Homework 4 for **MATH 435**

Due: Friday Sep 24

Problem 1

(a) Show that $(\mathbb{Q}, +)$ is not cyclic.

(b) Does there exist finite $X \subseteq \mathbb{Q}$ such that $< X >= \mathbb{Q}$ (as an additive group)?

Problem 2

Let G be a group and H a subgroup. Let $x \in G$. Let xHx^{-1} be the subset of G consisting of all elements xyx, $y \in H$.

Show that xHx^{-1} is a subgroup of G

Problem 3

Let G be a finite group, and suppose H is a subgroup of G such that $[G : H] = 2$. Show that for all $x \in G$, $xHx^{-1} = H$.

Problem 4

Book, p. 158, Exercise 2.62

Problem 5

Recall that the *alternating group of order* n, A_n, is defined as

$$A_n = \{\alpha \in S_n : \ \mathrm{sgn}(\alpha) = 1\}.$$

Show that A_n is generated by the $(n-2)$-many 3-cycles $(1\ 2\ 3), (1\ 2\ 4), \ldots, (1\ 2\ n)$, i.e.

$$A_n =< (1\ 2\ 3), (1\ 2\ 4), \ldots, (1\ 2\ n) > .$$