Homework 3 for **MATH 561**, Set Theory

Due: Thursday Feb 23

Problem 1 – Well-founded relations

Show (in ZF) that if **R** is a well-founded relation on a class **A** (but possibly not set-like), then every non-empty subclass **X** of **A** has an **R**-minimal element.

Problem 2 – V_ω

Define $E \subseteq \omega \times \omega$ as follows: $n\,E\,m$ iff there is a 1 in the n-th place (counting from the right) in the binary representation of m. Show that $\langle \omega, E \rangle \cong \langle V_\omega, \in \rangle$.

Problem 3 – Inaccessible cardinals

The previous exercise yields, in particular, a one-one map between ω and V_ω. After ω, the cardinality of V_ω increases exponentially. Does there exist a stage again where

$$\kappa = |V_\kappa| \,?$$

Read in Chapter 3 and 5 about inaccessible cardinals and show that those have the property in question.

Problem 4 – Gödel's 2nd Incompleteness Theorem

Using the Fixed Point Lemma and the existence of a provability predicate for ZF, show that if T is a recursive, consistent extension of ZF, then

$$T \nvdash \mathrm{Con(T)}\,.$$