Homework 3 for
MATH 574, Topics in Logic

Due: Wednesday Feb 26

Problem 1

Let $A = \{0, 1, 2, \ldots, 9\}$. Show that there exists a finite automaton that accepts the set

$$L = \{w \in A^{<\mathbb{N}} : \ w \text{ is the decimal representation of a number divisible by 3}\}$$

(*Note:* String other than 0 that start with a sequence of one or more 0's should be rejected.)

Problem 2

Suppose $L_1, L_2 \subseteq \{0,1\}^{<\mathbb{N}}$ are regular languages (i.e. each is accepted by some finite automaton). Show that the following languages are also regular:

1. $L_1{}^\frown L_2 := \{w : w = v_1{}^\frown v_2,\ v_1 \in L_1, v_2 \in L_2\}$,
2. $L_1 \cup L_2$,
3. $L_1^* := \{w : w = v_1{}^\frown v_2{}^\frown \ldots{}^\frown v_n \text{ for some } v_1, \ldots, v_n \in L_1\}$

Hint: Thinking about this week's *Weekly Challenge* first makes this problem considerably easier.

Problem 3

Write a Turing machine program that accepts the language

$$\{0^n : n \geq 1\}$$

(over the binary alphabet). You do not need to verify or prove that your program works, but you should group and comment your instructions so that the idea of your algorithm becomes clear to the reader.

Problem 4

Consider a Turing machine M with alphabet $A = \{0, 1\}$ and three states (including the halting state q_F). We require that M halts on input 0. What is the maximum number of 1's that such an M can output on input 0?