# Homework 7 for **MATH 435**

Due: Friday Oct 22

**Problem 1**

Book, p. 206, Exercise 2.211 (i)-(vii).
(Give the reasons.)

**Problem 2**

Show that if G acts transitively on X, then $\mathcal{O}(x) = X$ for all $x \in X$.
This means if one orbit is all of X, then all orbits are.

**Problem 3**

This exercise gives two applications of the *orbit-stabilizer relation*.

(a) Let $\mathcal{O}$ be the symmetry group of the cube. $\mathcal{O}$ acts on the set $\{v_1, \ldots, v_8\}$ of vertices of a cube. Show that the action is transitive. Determine the stabilizer group of a vertex. Apply the orbit-stabilizer relation to compute the order of $\mathcal{O}$.

(b) The symmetric group $S_n$ acts on $\{1, 2, \ldots, n\}$. Show that the action is transitive. Furthermore, show that the stabilizer of $n$ is isomorphic to $S_{n-1}$. Apply the orbit-stabilizer relation to show that $|S_n| = n|S_{n-1}|$. Use this to give an “algebraic” proof that $|S_n| = n!$.

**Problem 4**

Recall the definition of a *solvable group* from Homework 6. Show that if G is a group such that $|G| = p^n$, where $p$ is a prime and $n \in \mathbb{N}$, then G is solvable.