Sample Midterm 2 for MATH 104

Problem 1

[Review all important definition and results of the relevant material. You will be asked to state a few of them precisely. Among those are: *series, convergence, absolute convergence, Cauchy criterion, comparison test, root test, ratio test, alternating series, metric spaces, completenes, open and closed sets, compact sets, Heine-Borel, continuity, continuous functions on compact sets, intermediate value theorem, uniform continuity, continuity on metric spaces, power series, radius of convergence, pointwise and uniform convergence of power series, continuity and uniform convergence, Weierstrass M-test, uniform convergence of power series, Abel's theorem.*]

Problem 2

If the followings statements are true, answer "TRUE". If not, give a brief explanation why.

(1) $\sum_{n=2}^{\infty}(n+(-1)^n)^{-2}$ converges.

(2) $d(x,y) = |x_1 - y_1| + |x_2 - y_2|$ is a metric on $\mathbb{R}^2$.

(3) $[0,1] \setminus \{\frac{1}{n} : n \in \mathbb{N}\}$ is compact.

(4) If $f_n \to f$ pointwise on a compact set, and all f_n and f are continuous, then $f_n \to f$ uniformly.

(5) A power series converges uniformly on its interval of convergence.

Problem 3

Let $K \subseteq \mathbb{R}$ be compact. Show that $\sup K$ is finite and belongs to K.

Problem 4

Let $f_n(x) = x^n/n$. Show that f_n converges uniformly on $[-1,1]$ and specify the limit function.

Problem 5

Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and that there exist $a, b \in \mathbb{R}$ such that $f(a)f(b) < 0$. Show that there exists an x such that $f(x) = 0$.