

The Model Existence Theorem

Key Concepts

- Extend T to a Henkin theory T_H , then complete it to a theory T' .
- T' is a complete Henkin theory for the extended language \mathcal{L}_H .
- For T' , its term model $\mathcal{A}_{T'}$ satisfies

$$\mathcal{A}_{T'} \models \sigma \iff T' \vdash_{\mathcal{L}_H} \sigma$$

for all \mathcal{L}_H -sentences σ .

Problems

Exercise 0.1.

How do we obtain a model for T from a model for T' ?

Exercise 0.2.

Why is $\mathcal{A}_{T'}$ countable if \mathcal{L} is countable?

Exercise 0.3.

Let X be the set of all maximally consistent \mathcal{L} -theories. Recall that the sets

$$\langle \sigma \rangle = \{T \in X : \sigma \in T\} \quad (\sigma \text{ } \mathcal{L}\text{-sentence})$$

generate a Hausdorff topology on X .

Show that the topology is compact.