Homework 3 for MATH 185

Due: Wednesday February 14, 3:10 pm in class

Problem 1

Let $\alpha(t) = \exp(2\pi i t)$, $0 \leqslant t \leqslant 1$, be the unit circle. Show that

$$\left| \int_\alpha \frac{\sin(\xi)}{\xi^2} d\xi \right| \leqslant 2\pi e.$$

Problem 2

Evaluate the following integrals:

(a) $\int_\alpha \bar{z} dz$, where α is the straight line segment from -1 to 1;

(b) $\int_\alpha \bar{z} dz$, where α is the circle segment of the unit disk around zero between -1 and 1;

(c) $\int_\alpha 1/z \, dz$, where $\alpha(t) = \cos(t) + 2i \sin(t)$, $0 \leqslant t \leqslant 2\pi$;

(d) $\int_\alpha 1/z^2 \, dz$, where α is as in (c).