 任务

1. 列举出常用的单位根检验方法（尽可能全面）。

2. 根据上述模型1和模型2生成模拟数据，对第 2 节中描述的检验方法以及第 1 问中提到的单位根检验方法进行蒙特卡洛模拟，对比这些方法的 size 表现（即将模型中的 ϕ 在生成数据时候设置为 1）。蒙特卡洛模拟应最少重复 1000 次。

最后需要展示的是：第二节检验方法部分所描述方法的程序；0.05 显著性水平下 size 表现对比结果，用表格呈现。

3. 根据上述模型1和模型2生成模拟数据，对比第 2 节中描述的检验方法以及第 1 问中提到的单位根检验方法的 power 表现。(在生成数据时将备则假设中 ϕ 的值代入)。模拟次数同样至少重复 1000 次。最后结果要以图形的形式展现。

提示：横坐标是 ϕ 的值，范围设置为 [0.92,1]，间隔为 0.1。纵坐标是 power 的值 (0.05 显著性水平下)。图上应该有几条曲线，分别代表不同的检验方法。