Lesson 2

Computability

Math 574 - Topics in Logic
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Jan Reimann

2-2

Turing Machines

Finite Automata:

- model of restricted computational power

$$L = \{0^n 1^n : n \geq 0\}$$

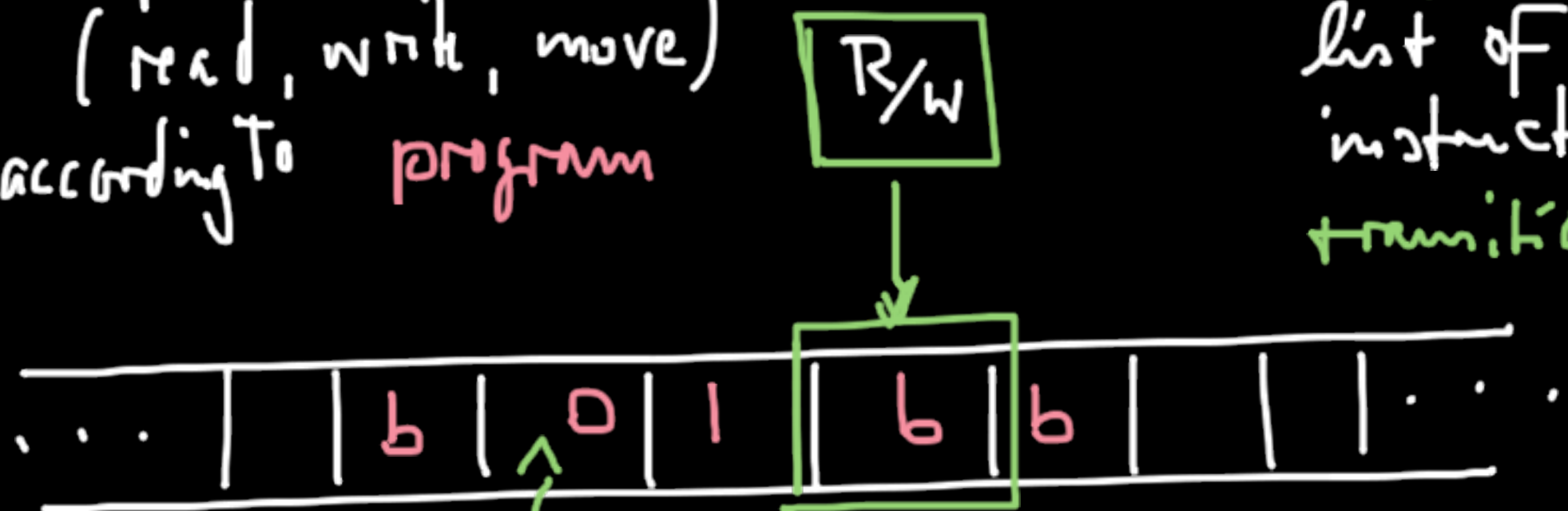- FA have fixed, finite amount of memory

Turing Machine (TM)

- finitely many states ($\rightarrow$ program)

+ memory/work tape on which
machine can read & write

CPU $\rightarrow$ memory

performs operations
(read, write, move)
according to   program

program :
list of
instructions
transition function

R/w

. . . . | b | b | 0 | 1 | b | b | b | | | . . . .

tape

Cell

current cell

holds entry from
a finite alphabet A
or a special symbol "b" ∉ A
BLANK

Formal Definition of TM

- set of states Q (finite)

  - $q_0 \in Q$ initial state

  - $q_F \in Q$ halting state

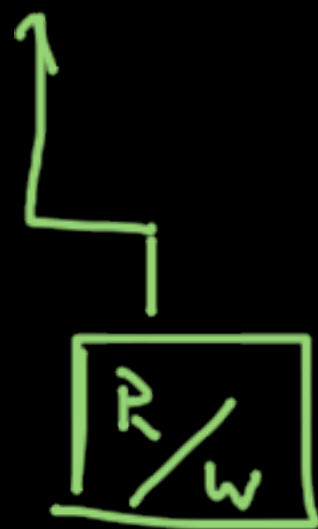- Alphabet (finite) A, Special ("blank") symbol "b" $\notin A$

- transition function

$$\delta : Q \times A \cup \{b\} \longrightarrow A \cup \{b\} \times \overset{\text{direction}}{D} \times Q$$

$$D = \{L, R, S\}$$

Running a TM

- input $w = w_0 \ldots w_{n-1}$, $w_i \in A$

Set the current state $= \underline{q_0}$

initial state

initial configuration of TM

Computation Step

- Given : configuration ("snapshot")

current symbol
x

current state
q

- look up transition function

$$\delta(q, x) = (y, d, p)$$

new state

$\in A \times \{b\}$

$L, R \text{ or } S$

current state = 3

$$\delta(3, 0) = (1, R, 5)$$

new current state = 5

Halting & Output

- TM **halts** if we reach state q_F

- If this happens, TM
  outputs the string (over A)
  immediately to the right of
  current cell

output:

011

| | 1 | b | 1 | 1 | 0 | 1 | 1 | b | ... |