# Homework 6 for
# **MATH 561**, Set Theory

Due: Thursday Mar 29

In the following, $M$ is always a countable transitive model of ZFC.

### Problem 1 – Generic filters

Show that the following properties are equivalent to being a $\mathbb{P}$-generic filter over $M$:

(a) $G \cap D \neq \emptyset$ for all $D \in M$ which are dense and *open* (i.e. $p \in D$ implies $q \in D$ for all $q \leq p$).

(b) $G \cap D \neq \emptyset$ for all $D \in M$ which is a *maximal antichain* in $\mathbb{P}$ (i.e. the elements of $D$ are pairwise incompatible, and $D$ is maximal with this property).

### Problem 2 – Generics in the ground model

We have seen in class that if the p.o. $\mathbb{P}$ satisfies

$$\forall p \exists q, r\, (q \leq p \ \wedge\ r \leq p \ \wedge\ q \perp r) \tag{*}$$

then any $\mathbb{P}$-generic over $M$ is not an element of $M$. Show that if there exists $p \in \mathbb{P}$ for which $(*)$ fails (such $p$ is called an *atom* of $\mathbb{P}$) and $\mathbb{P} \in M$, then there exists a filter $G \in M$ with $p \in G$ so that $G$ intersects *all* dense subsets of $\mathbb{P}$.

### Problem 3 – Not every subset of $\mathbb{P}$ extends a model

Assume that $\mathbb{P} \in M$ and $\mathbb{P}$ is infinite. Show that there exists an $H \subseteq \mathbb{P}$ such that $M[H]$ is not a model of ZF.

### Problem 4 – Number of generics

Show that if $\mathbb{P} \in M$ and $\mathbb{P}$ is non-atomic, then

$$\{G \colon G \text{ is } \mathbb{P}\text{-generic over } M\}$$

has cardinality $2^{\aleph_0}$.