

Homework 10 for MATH 185

Due: Wednesday April 18, 3:10 pm in class

Problem 1 [**]

Find the Laurent series expansion of the following functions around $a = 0$:

(a) $\sin(1/z)$ (b) $1/(z(z+1))$ (c) $z/(z+1)$ (d) e^z/z^2 .

Problem 2 [**]

Find the Laurent series of the function

$$f : \mathbb{C} \setminus \{1, -2\} \rightarrow \mathbb{C}, \quad z \mapsto \frac{1}{(z-1)(z+2)}$$

for the annuli

(a) $\mathcal{A}_1 = \{z : 0 < |z| < 1\}$ (b) $\mathcal{A}_2 = \{z : 1 < |z| < 2\}$ (c) $\mathcal{A}_3 = \{z : 2 < |z|\}$

Problem 3 [**]

Let $0 < \varepsilon < 2\pi$ and define $f : \dot{U}_\varepsilon(0) \rightarrow \mathbb{C}$ by

$$f(z) = \frac{\sin(z)}{\cos(z^3) - 1}.$$

Show that f has a pole of order 5 in 0 (i.e. $\text{ord}(f; 0) = -5$) and determine the coefficient a_{-1} of the Laurent series of f around 0.

Problem 4 [**]

Let $D \subseteq \mathbb{C}$ be open. Suppose $a \in D$ and $f : D \setminus \{a\} \rightarrow \mathbb{C}$ is analytic and one-one. Prove the following statements.

- (a) f has in a a non-essential singularity.
- (b) If f has a pole in a , then it is a pole of order 1.
- (c) If f has a removable singularity in a , then the analytic extension of f to D is one-one, too.