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Problem 1. ***b. 1000***

$$
\begin{aligned}
0.03 &\geq 2Me^{-2\epsilon^2 N} \\
0.03 &\geq 2(1)e^{-2(0.05)^2 N} \\
0.03 &\geq 2e^{-0.005N} \\
\ln(0.03/2) &\geq -0.005N \\
N &\geq (\ln(0.03/2))/(-0.005) \\
N &\approx 840 && \textit{closest to 1000}
\end{aligned}
$$

Problem 2. ***c. 1500***

$$
\begin{aligned}
0.03 &\geq 2Me^{-2\epsilon^2 N} \\
0.03 &\geq 2(10)e^{-2(0.05)^2 N} \\
0.03 &\geq 20e^{-0.005N} \\
\ln(0.03/20) &\geq -0.005N \\
N &\geq (\ln(0.03/20))/(-0.005) \\
N &\approx 1300 && \textit{closest to 1500}
\end{aligned}
$$

Problem 3. ***d. 2000***

$$
\begin{aligned}
0.03 &\geq 2Me^{-2\epsilon^2 N} \\
0.03 &\geq 2(100)e^{-2(0.05)^2 N} \\
0.03 &\geq 200e^{-0.005N} \\
\ln(0.03/200) &\geq -0.005N \\
N &\geq (\ln(0.03/200))/(-0.005) \\
N &\approx 1760 && \textit{closest to 2000}
\end{aligned}
$$

Problem 4. ***b. 5***
As shown in class, the break point for $\mathbb{R}^2$ is 4 points which is a line. We only need one more point for there to be a break point in $\mathbb{R}^3$ which is a plane.

Problem 5. ***b. i, ii, v***
If the function is neither a polynomial nor 2^n, it is not a possible growth function. Lets go through each possibility.

1. *$1 + N$ is a polynomial and thus a possible growth function.*
2. *$1+N+\binom{N}{2} = 1+N+\frac{N!}{2!(N-2)!} = 1+N+\frac{N(N-1)}{2} = \frac{1}{2}N^2+\frac{1}{2}N+1$. This is a polynomial and thus a possible growth function.*

3. $\sum_{i=1}^{\lfloor\sqrt{N}\rfloor}\binom{N}{i}$ *has a maximum power of* N^{k-1} *when the sum goes to* $k-1$*. Since this sum goes to* $\lfloor\sqrt{N}\rfloor$*, it has a maximum power of* $N^{\lfloor\sqrt{N}\rfloor}$*. This is neither a polynomial nor* 2^N *and therefore not a possible growth function.*

4. $2^{\lfloor\sqrt{N}\rfloor}$ *is neither a polynomial nor* 2^N *and therefore not a possible growth function.*

5. 2^N *is a possible growth function.*

Problem 6. ***c. 5***
Where $h:\mathbb{R}\to\{-1,+1\}$ *and* $h(x)=+1$*, the breakpoint is 5 because there is no way of creating two positive intervals with 5 points as shown:* $\{+1,-1,+1,-1,+1\}$.

Problem 7. ***c.*** $\binom{N+1}{4}+\binom{N+1}{2}+1$
The breakpoint for this problem is $N=5$*. The only function for which all* $N<5$ *are satisfied is* $\binom{N+1}{4}+\binom{N+1}{2}+1$.

Problem 8. ***d. 2M + 1***
In the general case with M *intervals with* $h:\mathbb{R}\to\{-1,+1\}$ *and* $h(x)=+1$ *if the point falls in the* M *intervals or* $h(x)=-1$ *otherwise, the smallest breakpoint is* $2M+1$*. There must be a negative point between every positive point. Positive points will always be on the ends* $(+1,-1,+1,...+1,-1,+1)$*. Thus, if we have* M *negative points, there will be* $M+1$ *positive points. Therefore, we need* $M+M+1=2M+1$ *to establish a break point.*

Problem 9. ***d. 7***
We can put points on the edge of a circle, to see if triangles can be drawn to classify the points. If the points are all alternating $+1$ *and* -1*, a triangle can always be drawn around all the* $+1$*'s up until* $N=7$*. When* $N=7$*, there will be two* $+1$ *points next to each other and therefore can be treated as one point. With that point treated as one, there are two others around the circle (since there would be 4* $+1$*'s and 3* -1*'s). However, past* $N=7$*, a triangle can not always be drawn. Therefore,* $N=7$ *is the breakpoint. This makes sense because for a triangle, M = 3 points. From problem 8, we determined that the maximum number of points that the set can support is* $2M+1=7$.

Problem 10. ***b.*** $\binom{N+1}{2}+1$
Since we know that x_1^2 *and* x_2^2 *are always positive and bounded by* a^2 *and* b^2 *which are also always positive, we can treat the problem as a one interval problem. The growth function for the one interval problem is* $\binom{N+1}{2}+1$.