Homework 6 for MATH 185

Due: Wednesday March 7, 3:10 pm in class

Problem 1

Let $U \subseteq \mathbb{C}$ be a domain, and let $g : U \to \mathbb{C}$. Show that if $h(z) := g(z^n)$ is analytic in U, then g is analytic in U.

Problem 2

Find all entire functions f such that $f + f'' = 0$

Problem 3

Give a formal proof that the Riemann ζ function

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

is normally convergent on $D = \{s : \mathrm{Re}(s) > 1\}$. Conclude that ζ is analytic in D. Show that ζ does not converge uniformly on D. Compute the Taylor expansion of ζ around $s = 2$.

Problem 4

Compute the Taylor expansion of $f(z) = \mathrm{Log}(1 + z)$ around 0. Determine the radius of convergence R. (How can this be done without resorting to the usual formula for R?) Show that the Taylor series converges for all $|z| = R$ except for one point.