Sample Midterm 1 for MATH 104

Problem 1

Show that every real number is the limit of a sequence of rational numbers.

Problem 2

Let (a_n) and (b_n) be two sequences. Assume that $\lim a_n = a$, $\lim b_n = b$, and that $a_n \leqslant b_n$ for all $n \in \mathbb{N}$.

Show that $a \leqslant b$.

Problem 3

Let (s_n) and (t_n) be two bounded sequences. Show that

$$\limsup_n (s_n + t_n) \leqslant \limsup_n s_n + \limsup_n t_n.$$

Give an example of two bounded sequences (s_n), (t_n) such that

$$\limsup_n (s_n t_n) \neq \limsup_n s_n \limsup_n t_n.$$

Problem 4

Show that a countable union of countable sets is countable. More precisely, let $\{E_i : i \in \mathbb{N}\}$ be a countable family such that each E_i is countable. Show that

$$\bigcup_{i \in \mathbb{N}} E_i = \{x : \text{exists } i \in \mathbb{N} \text{ such that } x \in E_i\}$$

is countable.