$H(\vec{p}) = -\sum p_i \log p_i$

max if $p_1 = p_2 = \cdots = p_n$ prove this

#1: • Start with $n=2$ $\vec{p} = (p, 1-p)$

use calculus

• $n \geq 3$: use "2nd year" calculus

Optimization under constraints.

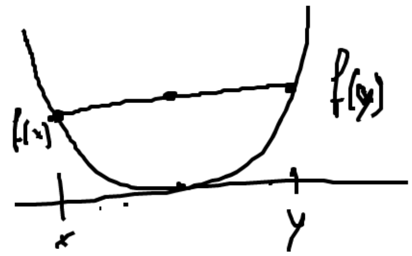
max $f(\vec{x})$ given $g(\vec{x}) = c$

$\Lambda = f - \lambda(g-c)$ Lagrange multipliers

max $H(\vec{p})$ subject to $\sum p_i = 1$

Jensen's inequality:

f convex



$$\lambda f(x) + (1-\lambda) f(y) \geq f(\lambda x + (1-\lambda) y) \qquad \lambda \in [0,1]$$

strictly convex : $=$ $\Rightarrow$ $\lambda = 0, 1$

$-\log$ convex

$$\mathbb{E}(f(X)) \geq f(\mathbb{E}X)$$

$$\mathbb{E}(fX) = f(\mathbb{E}X)$$

$$\mathbb{E}(fX) = f(\mathbb{E}X) \qquad X \text{ binary}$$

$$\underbrace{p \cdot f(x_1) + (1-p) f(x_2)}_{\mathbb{E} f(X)} = f(p x_1 + (1-p) x_2)$$

f strictly convex

$\Rightarrow p = 0, 1$

$\Rightarrow X$ is constant $\Rightarrow \underline{\mathbb{E}X = X}$

use induction to generalize to n-dim. X

Jensen's inequ. is used to show

$\underline{D(p \| q) \geq 0}$ | $D(p \| q) = 0$ iff $p = q$

later this

Show that $D(p \| u) = \log |A| - H(X)$

≥ 0 ... uniform ... X - A-valued

iff $p = u$

$g : \underbrace{\text{im}(X)}_{A} \longrightarrow \mathbb{R}$

$H(Y|X)$ definition

$$\boxed{H(g(X)) \leq H(X)}$$

$$H(X, g(X)) = H(X) + \underbrace{H(g(X)|X)}_{?}$$

$$H(X, g(X)) = H(g(X)) + H(X|g(X))$$

$$\geq \ldots$$

$$\begin{tikzcd} & p \arrow[r] & \underline{\underline{\sigma}} \\ \underline{q} \arrow[r] \arrow[urr, bend left] & p \end{tikzcd}$$

$C(\sigma) = |p|$

$C(p) = |q|$

If $|q| \ll |p| \longrightarrow C(\sigma) \ll |p|$ $\lightning$

$$K(\sigma) \stackrel{+}{\leq} |\sigma| + 2\log|\sigma|$$

$$K(\sigma) \stackrel{+}{\leq} 2|\sigma|$$

prefix-free copy machine

$$0^{|\sigma|}1\sigma \longmapsto \sigma$$

$2|\sigma|+1$

$|\sigma| = n \leftarrow$ binary repres. s_n

$|s_n| = \log n$

$$0^{|s_{|\sigma|}|}1 \, s_{|\sigma|} \, \sigma \longmapsto \sigma$$

$$|s_{|\sigma|}| + 1 + |s_{|\sigma|}| + |\sigma| = |\sigma| + 2\log|\sigma| + 1$$