

Graduate Course on “Algorithmic Information Theory, Complexity, and Data Compression”

1. Basics: Information and Computation

- Information and codes
- Random variables and dynamical systems
- Computability
- Turing machines
- Finite automata

2. Entropy

- Origins of entropy in physics: thermodynamics
- Shannon’s information theoretic entropy
- Entropy in dynamical systems
- Algorithmic entropy: Kolmogorov complexity

3. Complexity

- Algorithmic randomness: finite strings
- Algorithmic randomness: infinite strings
- Randomness vs entropy
- Typicality: generic points in dynamical systems
- Randomness vs typicality
- Normality and finite automata

4. Coding and Data Compression

- Universal codes
- Compression and entropy
- Lossless data compression: Lempel-Ziv compression
- Lossy compression: rate distortion theory
- Compressors as approximations to Kolmogorov complexity

5. Applications

- Fractal dimension, entropy, and Kolmogorov complexity
- Inductive inference via Kolmogorov complexity, Minimum Description Length principle
- Clustering by data compression

Bibliography

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- T. M. Cover and J. A. Thomas. Elements of information theory. Wiley-Interscience, 2006.
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- M. Li and P. Vitányi. An introduction to Kolmogorov complexity and its applications. Springer, 1997.
- P. C. Shields. The ergodic theory of discrete sample paths. American Mathematical Society, 1996.