Homework 1 for **MATH 104**

Due: Tuesday, September 12, 9:30am in class

Problem 1

Determine whether the following sets are bounded (from below, above, or both). If so, determine their infimum and/or supremum and find out whether these infima/suprema are actually minima/maxima.

(1) $S_1 = \{\, 1 + (-1)^n : \; n \in \mathbb{N}\}$;

(2) $S_2 = \{\frac{1}{m} + \frac{1}{n} : \; m, n \in \mathbb{N}\}$;

(3) $S_3 = \{x \in \mathbb{R} : \; x^2 + x + 1 \geqslant 0\}$;

(4) $S_4 = \{\cos(\frac{n\pi}{3}) : \; n \in \mathbb{N}\}$.

Problem 2

Prove that in any ordered field F, the following hold:

(1) $0 < 1$;

(2) if $0 < a < b$, then $0 < b^{-1} < a^{-1}$ for $a, b \in F$.

Problem 3

Let A and B sets of real numbers such that

(a) $A \cup B = \mathbb{R}$,

(b) if a is in A and b is in B, then $a < b$,

(c) A contains no largest element (maximum).

Prove that B contains a smallest element (minimum).

Problem 4

Let A and B nonempty sets of reals which are both bounded from above. Define the set $A + B$ as

$$A + B = \{a + b : \; a \in A \text{ and } b \in B\}.$$

Show that $\sup A + B = \sup A + \sup B$.

Bonus Problem

Can there be a field of exactly six elements?