Homework 8 for MATH 185

Due: Wednesday March 21, 3:10 pm in class

Problem 1

Give two new proofs of the fundamental theorem of algebra:

(a) Using the open mapping theorem. (Show that the image $P(\mathbb{C})$ of a complex polynomial P is open and closed.)

(b) Using the mininmal modulus principle.

Problem 2

Let T be a Möbius transformation, i.e. T is of the form

$$T(z) = \frac{az+b}{cz+d},$$

where $a, b, c, d \in \mathbb{C}$ such that $ad - bc \neq 0$.

(a) Show that T can be written as a composition $T = T_4 \circ T_3 \circ T_2 \circ T_1$, where T_1 and T_4 are *translations*, T_2 is an *inversion*, and T_3 is a *rotation-dilation*.

(b) Use part (a) to show that if $L \subset \mathbb{C}$ is a straight line and $S \subset \mathbb{C}$ is a circle, then $T(L)$ is either a straight line or a circle, and $T(S)$ is either a straight line or a circle.