Math 557 Sep 22

The Model Existence Theorem

Key Concepts

- Extend T to a Henkin theory T_H, then complete it to a theory T'.
- T' is a complete Henkin theory for the extended language $\mathcal{L}_H$.
- For T', its term model $\mathcal{A}_{T'}$ satisfies
$$\mathcal{A}_{T'} \models \sigma \iff T' \vdash_{\mathcal{L}_H} \sigma$$
for all $\mathcal{L}_H$-sentences σ.

Problems

Exercise 0.1.
How do we obtain a model for T from a model for T'?

Exercise 0.2.
Why is $\mathcal{A}_{T'}$ countable if $\mathcal{L}$ is countable?

Exercise 0.3.
Let X be the set of all maximally consistent $\mathcal{L}$-theories. Recall that the sets
$$\langle \sigma \rangle = \{T \in X : \sigma \in T\} \quad (\sigma \ \mathcal{L}\text{-sentence})$$
generate a Hausdorff topology on X.

Show that the topology is compact.