Sample Midterm 1 for MATH 185

Problem 1

If the followings statements are true, answer "TRUE". If not, give a brief explanation why.

(1) If $f : \mathbb{C} \to \mathbb{C}$ is complex differentiable at a, so is the function $g(z) := \overline{f(\overline{z})}$.

(2) The set $\{z \in \mathbb{C} : |z^2 - 3| < 1\}$ is a domain.

(3) The set $\{z \in \mathbb{C} : |z| < 1 \wedge |z + 1| < \sqrt{2}\}$ is a star-shaped domain.

(4) The function $f(z) = -1/z^4$ has an anti-derivative in $D = \mathbb{C}^\bullet$.

(5) $\int_\alpha \frac{2z-1}{z^2-8z+15} = 0$, where α is the unit circle around 0.

Problem 2

Compute the real and imaginary part of $((1 + i)/\sqrt{2})^k$ for $k \in \mathbb{Z}$.

Problem 3

Determine in which points the function $f(x+iy) = \sin^2(x+y)+i\cos^2(x+y)$ is complex differentiable and compute the derivative at those points.

Problem 4

Compute

$$\int_\alpha \frac{1}{|z|^2} dz,$$

where α is the circle of radius 3 around 0.