Finite element method is a classic numerical method for solving partial differential equations. In this chapter, we will give a brief introduction to this method. discuss its basic properties and error estimates. In later chapters, we will show that the neural network functions can be viewed as an extension of finite element function. In this chapter, we discuss the classical linear finite element spaces, the error estimate of the finite element method and adaptivity method to improve the approximation. For shape-regular mesh, we will establish both the upper and lower bound of the approximation error.

Linear finite element spacesFEspace In this section, we introduce linear finite element spaces. We will walk through the basic setup, and derive some error estimates.

Triangulations

Given a bounded polyhedral domain $\subset R^d$, a geometric triangulation (also called mesh or grid) $T_h = \{\tau\}$ of Ω is a set of d-simplices such that enumerate

[(1)] $\overline{\Omega} = \cup\tau$, where $\overline{\Omega}$ denotes the closure of Ω.

[(2)] if τ_1 and τ_2 are distinct elements in T_h then $\circ\tau_1 \cap \circ\tau_2 =$, where $\circ\tau_i$ denotes the interior of $\tau_i, i = 1, 2$

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