Homework 6 for
MATH 574, Topics in Logic

Due: Friday, May 2

Problem 1

Show that the entropy $H(\vec{p})$ of an n-dimensional probability vector $p = (p_1, \dots, p_n)$ is maximal if $p_1 = \cdots = p_n = 1/n$.

Problem 2

We have seen that algorithmic entropy (Kolmogorov complexity) cannot be increased by a computable function ($C(h(\sigma)) \leq^+ C(\sigma)$). Show that a similar statement is true for probabilistic entropy: If X is a discrete random variable, and g is a real-valued function defined on the range of X, then for the random variable $g(X)$ it holds that

$$H(g(X)) \leq H(X).$$

Problem 3

What is the complexity of a string that is a shortest program for some other string?

Show that there exists a constant b such that, whenever p is a string such that $U(p) = \sigma$ and $|p| = C(\sigma)$, then $C(p) \geq |p| - b$.

This means shortest programs are incompressible (up to a constant).

Problem 4

We saw in Lesson 4-4 that $K(\sigma) \leq^+ |\sigma| + K(|\sigma|)$. Can we give an upper bound better than $K(\sigma) \leq^+ 2|\sigma|$ that does not mention K on the right hand side?

Prove that $K(\sigma) \leq^+ |\sigma| + \log|\sigma| + 2\log\log|\sigma|$.