# Homework 1 for **MATH 574**

Due: Wednesday January 26

**Problem 1**

Show that the product of a countable sequence of Polish spaces is Polish.

**Problem 2**

Verify $(0,1)$ (as a topological subspace of $\mathbb{R}$) is Polish.

**Problem 3**

Show that the metric on $A^{\mathbb{N}}$ introduced in class is an *ultrametric*, i.e.

$$d(\alpha,\beta) \leqslant \max\{d(\alpha,\xi), d(\beta,\xi)\} \quad \text{for all } \xi \in A^{\mathbb{N}}.$$

Furthermore, show that in any ultrametric space, any ball is clopen (i.e. open and closed).

**Problem 4**

Verify that $A^{\mathbb{N}}$ is compact if and only if $A$ is finite.

**Problem 5**

Show that $(A^{\mathbb{N}})^{\mathbb{N}}$ (with the product topology of countably many copies of $A^{\mathbb{N}}$, which in turn carries the topology introduced in class) is homeomorphic to $A^{\mathbb{N}}$.