Homework 2 for
MATH 561, Set Theory

Due: Thursday Feb 2

Problem 1 – More on ultrafilters

Jech, Exercises # 7.5, 7.10, 7.11, 7.12

Problem 2 – Ultraproducts

(a) Show that if $\mathcal{U}$ is a principal ultrafilter over S, then the embedding $j : \mathcal{M} \to \Pi_{\mathcal{U}} \mathcal{M}_s$ is an isomorphism.

(b) Show that if $\mathcal{U}$ is an ultrafilter over S, and $(\mathcal{M}_s : s \in S), (\mathcal{N}_s : s \in S)$ are structures such that $\mathcal{M}_s \preceq \mathcal{N}_s$ for all $s \in S$, then $\Pi_{\mathcal{U}} \mathcal{M}_s \preceq \Pi_{\mathcal{U}} \mathcal{N}_s$.

Problem 3 – Definability

Marker, Exercises # 1.4.15 and 2.5.25 (for Part (a) of 2.5.25 you can argue informally)

Problem 4 – Comprehension

Show that the following set cannot exist:

$$\{x : \neg \exists u \, (u \in x \ \wedge \ x \in u)\}.$$