Math 574 - Topics in Logic

Spring 2014, Pennsylvania State University

Basic info

- Instructor: Jan Reimann
- Class: MWF 9:05–9:55, 104 Osmond AND Online
- Credits: 3 (more credits possible by completing an additional project)
- Course grade: Based on homework and online activity (see "Student activities" below)
- Course Topics: Algorithmic Information Theory, Complexity, and Data Compression
- Prerequisites: Some general knowledge in upper-division math, particularly analysis and topology. No previous knowledge in logic is required.

Course outline

1. Basics: Information and Computation
   - Information and codes
   - Random variables and dynamical systems
   - Computability
   - Turing machines
   - Finite automata
2. Entropy
   - Origins of entropy in physics: thermodynamics
   - Shannon's information theoretic entropy
   - Entropy in dynamical systems
   - Algorithmic entropy: Kolmogorov complexity
3. Complexity
   - Algorithmic randomness: finite strings
   - Algorithmic randomness: infinite strings
   - Randomness vs entropy
   - Typicality: generic points in dynamical systems
   - Randomness vs typicality
   - Normality and finite automata
4. Coding and Data Compression
   - Universal codes
   - Compression and entropy
   - Lossless data compression: Lempel-Ziv compression
   - Lossy compression: rate distortion theory
   - Compressors as approximations to Kolmogorov complexity
5. Applications
   - Fractal dimension, entropy, and Kolmogorov complexity
   - Inductive inference via Kolmogorov complexity, Minimum Description Length principle
   - Clustering by data compression

Bibliography

There is no textbook required for the course, but all material covered in class can be found in one of the following texts:

- P. Billingsley. Ergodic theory and information. John Wiley & Sons Inc., 1965
- T. M. Cover and J. A. Thomas. Elements of information theory. Wiley-Interscience, 2006.
- R. G. Downey and D. R. Hirschfeldt. Algorithmic randomness and complexity. Springer, 2010.
- M. Li and P. Vitányi. An introduction to Kolmogorov complexity and its applications. Springer, 2008.
- P. C. Shields. The ergodic theory of discrete sample paths. American Mathematical Society, 1996.

The book by Li and Vitányi is particularly helpful. It is available online for Penn State members at http://link.springer.com/book/10.1007%2F978–0–387–49820–1 (You have to be on a campus computer or connected through the Penn State VPN.)

Course resources

The following web services will be used for this class:

- This course page: http://www.personal.psu.edu/jsr25/Spring_14/AIT_syllabus_long.html
- The Angel page for the course.
- The Piazza page for the course. Piazza is an online discussion board for courses and has math editing capability built in.
- Penn State's online meeting platform: https://meeting.psu.edu. Here we will meet during off-weeks for discussions and office hours.

Course schedule

All lectures for this class will be online, posted on this page, on Angel, and on Piazza. The material will be presented in lessons (roughly 10–12). Each lesson comprises a number of short lecture videos (5 - 15 minutes each). Students are expected to complete one lesson each week (roughly).

We will use the class meetings for discussions, problem sessions, and some complementary material. *Class meetings will be bi-weekly*, and announced in a timely manner. In off weeks, we will meet on Penn State's online meeting platform: https://meeting.psu.edu

You can subscribe to the Google calendar for the course to keep track of the schedule.

Student activities

To successfully complete the course, students are expected to do the following:

- Hand in solutions to homework assignments (biweekly).
- Take short quizzes on Angel related to each lecture.
- Participate in the weekly challenge on Piazza.
- Participate in Piazza discussions. Ask good questions and answer questions by others.
- Produce a nice write-up of one lesson (in LaTeX) by the end of the semester.

Academic Integrity

All Penn State Policies regarding ethics and honorable behavior apply to this course.