Homework 11 for **MATH 185**

Due: Friday April 27, 3:10 pm in class

Problem 1

Let $D \subseteq \mathbb{C}$ be a domain, $a \in D$, and suppose $f, g : D \setminus \{a\} \to \mathbb{C}$ are analytic functions with non-essential singularities in a. Show that the following assertions hold.

(a) If a is a pole of order k (i.e. $\operatorname{ord}(f; a) = -k$), then

$$\operatorname{Res}(f; a) = \lim_{z \to a} \frac{h^{(k-1)}(z)}{(k-1)!}, \quad \text{where } h(z) = (z-a)^k f(z).$$

(b) If $\operatorname{ord}(f; a) = l$ and $\operatorname{ord}(g; a) = l + 1$, $l \geq 0$, then

$$\operatorname{Res}(f/g; a) = (l+1)\frac{f^{(l)}(a)}{g^{(l+1)}(a)}.$$

(c) If $f \not\equiv 0$, then $\operatorname{Res}(f'/f; a) = \operatorname{ord}(f; a)$.

Problem 2

Compute the residues of the following functions at the indicated points:

$(a)\ \dfrac{\exp(z^2)}{z-1}, a = 1$	$(d)\ \dfrac{z^2}{z^4-1}, a = \exp(\pi i/2)$	$(g)\ \dfrac{z+2}{z^2-2z}, a = 0$
$(b)\ \dfrac{\exp(z^2)}{(z-1)^2}, a = 1$	$(e)\ \dfrac{\exp(z)-1}{\sin(z)}, a = 0$	$(h)\ \dfrac{1+\exp(z)}{z^4}, a = 0$
$(c)\ \left(\dfrac{\cos(z)-1}{z}\right)^2, a = 0$	$(f)\ \dfrac{1}{\exp(z)-1}, a = 0$	$(i)\ \dfrac{\exp(z)}{(z^2-1)^2}, a = 1$

Problem 3

Evaluate the integral

$$\oint_{|z|=7} \frac{1+z}{1-\cos(z)} dz.$$

Problem 4

Do exercise III.6.2 on page 172. Use the hint. Justify your steps carefully and precisely.