Statistics & Probability | 3rd Quarter

WW5

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A population consists of the five measurements 2, 6, 8, 0, and 1.

1. What are the mean (μ) and standard deviation (σ) of the population?

$$\mu = 3\frac{2}{5} = 3.4$$
$$\sigma = 3.07$$

2. How many different samples of size $n=2$ can be drawn from the population? List them with their corresponding means.

Sample	Mean
2, 6	4
2, 8	5
2, 0	1
2, 1	1.5
6, 8	7
6, 0	3
6, 1	3.5
8, 0	4
8, 1	4.5
0, 1	0.5

3. Construct the sampling distribution of the sample means.

Sample Mean $\bar{X}$	**Frequency**	**Probability** $P(\bar{X})$
4	2	1/5
5	1	1/10
1	1	1/10
1.5	1	1/10
7	1	1/10
3	1	1/10
3.5	1	1/10
4.5	1	1/10
0.5	1	1/10

4. What is the mean of ($\mu_{\bar{X}}$) of the sampling distribution of the sample means? Compare this to the mean of the population.

$\bar{X}$	$P(\bar{X})$	$\bar{X} \cdot P(\bar{X})$
4	$\frac{1}{5}$	0.80
5	$\frac{1}{10}$	0.50
1	$\frac{1}{10}$	0.10
1.5	$\frac{1}{10}$	0.15
7	$\frac{1}{10}$	0.70
3	$\frac{1}{10}$	0.30
3.5	$\frac{1}{10}$	0.35
4.5	$\frac{1}{10}$	0.45
0.5	$\frac{1}{10}$	0.05

$\mu_{\bar{X}} = 3.4$

$\mu = 3.4$

They are the same and equal.

5. What is the standard deviation ($\sigma_{\bar{X}}$) of the sampling distribution of the sample means? Compare this to the standard deviation of the population.

$\bar{X}$	$P(\bar{X})$	$\bar{X}-\mu$	$(\bar{X}-\mu)^2$	$P(\bar{X})\cdot(\bar{X}-\mu)^2$
4	$\frac{1}{5}$	0.6	0.36	0.072
5	$\frac{1}{10}$	1.6	2.56	0.256
1	$\frac{1}{10}$	-2.4	5.76	0.576
1.5	$\frac{1}{10}$	-1.9	3.61	0.361
7	$\frac{1}{10}$	3.6	12.96	1.296
3	$\frac{1}{10}$	-0.4	0.16	0.016
3.5	$\frac{1}{10}$	0.1	0.1	0.010
4.5	$\frac{1}{10}$	1.1	1.21	0.121
0.5	$\frac{1}{10}$	-2.9	8.41	0.841

$\sigma=\sqrt{\sum(X-\mu)^2\cdot P(X)}$
$\sigma=\sqrt{3.549}$
$\sigma_{\bar{X}}=1.88$
$\sigma=3.07$

They are different, and the standard deviation of the population is higher than that of the sampling distribution of the sample means.

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Given the population 1, 3, 4, 6, and 8. Suppose samples of size 3 are drawn from this population.

1. What are the mean (μ) and standard deviation (σ) of the population?

$\mu=4.4$
$\sigma=2.42$

2. How many different samples of size $n=3$ can be drawn from the population? List them with their corresponding means.

Sample	Mean
1,3,4	2.67
1,3,6	3.33
1,3,8	4
1,4,6	3.67
1,4,8	4.33
1,6,8	5
3,4,6	4.33
3,4,8	5
3,6,8	5.67
4,6,8	6

3. Construct the sampling distribution of the sample means.

Sample Mean $\bar{X}$	Frequency	Probability $P(\bar{X})$
2.67	1	$\frac{1}{8}$
3.33	1	$\frac{1}{8}$
4	1	$\frac{1}{8}$
3.67	1	$\frac{1}{8}$
4.33	2	$\frac{1}{4}$
5	2	$\frac{1}{4}$
5.67	1	$\frac{1}{8}$
6	1	$\frac{1}{8}$

4. What is the mean ($\mu_{\bar{X}}$) of the sampling distribution of the sample means? Compare this to the mean of the population.

$\bar{X}$	$P(\bar{X})$	$\bar{X} \cdot P(\bar{X})$
2.67	$\frac{1}{8}$	0.3338
3.33	$\frac{1}{8}$	0.4162
4	$\frac{1}{8}$	0.5000
3.67	$\frac{1}{8}$	0.4588
4.33	$\frac{1}{4}$	1.0825
5	$\frac{1}{4}$	1.2500
5.67	$\frac{1}{8}$	0.7088
6	$\frac{1}{8}$	0.7500

$\mu_{\bar{X}}=5.5$
$\mu=4.4$

5. What is the standard deviation ($\sigma_{\bar{X}}$) of the sampling distribution of the sample means? Compare this to the standard deviation of the population.

$\bar{X}$	$P(\bar{X})$	$(\bar{X}-\mu)$	$(\bar{X}-\mu)^2$	$P(\bar{X})\cdot(\bar{X}-\mu)^2$
2.67	$\frac{1}{8}$	-2.83	8.0089	1.0011125
3.33	$\frac{1}{8}$	-2.17	4.7089	0.5886125
4	$\frac{1}{8}$	-1.5	2.25	0.28125
3.67	$\frac{1}{8}$	-1.83	3.3489	0.4186125
4.33	$\frac{1}{4}$	-1.17	1.3689	0.342225
5	$\frac{1}{4}$	-0.5	0.25	0.0625
5.67	$\frac{1}{8}$	0.17	0.0289	0.0036125
6	$\frac{1}{8}$	0.5	0.25	0.03125

$$\sigma_{\bar{X}}=\sqrt{\sum P(\bar{X})\cdot(\bar{X}-\mu)^2}$$
$$\sigma_{\bar{X}}=\sqrt{2.729175}$$
$$\sigma_{\bar{X}}=1.652$$
$$\sigma=2.42$$

Obviously, the standard deviation of the population is significantly bigger than the standard deviation of the sampling distribution of the samples.