Bonus Homework for **MATH 185**

Due: Monday May 7, 3:10 pm in class

Problem 1

Prove the *integral representation* of the Laurent series coefficients: If $f(z) = \sum_{n=-\infty}^{\infty} a_n(z-a)^n$ in some annulus $\mathcal{A} = \{z \in \mathbb{C} : \ r < |z-a| < R\}$, then

$$a_n = \frac{1}{2\pi i} \oint_{|z-a|=\rho} \frac{f(z)}{(z-a)^{n+1}}\, dz,$$

for every $n \in \mathbb{Z}$ and $r < \rho < R$.

Problem 2

Does there exist a closed, piecewise smooth curve $\alpha : [0,1] \to \mathbb{C}$ such that the winding number χ, interpreted as a function $\chi_\alpha(a) = \chi(\alpha; a)$ from $\mathbb{C}$ into $\mathbb{Z}$ takes infinitely many values, i.e. such that the set

$$\{\chi_\alpha(z) : \ z \in \mathbb{C}\} \subseteq \mathbb{Z}$$

is infinite? Justify your answer.

Problem 3

Let $a \in \mathbb{R}$, $a > 1$. Set $f_a(z) = z + a - e^z$.

(a) Show that f_a has exactly one zero in the left half-plane $\{z \in \mathbb{C} : \ \mathrm{Re}(z) < 0\}$.

(b) Show that this zero is on the real line.