

# Math 557 August 27

## Problems

### Exercise 0.1.

The set of  $L$ -terms  $\mathcal{T}^{\mathcal{L}}$  is defined as the *smallest* set (with respect to  $\subseteq$ ) containing variables and constant symbols and that is closed under function symbol application.

- Why do this set exist?
- Does a similar argument work for the set of  $\mathcal{L}$ -formulas?
- Find an example of a (non-empty) family of sets and a property  $P$  (applicable to the sets in the family) such that a  $\subseteq$ -smallest set with property  $P$  does not exist.

### Exercise 0.2.

Prove the **Readability Lemma**: For any term  $t$ , exactly one of the following holds.

1. There exists a  $v \in \mathcal{V}$  such that  $t \equiv v$ .
2. There exists a  $c \in \mathcal{C}^{\mathcal{L}}$  such that  $t \equiv c$ .
3. There exists a number  $n \in \mathbb{N}$ , a function symbol  $f \in F_n^{\mathcal{L}}$ , and terms  $t_1, \dots, t_n$  such that  $t \equiv ft_1 \dots t_n$ .

### Exercise 0.3.

Show that no proper initial segment of a term is a term. (*Use induction on the length of a term.*)

Deduce **unique readability**: The decomposition in (3.) above is unique.

### ! Take-home Problem

Prove **unique readability** for the set of  $\mathcal{L}$ -formulas.

### Exercise 0.4.

The set  $C$  is defined as the smallest set of finite strings over the alphabet  $\{\times, \circ\}$  such that

- (C1)  $\times \in C$ ,
- (C2)  $\circ \in C$ ,
- (C3) if  $s, t \in C$  and the last symbol in  $s$  differs from the first symbol in  $t$ , then  $st \in C$  (where  $st$  is the concatenation of  $s$  and  $t$ ).

Formulate the property of unique readability for the system (C1)-(C3) and argue that it does not hold for this system.

### Exercise 0.5.

Let  $\mathcal{L} = (0, <)$  be a language with one constant symbol  $0$  and one binary relation symbol  $<$ . In the following, for sake of (human) readability, we write  $0 < v_1$  instead of  $< 0v_1$ .

Consider the expression

$$\phi \equiv (\exists v_2 \ v_1 < v_2 \wedge \exists v_1 \ v_2 < 0).$$

- Is this a correct  $\mathcal{L}$ -formula?
- Characterize every variable occurrence in  $\phi$  as free or bound.