Sample Midterm 2 for MATH 185

Problem 1

If the followings statements are true, answer "TRUE". If not, give a brief explanation why.

(1) If $f(z)$ is analytic on a domain $D \subseteq \mathbb{C}$, and α is aclosed path in D, then $\int_\alpha f(z)dz = 0$.

(2) If f is analytic on the unit disk $\mathbb{E} = \{z : |z| < 1\}$, then there exists an $a \in \mathbb{E}$ such that $|f(a)| \geqslant |f(0)|$.

(3) If $\sum_n a_n z^n$ has radius of convergence R, then $\sum_n \mathrm{Re}(a_n) z^n$ has radius of convergence $\geqslant R$.

(4) If f and g are analytic on D, and if they agree on a non-empty set S which is closed in D, then $f = g$ in D.

Problem 2

Compute the integral

$$\oint_{|z|=3} \frac{\cos(\pi z)}{z^2 - 1}.$$

Problem 3

Let $f : \mathbb{C} \to \mathbb{C}$ be a non-comstant, entire function. Show that $f(\mathbb{C})$ is dense in $\mathbb{C}$, i.e. for every $a \in \mathbb{C}$ and for every $\varepsilon > 0$, $U_\varepsilon(a)$ contains a point from $f(\mathbb{C})$.

Problem 4

Expand $\frac{1}{z^2-1}$ in a Taylor series around $z = 0$ and determine the radius of convergence.