Lesson 2

Computability

Math 574 - Topics in Logic
Penn State, Spring 2014

Jan Reimann

PENNSTATE
Eberly College of Science

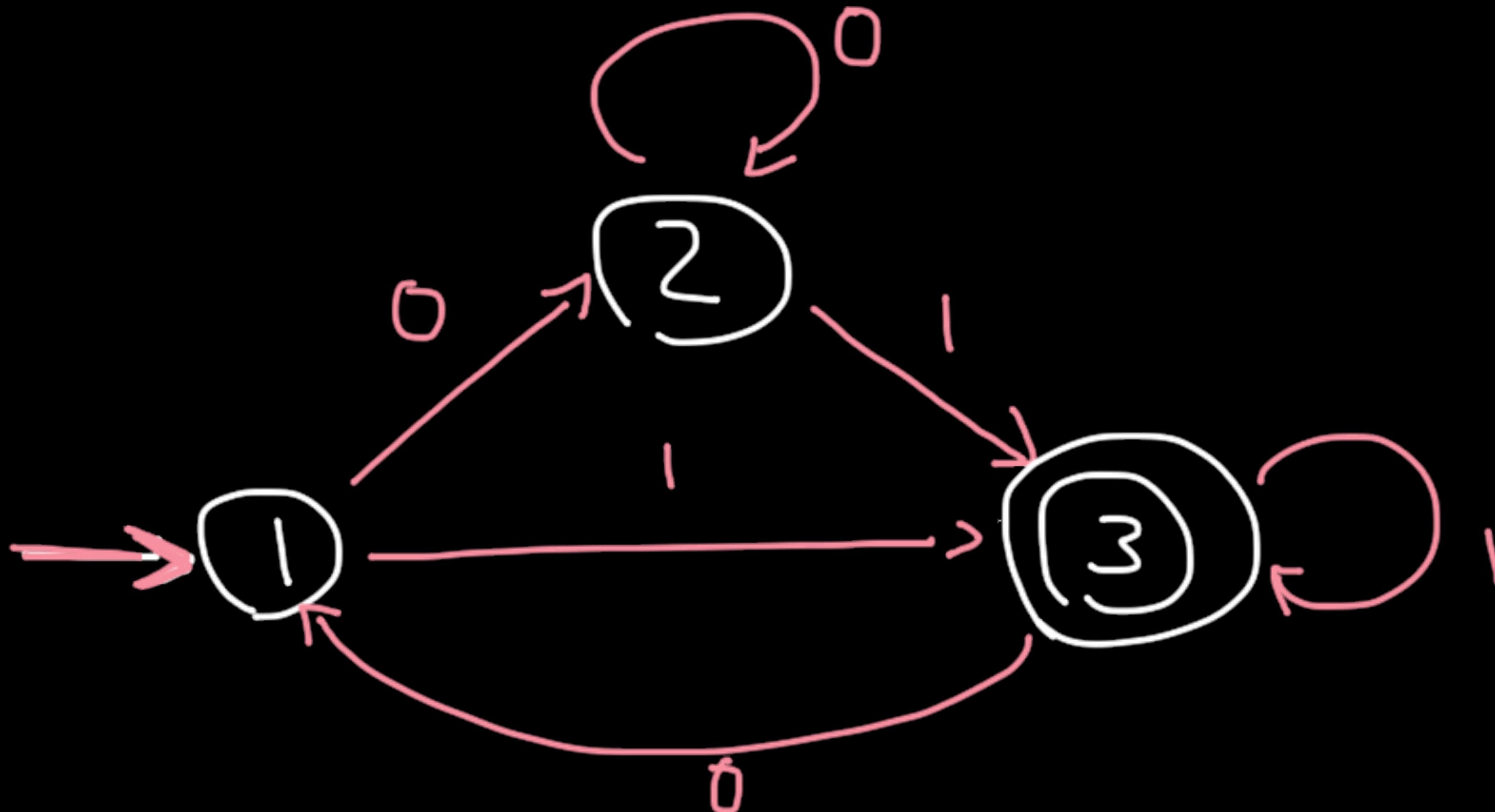
2-1

Finite Automata

Finite Automata

Quintuple $M = (Q, A, q_0, F, \delta)$

<u>Finite</u> set of states



$Q = \{1, 2, 3\}$ $q_0 = 1$ $F = \{3\}$

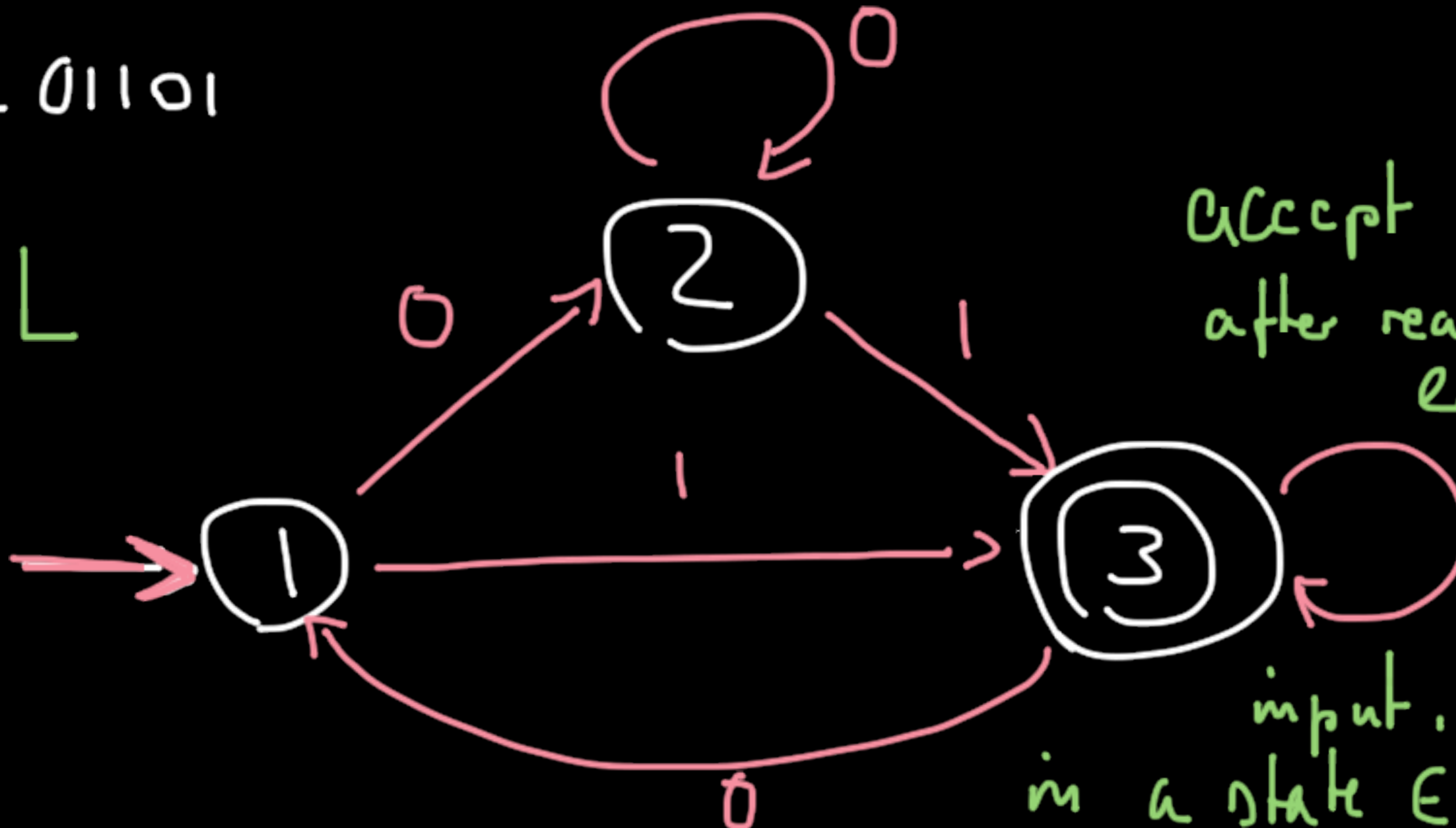
$A = \{0, 1\}$ $\delta : Q \times A \to Q$

• A finite automaton M accepts a set $L \subseteq A^{<\mathbb{N}}$.

language

$w = 01101$

$\in L$



accept if after reading last bit of input, we are in a state $\in F$

Input: $w = w_0 \cdots w_{n-1} \in A^{<\mathbb{N}}$

State function for w:

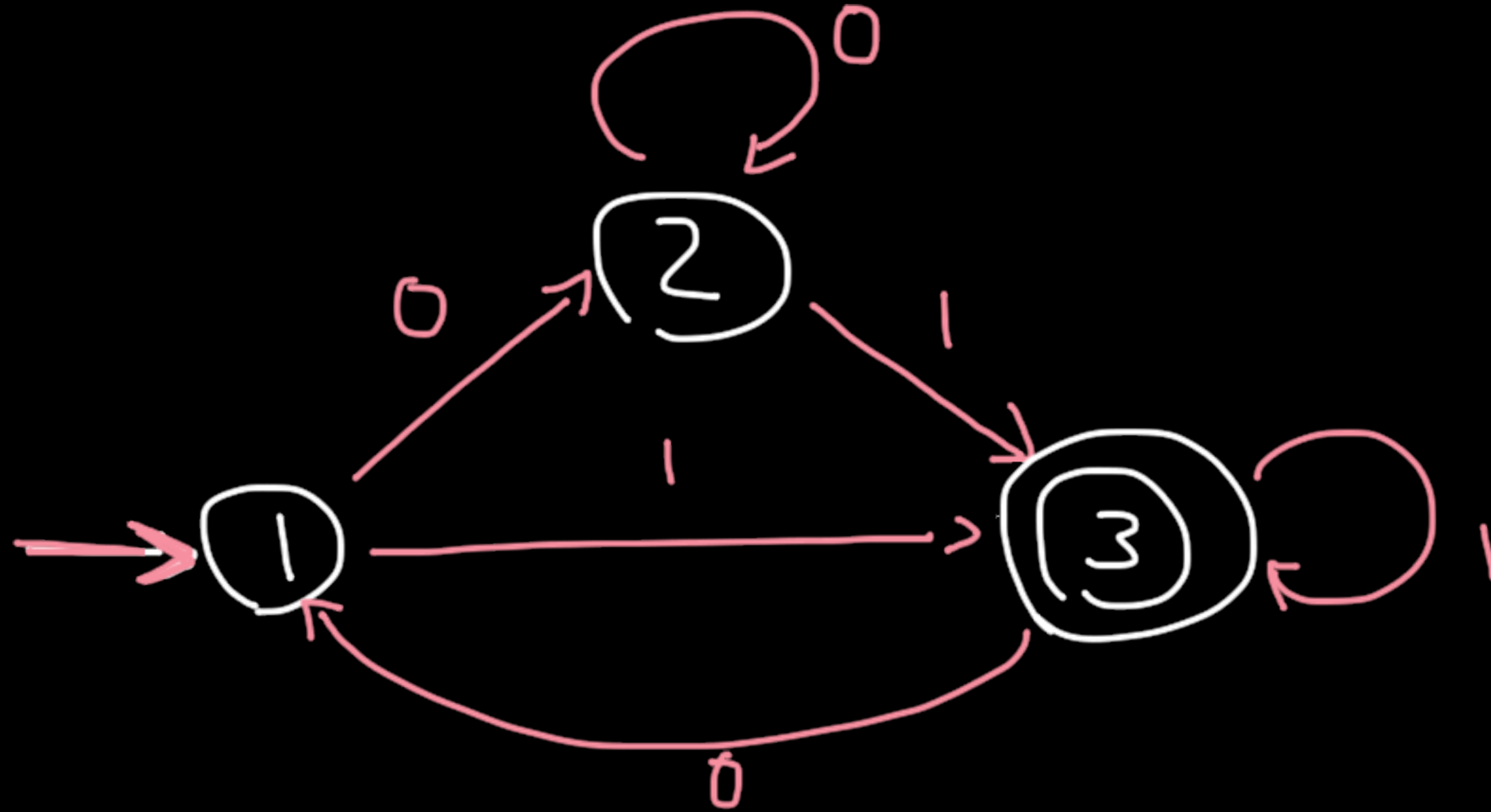
$$\tau_w(0) = q_0$$

$$\tau_w(k+1) = \delta\left(\tau_w(k), w_k\right)$$

DEF: M accepts $w \in A^{<\mathbb{N}}$, $w = w_0 \cdots w_{n-1}$

iff $\tau_w(n) \in F$

$$L_M = \{ w \in A^{<\mathbb{N}} : M \text{ accepts } w \}$$



other accepted strings

001

1

11

00011011

M accepts precisely those string that end with 1

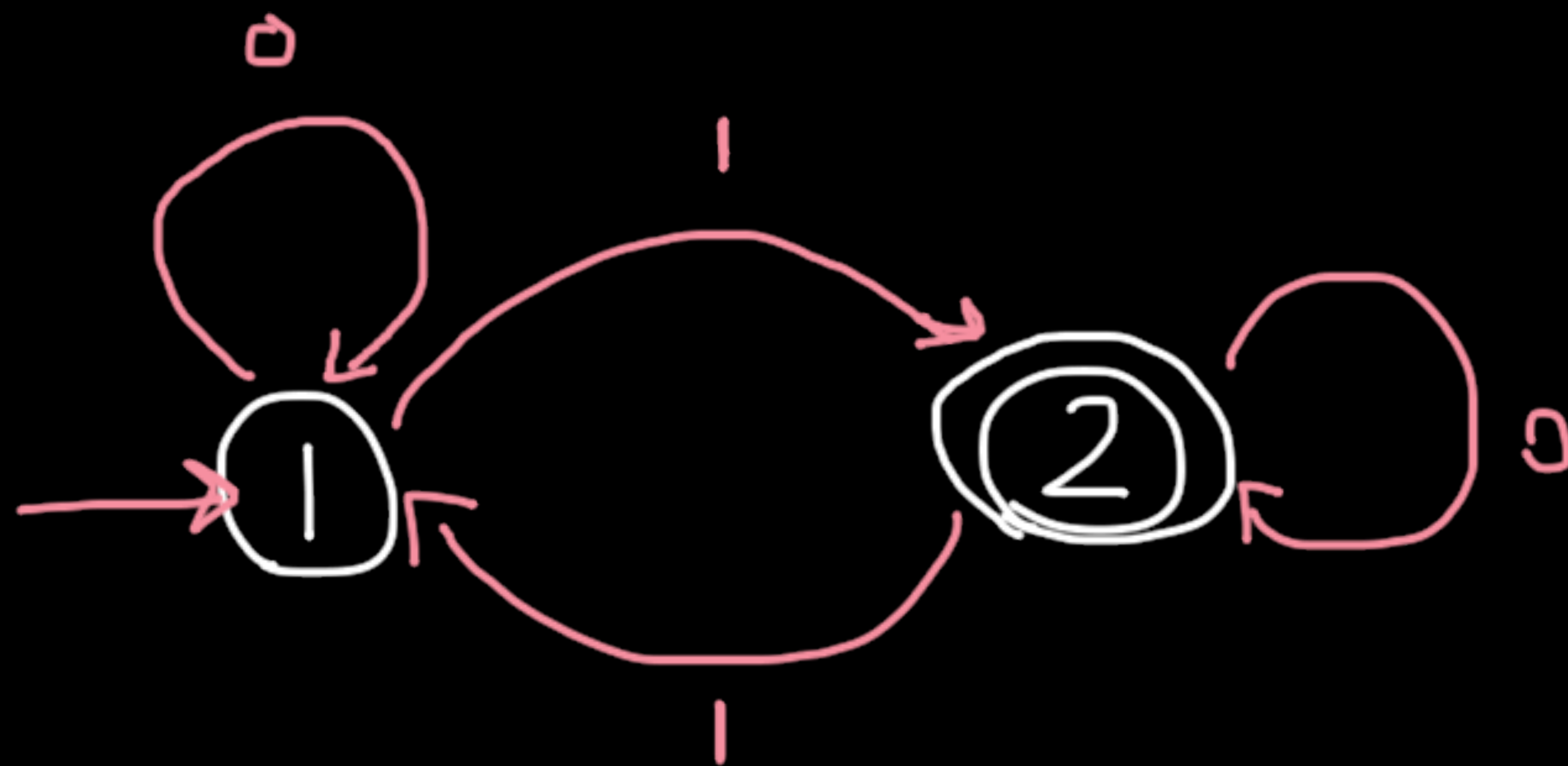
not accepted

0

00

0010

⋮



Accepted		not accepted
1	precisely the string with an odd # of 1's	0
10		11
01		110
001		000

- The languages that can be accepted by a FA can be characterized as the collection of regular languages:
  - $\emptyset$ is regular, and for each $a \in A$, $\{a\}$ is reg.
  - If $A, B \subseteq A^{<\mathbb{N}}$ are regular, then

$$A \cup B, \quad A^\frown B, \quad A^* \text{ are regular.}$$

$\{a^\frown b : a \in A, b \in B\}$
concatenation

all words that can be formed by concatenating finitely many words from A

  - No other language is regular

EXA:

- $\{w :\ w$ ends with $1\}$

  $\{w :\ w$ contains odd number of 1's$\}$

  are regular

- $\{w :\ w = 0^n 1^n$ for some $n \geq 0\}$

  $\{w :\ w$ is a palindrome$\}$ 01110

  are not regular

  (can be shown using Pumping Lemma)