

Chapter 1

Electric Forces and Fields

Electric and Magnetic Phenomena

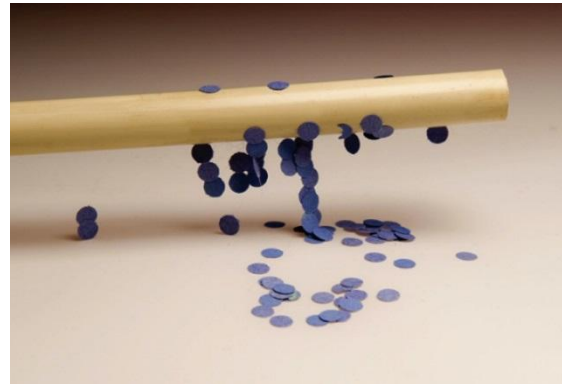
- Electric and magnetic forces act on electric charges and currents
- Electric charges and currents also act as sources of electric and magnetic fields
- Electricity and magnetism are really a single unified phenomena
 - Called electromagnetism
 - Leads to a theory of electromagnetic radiation
 - Light is an example
 - Electromagnetism forms the basis for the study of optics

Observational Facts

- Discovery of electricity is generally credited to the Greeks
 - About 2500 years ago
- They observed electric charges and the forces between them in many situations
- Used amber
 - A type of dried tree sap
 - After amber is rubbed with a piece of animal fur, the amber can attract small pieces of dust

Observational Facts, cont.

- The Greek word for amber was “elektron” from which we get the words electron and electricity (A)
- Modern experiments use plastic and paper (B)
- The force occurs even when the plastic and paper are not in contact (C)

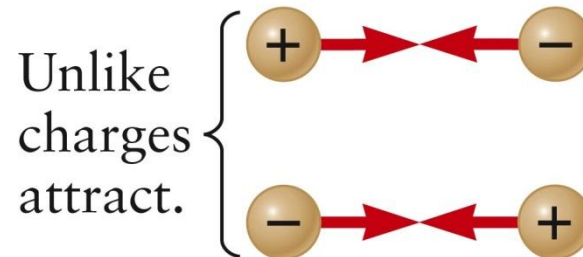
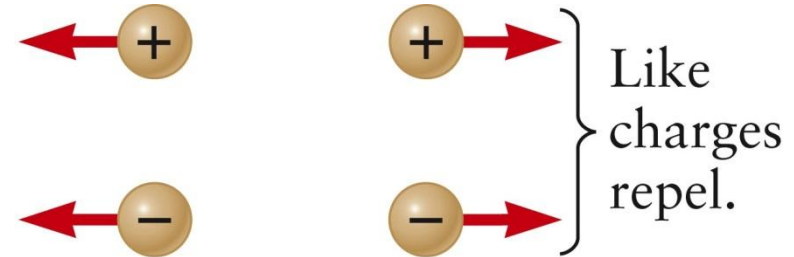


Basic Properties of Charges

- There are two types of electric charges
 - They are called positive and negative
 - Protons have positive charges
 - Electrons have negative charges
- Charge comes in quantized units
 - All protons carry the same amount of charge, $+e$
 - All electrons carry the same amount of charge, $-e$

Basic Properties, cont.

- Like charges repel each other, unlike charges attract
 - Like charges – both positive or both negative
 - Unlike charges – one positive and one negative
 - The “like” and “unlike” apply to the signs of the charges, not their magnitudes



Basic Properties, final

- Charge is conserved
 - The total charge on an object is the sum of all the individual charges carried by the object
 - Charge can move from place to place, and from one object to another, but the total charge of the universe does not change

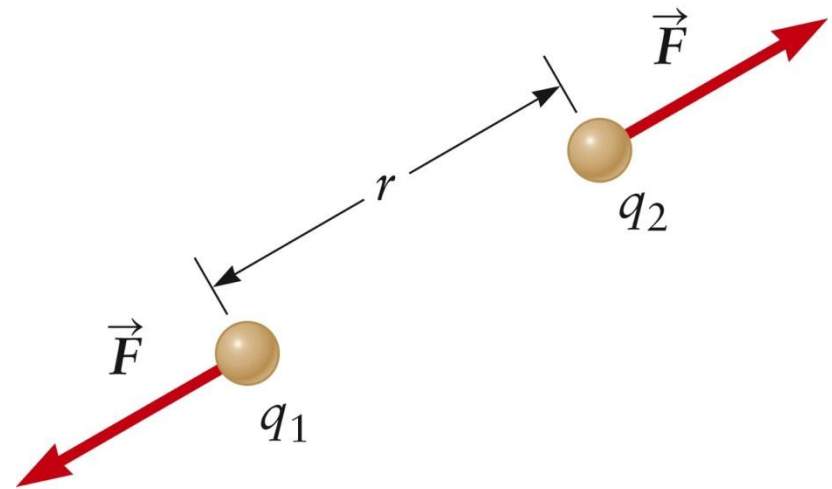
What is Electric Charge?

- Charge is a fundamental property of matter
 - The amount of charge on a particle determines how the particle reacts to electric and magnetic fields
 - An actual definition is not possible
- The SI unit of charge is the **Coulomb**
 - In honor of Charles de Coulomb
- Electron charge = $-e = -1.6 \times 10^{-19} \text{ C}$
- Proton charge = $+e = +1.6 \times 10^{-19} \text{ C}$
- The symbol e is used to denote the magnitude of the charge on an electron or proton
- The symbols q and Q are used to denote charge in general

Electric Forces and Coulomb's Law

- Electric force can be attractive or repulsive
- It is very large
- Assume two charged particles can be modeled as point particles
- The magnitude of the electric force between the two particles is given by Coulomb's Law

Like charges q_1 and q_2 repel.



COULOMB'S LAW:

$$|\vec{F}| = \frac{kq_1q_2}{r^2} = \frac{q_1q_2}{4\pi\epsilon_0r^2}$$

Coulomb's Law

- Coulomb's Law says the magnitude of the force between two electric charges is given by

$$F = k \frac{q_1 q_2}{r^2}$$

- The constant $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$
- The direction of the force is along the line that connects the two charges
- A repulsive force will give a positive value for F
 - Between like charges
- An attractive force will give a negative value for F
 - Between unlike charges

Coulomb's Law – Alternative

- Another way to write Coulomb's Law is

$$F = \frac{q_1 q_2}{4\pi \epsilon_o r^2}$$

- ϵ_o is another constant
- It is called the permittivity of free space
- It is equal to $8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{M}^2$
- The constants are related through

$$k = \frac{1}{4\pi\epsilon_o}$$

- Either form can be used to calculate the force between two particles

Features of Coulomb's Law

- The form of Coulomb's Law is similar to Newton's Law of Universal Gravitation
 - Both laws exhibit a $1/r^2$ dependence on the separation between particles
 - A negative charge can move in a circular orbit around a positive charge
 - This was the early model for the atom
 - Gravity is always attractive
 - Electric forces may be attractive or repulsive

Features of Coulomb's Law, cont.

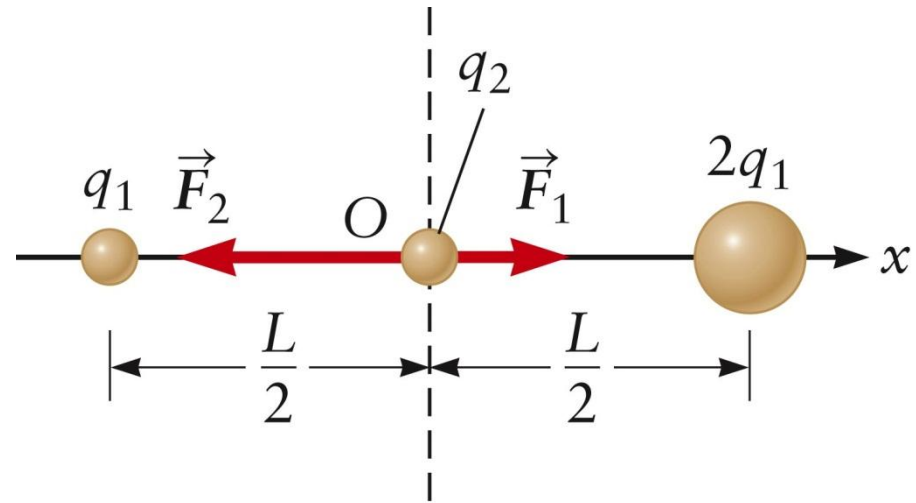
- The value of F given by Coulomb's Law is the magnitude on each of the particles
 - A force of F is exerted on q_1 and a force of equal magnitude and opposite direction is exerted on q_2
 - Newton's Third Law action-reaction pair

Size of the Electric Force

- Assume you have two boxes each containing one gram of electrons
- There would be 1.1×10^{27} electrons in each box
- The force between the boxes would be 3×10^{26} N
 - This is almost a million times larger than the force between the Sun and the Earth
- Ordinary matter consists of approximately equal numbers of electrons and protons
 - With equal numbers, $Q_{\text{total}} = 0$ and $F = 0$

Superposition of Forces

- When there are more than two charges in a problem, the ***principle of superposition*** must be used
- Find the forces on the charge of interest due to all the other forces
- Add the forces as vectors



Problem Solving Strategy

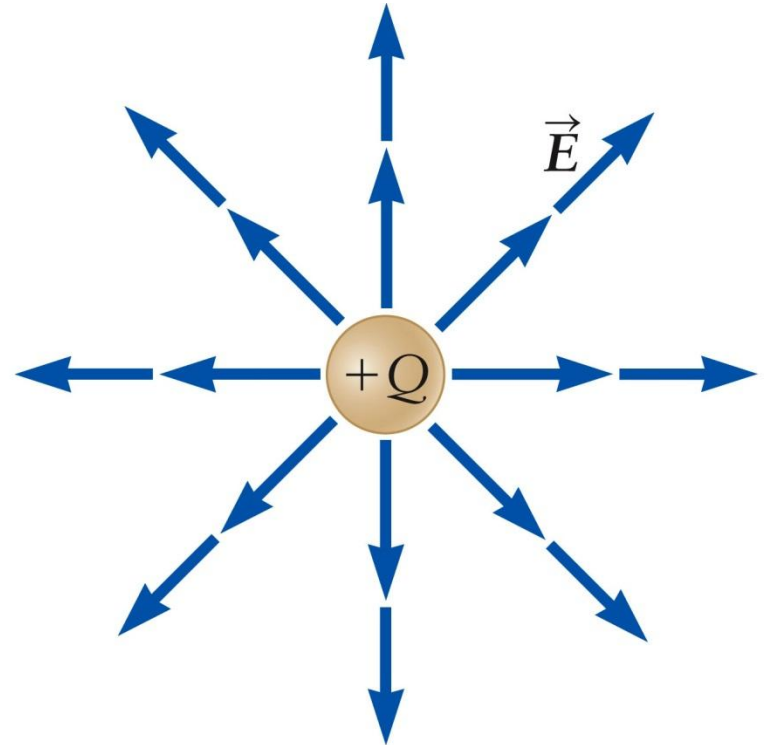
- ***Recognize the principle***
 - The electric force can be found using Coulomb's Law
 - The principle of superposition may also be needed
- ***Sketch the problem***
 - Construct a drawing and show the location and charge of each object
 - Include a coordinate system
 - Include the directions of all the electric forces acting on the particle of interest

Problem Solving Strategy, cont.

- ***Identify the relationships***
 - Use Coulomb's Law to find the magnitudes of the forces acting on the particle of interest
- ***Solve***
 - The total force on a particle is the sum of all the individual forces
 - Add the forces as vectors
 - It is usually easier to work in terms of the components along the coordinate system
- ***Check***
 - Consider what the answer means
 - Check if the answer makes sense

Electric Field

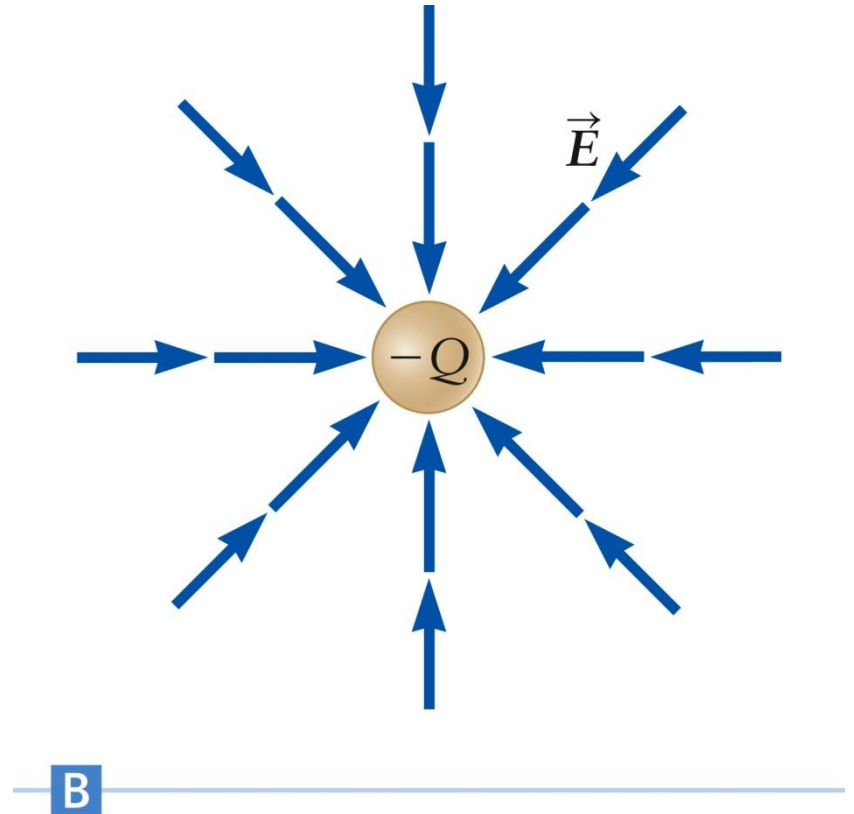
- An electric field gives another explanation of electric forces
- The presence of a charge produces an electric field
 - Shown by the arrows in the figure
- The electric field is similar to the gravitational field near an isolated mass



A

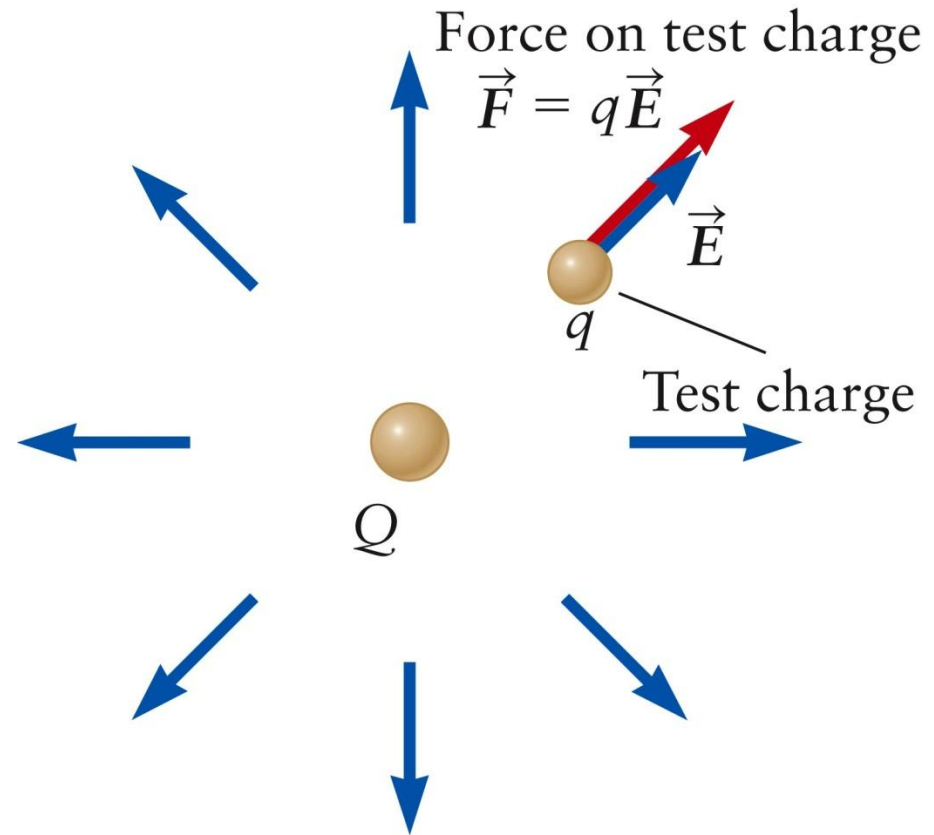
Electric Field, cont.

- A positive charge produces field lines that radiate outward
- For a negative charge the field lines are directed inward, toward the charge
- The electric field is a vector and denoted by \vec{E}



Electric Field and Test Charge

- Consider a point in space where the electric field is \vec{E}
- If a charge q is placed at the point, the force is given by $\vec{F} = q\vec{E}$
 - The charge q is called a test charge
- By measuring the force on the test charge, the magnitude and direction of the electric field can be inferred



Electric Field, cont.

- The electric force is either parallel or antiparallel to the electric field
 - Parallel if q is positive and antiparallel if q is negative
- SI units of the electric field are N/C
- Coulomb's Law can be used to find the magnitude of the electric field, where Q is the charge producing the field and q is the test charge

$$F = \frac{kQq}{r^2} = qE$$

$$E = \frac{kQ}{r^2}$$

Electric Field, final

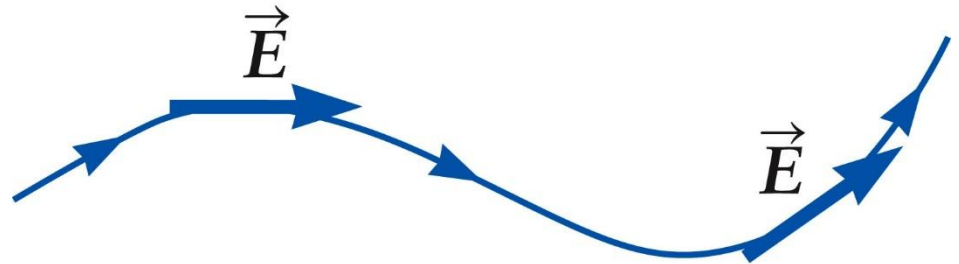
- The direction of the electric field is along the line connecting the charge producing the field to the point where the field is measured
- The electric field is directed away from Q when Q is positive
- The electric field is directed inward toward Q when Q is negative

Importance of the Electric Field

- An electric field is present even when there is no second (or test) charge present to experience the electric force
- Any charge or collection of charges will produce an electric field
- The electric field helps explain how the Coulomb force can act between two charges that are separated by large distances
- The electric field is essential for understanding electromagnetic waves

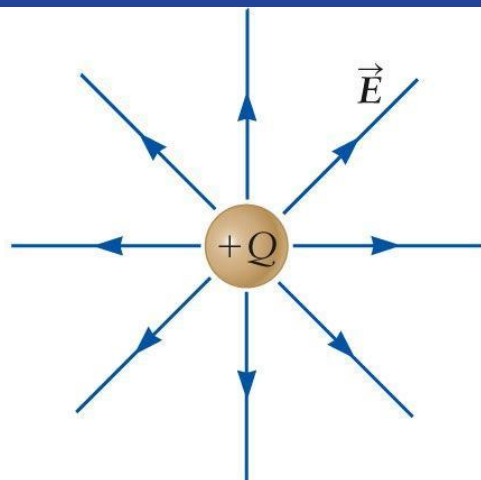
Drawing Electric Field Lines

- Another way to visualize an electric field is with electric field lines
- Field lines are a set of continuous lines that are always parallel to the electric field

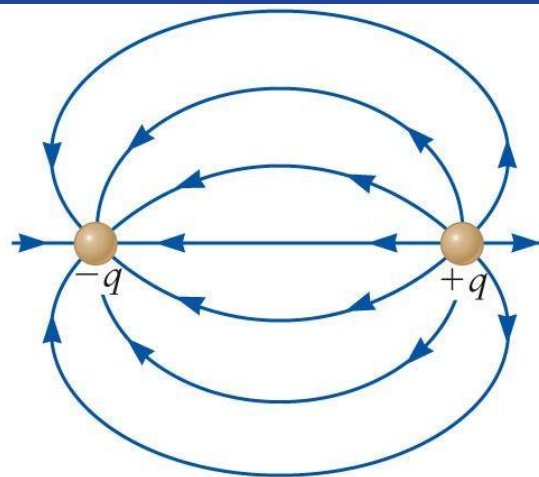


A

Examples of Electric Field Lines



B



C

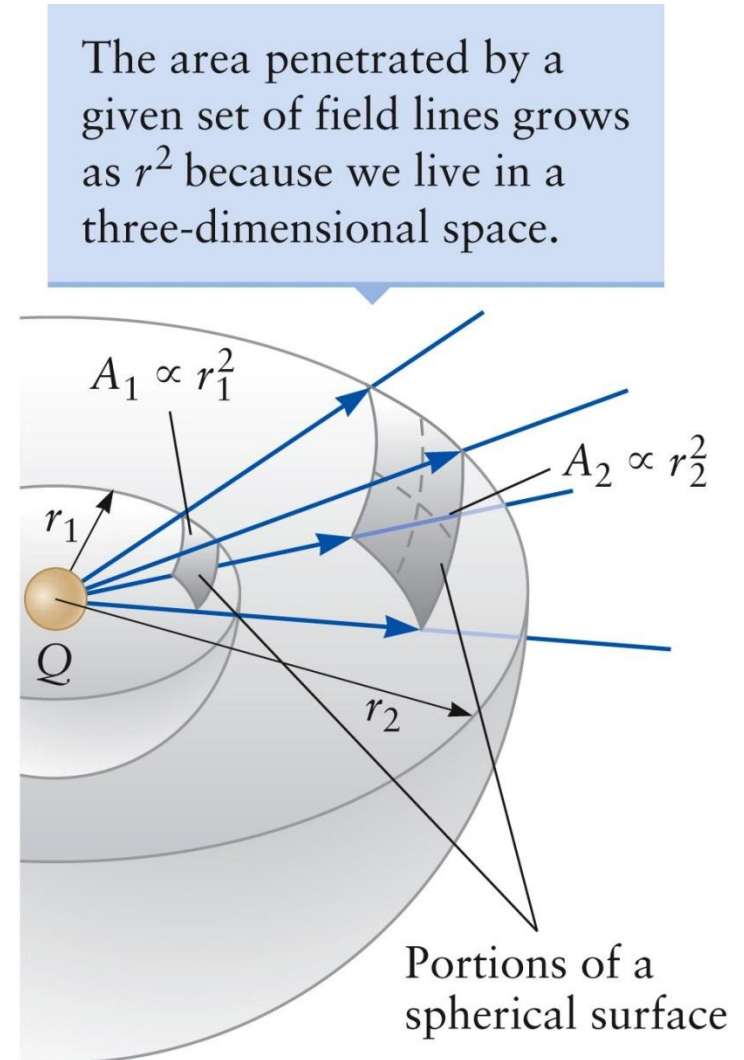
- The plot of the field lines does not show the magnitude of the field directly
- Changes in field strength can be inferred from the spacing of the field lines
 - The lines are most closely spaced where the field is the largest

Inverse Square Laws

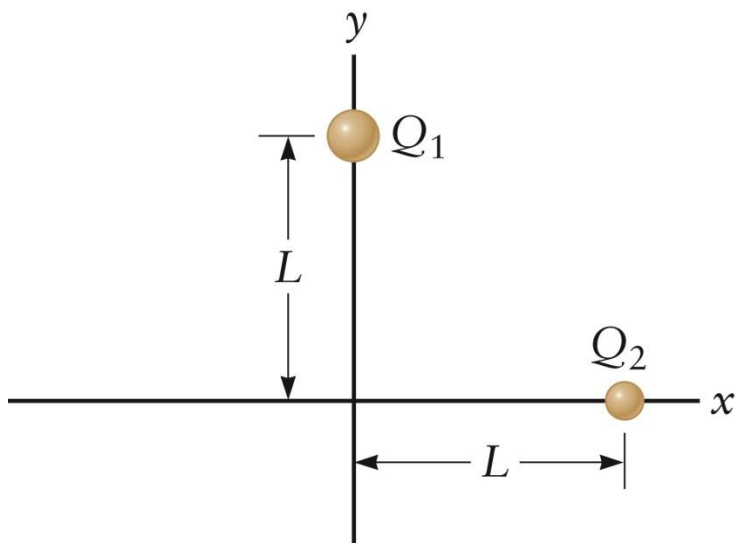
- The force between two point charges falls off as $1/r^2$
 - Where r is the separation between the charges
- The electric field also falls off as $1/r^2$
 - The electric force also obeys an inverse square law
- The charged particle “sets up” field lines in its neighborhood
 - The density of the field lines is proportional to the amount of charge on the particle
- The electric field produces a force on a nearby test charge and on any other nearby charges
 - The magnitude of the force is proportional to the density of field lines at the test charge

Inverse Square Laws, cont.

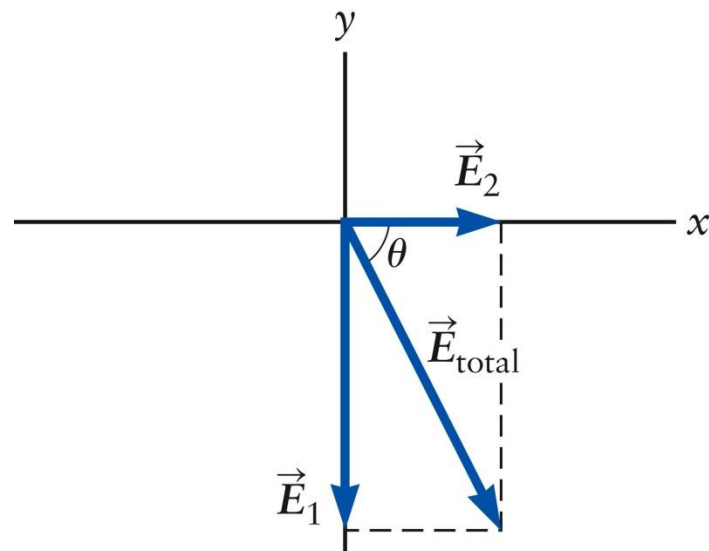
- The electric field lines emanate outward from a point charge
- They intercept larger and larger surface areas
- The surfaces are spherical
 - Their areas increase as $A \propto r^2$
 - The number of field lines per unit area falls as $1/r^2$



Electric Fields and Multiple Charges



A



B

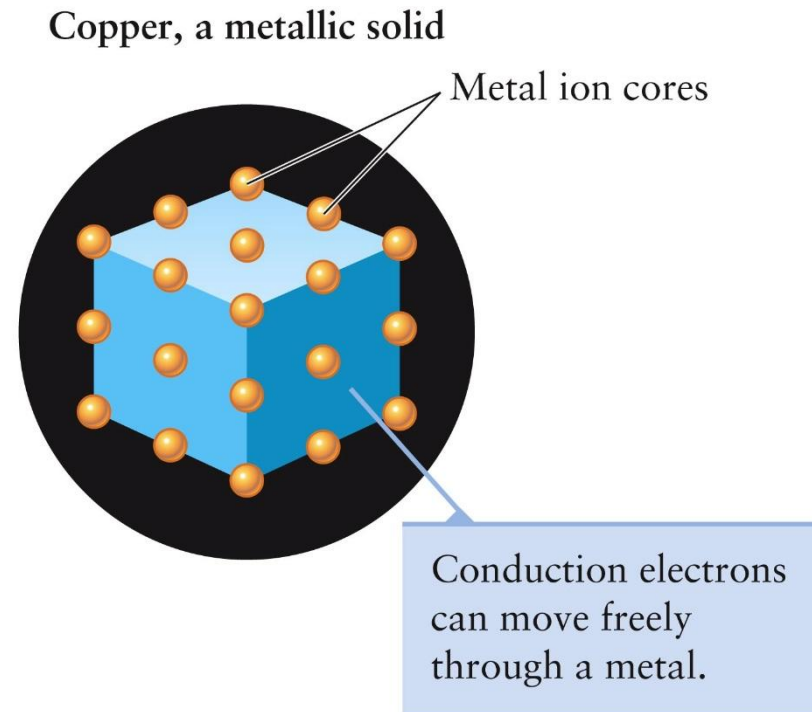
- To find the electric field due to multiple charges use the principle of superposition
 - Find the electric fields due to each charge
 - Add them as vectors

Drawing Electric Field Lines

- Field lines must always begin on positive charges
- Field lines must always end on negative charges
- Electric fields also obey the superposition principle
 - When adding the fields from multiple charges, always add them as vectors

Conductors

- Each atom by itself is electrically neutral
 - Equal numbers of protons and electrons
 - This example is copper
- When these atoms come together to form the piece of metal electrons are freed

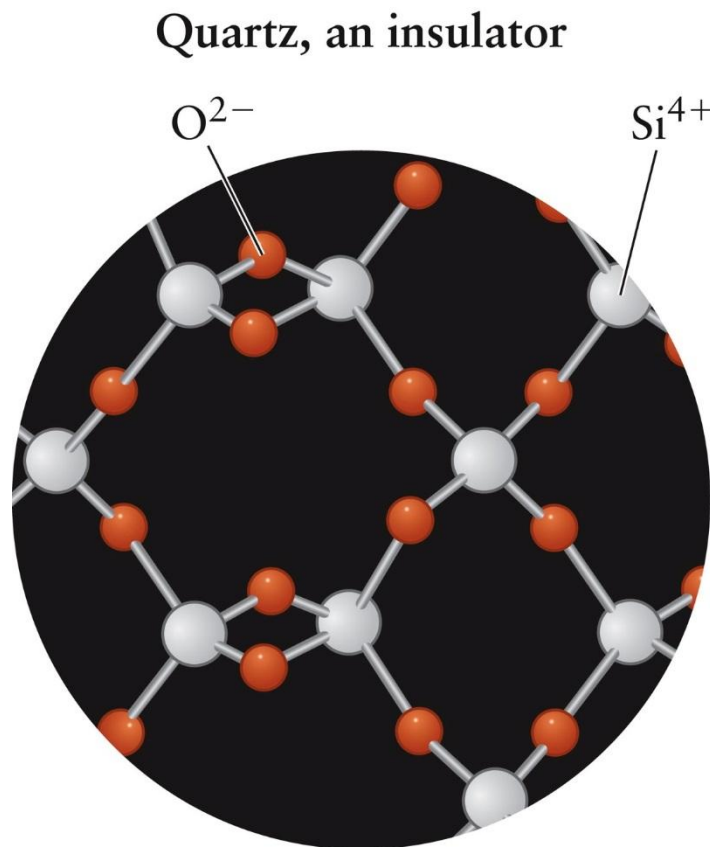


Conductors, cont.

- These electrons move freely through the entire piece of metal
- These electrons are called ***conduction electrons***
- The electrons leave behind positively charged ion cores that are bound and not mobile
- A piece of metal can also accept extra electrons or release some of its conduction electrons so the entire object can acquire a net positive or negative charge

Insulators

- In ***insulators*** the electrons are not able to move freely through the material
- Examples include quartz, plastic and amber
- This example is quartz (SiO_2)



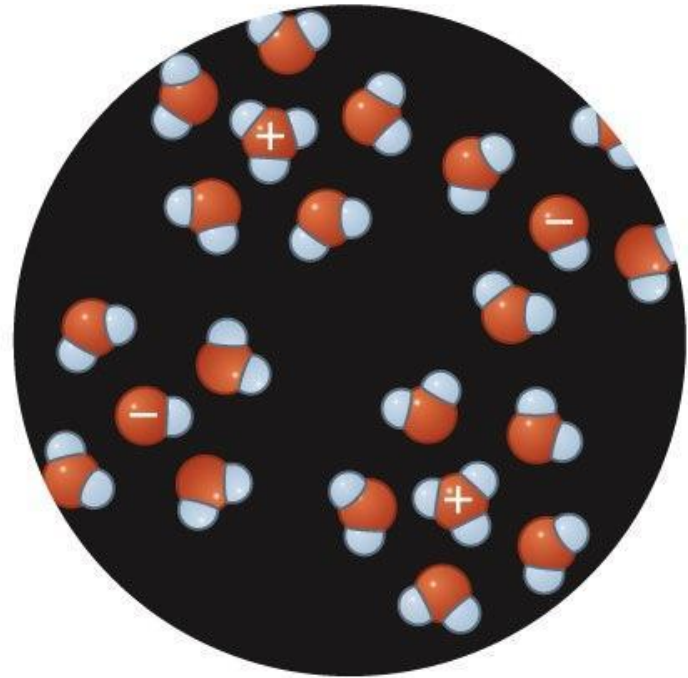
Insulators, cont.

- Electrons cannot escape from these ions
- There are no conduction electrons available to carry charge through the solid
- If extra electrons are placed on an insulator, they tend to stay in the place where initially placed

Liquids and Gases

- The total charge is zero and the sample is neutral
- A few molecules always dissociate into free ions
- These ions can carry charge from place to place similarly to the conduction electrons
- This example is water, but gases are similar

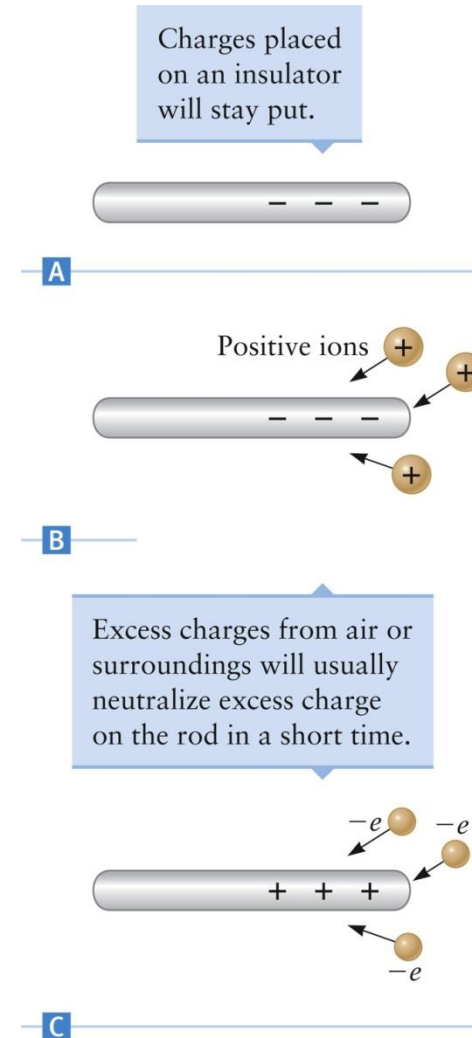
Water, a liquid



C

Charging an Insulator

- A few electrons are placed on the insulator
- They will tend to stay where they are placed
- The insulator will eventually be neutralized
 - The excess electrons will attract free ions from the air
 - They will neutralize the original charge



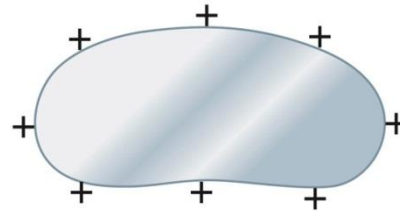
Excess Charge on a Metal

- Electrons can move easily through a metal
- Excess electrons will be distributed on the surface of the metal
- Eventually, all the charge carriers will come to rest and be in static equilibrium

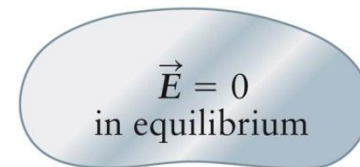
All excess charge sits on the surface of a metal.



A



B

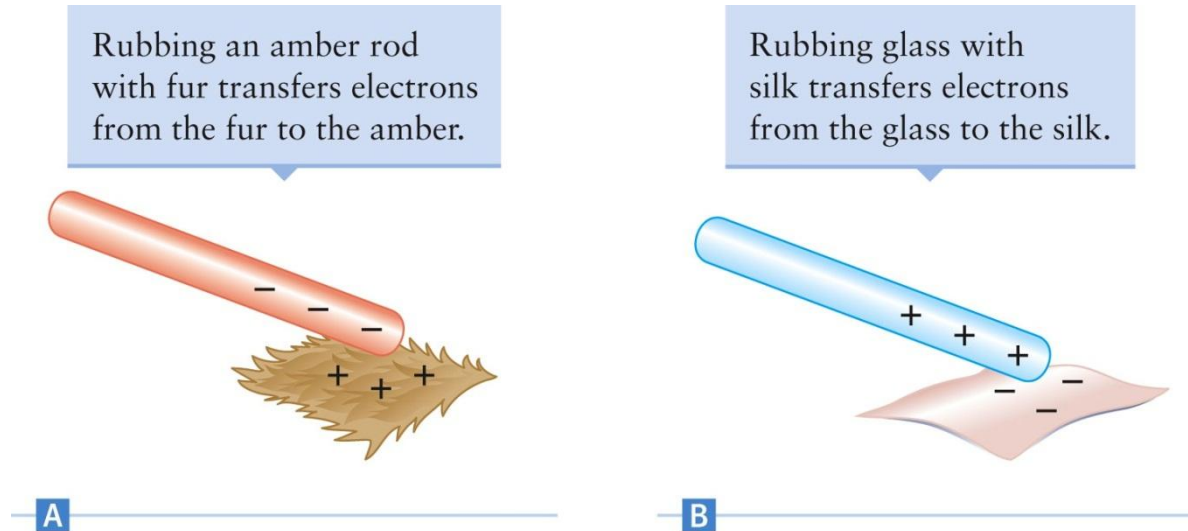


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Metals, cont.

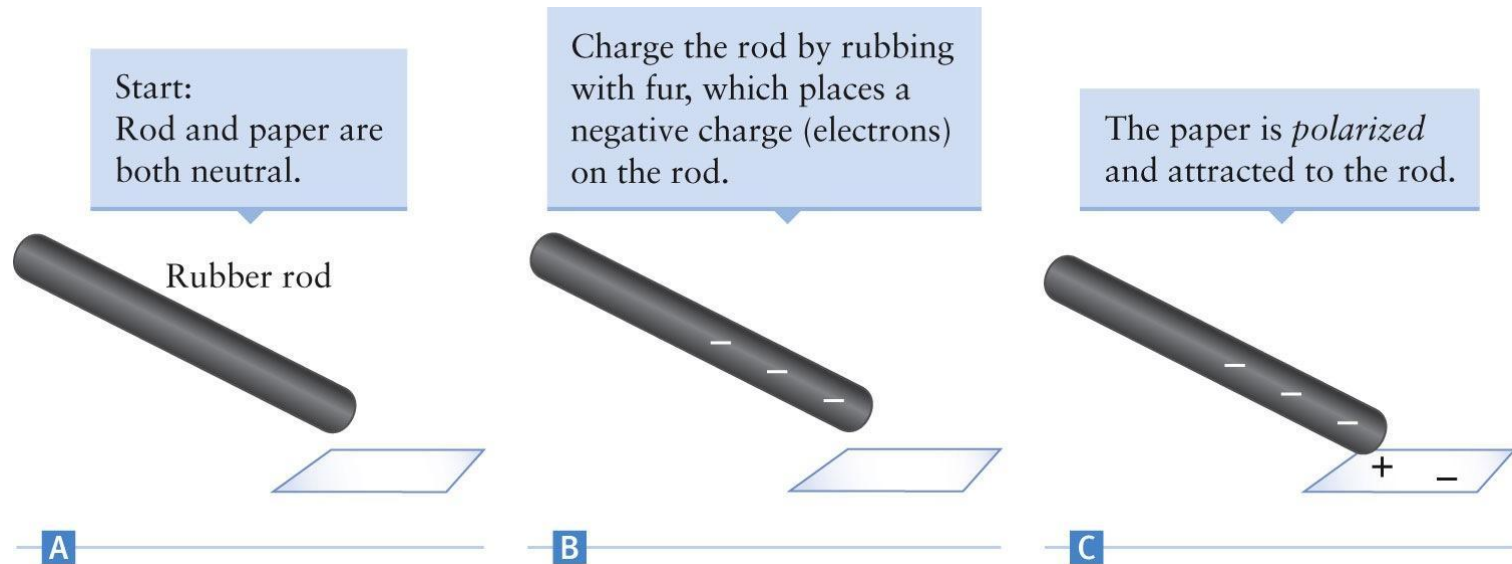
- For a metal in equilibrium
 - Any excess electrons must be at the surface
 - The electric field is zero inside a metal in equilibrium
- The net charge could also be positive
 - Electrons would have been removed from the metal
 - The deficit of electrons will be at the surface, so the surface will have a net positive charge
 - This does not correspond to placing positive ions on the metal

Charging an Object by Rubbing



- The act of rubbing causes some charge to be transferred from one material to another
 - Example: when rubbing amber with fur, electrons are moved from the fur to the amber
 - The amber acquires a net negative charge
 - The fur is left with a net positive charge
- Applies to many combinations of materials

Polarization

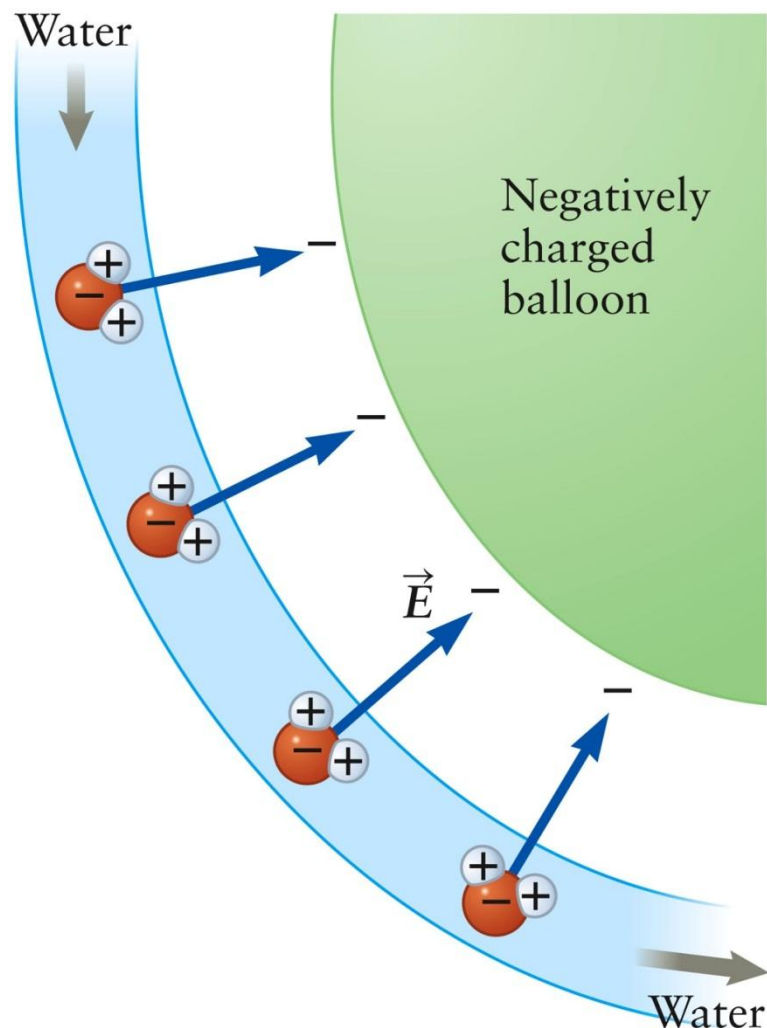


- The rod and paper are both neutral
- The rod is rubbed by the fur, obtaining a negative charge
- The presence of the rod causes the electrons in the paper to be repelled and the positive ions are attracted
- The paper is said to be ***polarized***

Polarization, cont.

- The paper is still electrically neutral
- The negative side of the paper is repelled
- The positive side of the paper is closer to the rod so it will experience a greater electric force
- The net effect is that the positive side of the paper is attracted to the negative rod
- There can be an electric force on an object even when the object is electrically neutral, provided the object is polarized

Polarization, Balloon and Water Example



B

Electrical “Ground”

- The idea of *electrical ground* plays an important role in many situations
- If you watch the excess electrons on a charged rod you would see them eventually flow to the ground
- The ground is generally moist, so it conducts well
- Because the ground is everywhere, it provides a common path that excess charge can use to flow from one spot to another
- The term *ground* is used to denote a path or destination of the excess charges, even if dirt is not actually involved

Charging by Contact

- Assume a negatively charged rubber rod
- Bring the rod into contact with the metal
- Some of the excess electrons will move to the metal
- The metal has been charged by contact

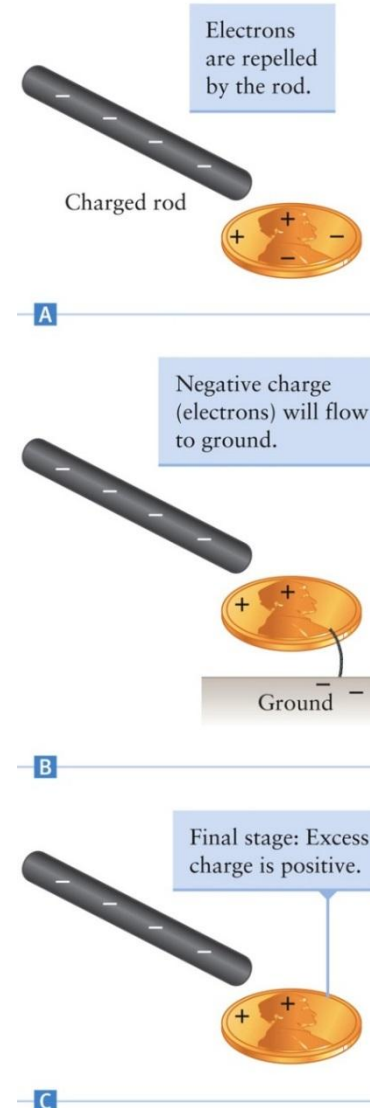
CHARGING AN OBJECT BY CONTACT



Charging by Induction

- Charging by induction makes use of polarization
- The negatively charged rod is first brought near the metal, polarizing the metal
- A connection is made from the piece of metal to electrical ground using a wire

CHARGING BY INDUCTION



Charging by Induction, cont.

- The electrons are able to use the wire to move even farther from the charged rod
- Some of the electrons will move off the original piece of metal and into the electrical ground region
- The final step is to remove the grounding wire
- The original piece of metal will be left with a net positive charge
 - The positive charge is produced by removing electrons

Electric Flux and Gauss' Law

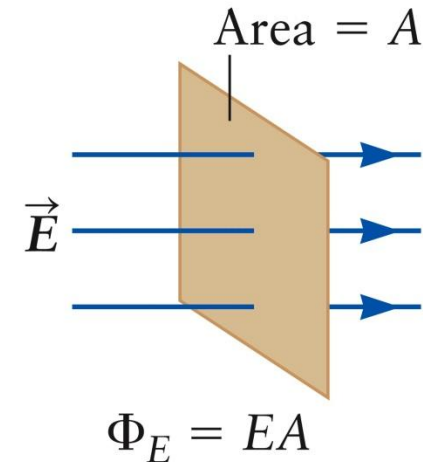
- Gauss' Law can be used to find the electric field of a complex charge distribution
 - Easier than treating it as a collection of point charge and using superposition
- To use Gauss' Law, a quantity called ***electric flux*** is needed
- The electric flux is equal the product of the electric field that passes through a particular surface and the area of the surface
 - The electric flux is denoted by Φ_E

Electric Flux Examples

- Fig. A

- The electric field is perpendicular to the surface of area A

- $\Phi_E = EA$

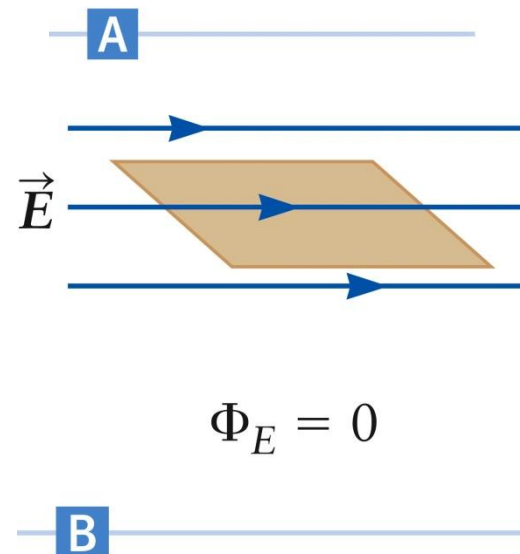


- Fig. B

- The electric field is parallel to the surface of area A

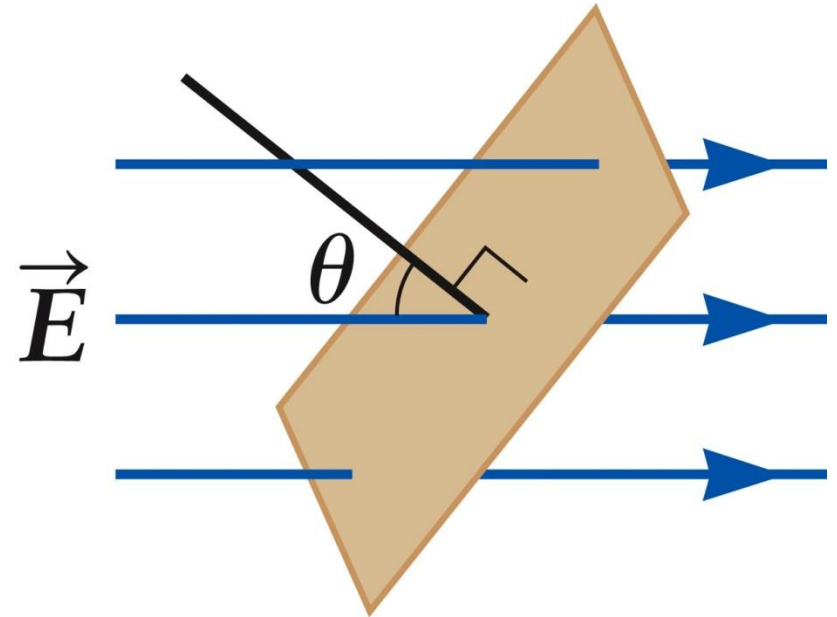
- $\Phi_E = 0$

- No field lines pass through the area



Electric Flux Examples, cont.

- Fig. C
 - The electric field makes an angle with the surface
 - $\Phi_E = E A \cos \theta$
- Flux is a scalar quantity

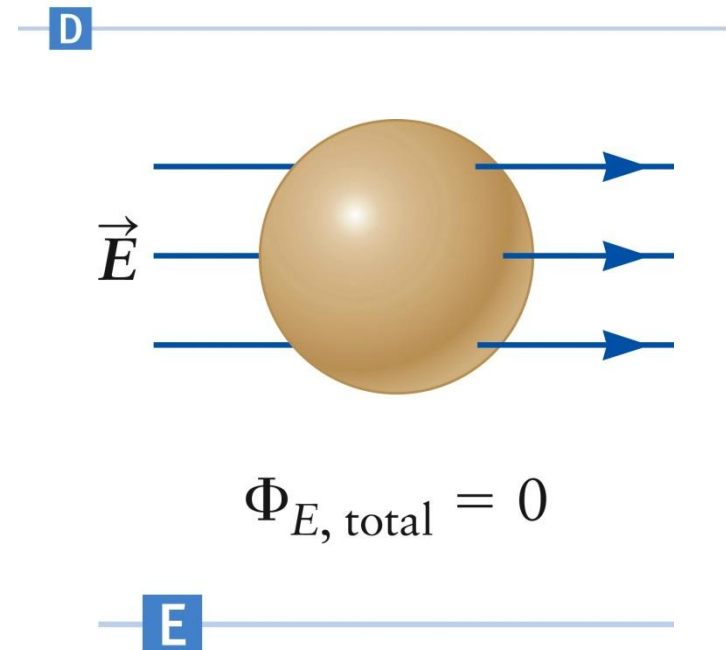
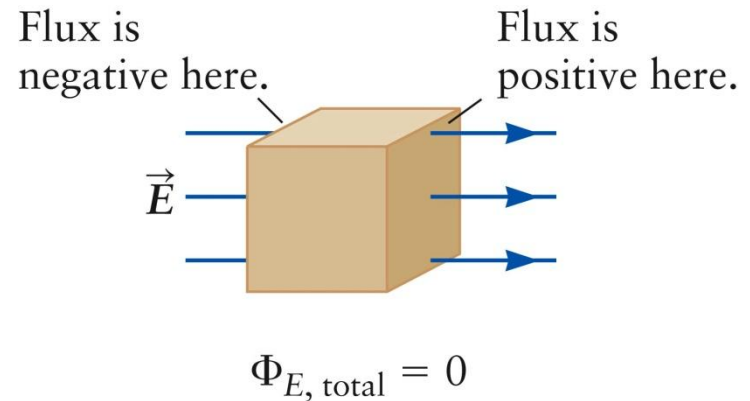


$$\Phi_E = E A \cos \theta$$



Electric Flux Examples, Closed Surface

- The flux is positive if the field is directed out of the region surrounded by the surface and negative if going into the region
- Fig. D and E
 - The total flux is the sum of the contributions of the fields going in and coming out
 - $\Phi_E = 0$



Gauss' Law

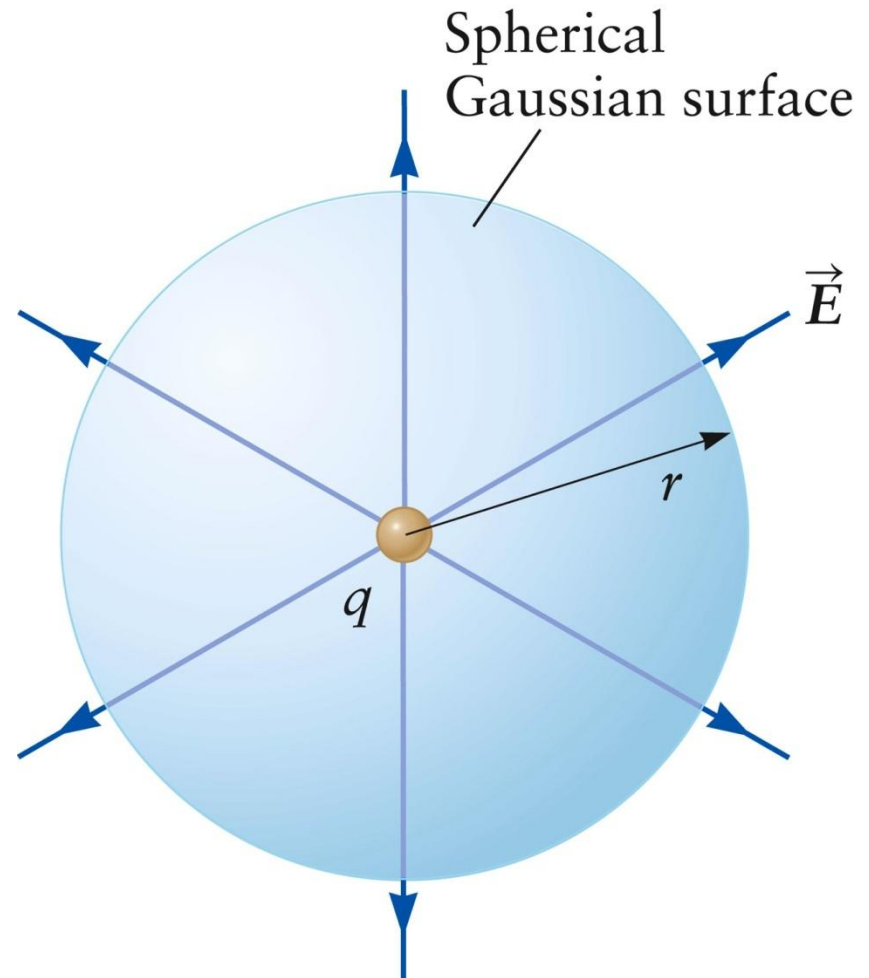
- Gauss' Law says that the electric flux through any closed surface is proportional to the charge q inside the surface

$$\Phi_E = \frac{q}{\epsilon_0}$$

- The constant ϵ_0 is the permittivity of free space
 - This was encountered in one form of Coulomb's Law
- Since the flux depends on the electric field, Gauss' Law can be used to find the field

Gauss' Law: Point Charge

- Choose a ***Gaussian surface***
 - Want a surface that will make the calculation as easy as possible
 - Choose a surface that matches the symmetry of the problem
- For a point charge, the field lines have a spherical symmetry



Gauss' Law: Point Charge, cont.

- The spherical symmetry means that the magnitude of the electric field depends only on the distance from the charge
- The electric field is directed radially
 - Either inward or outward
- A surface that matches this symmetry is a sphere centered on the charge
- Because of the symmetry, the magnitude of the field is the same at all points on the sphere
- The field is perpendicular to the sphere at all points where it intersects the surface

Gauss' Law: Point Charge, final

- Since the field is perpendicular to the area, the flux is the product of the field and the area: $\Phi_E = E A_{\text{sphere}}$
- A_{sphere} is the area of the Gaussian sphere
- With a radius of r , $A_{\text{sphere}} = 4 \pi r^2$
- Therefore, $\Phi_E = 4 \pi r^2 E$
- From Gauss' Law,

$$\Phi_E = 4\pi r^2 E = \frac{q}{\epsilon_0} \quad \text{and} \quad E = \frac{q}{4\pi\epsilon_0 r^2}$$

- This agrees with the result from Coulomb's Law

Problem Solving Strategy

- ***Recognize the principle***
 - Calculate the electric flux through a Gaussian surface
 - The choice of the Gaussian surface is key!
- ***Sketch the problem***
 - Draw the charge distribution
 - Use the symmetry of the distribution to sketch the electric field
 - The Gaussian surface should match the symmetry of the electric field

Problem Solving Strategy, cont.

- ***Identify the relationships***

- Your Gaussian surface should satisfy one or more of the following conditions:
 - The field has constant magnitude over all or much of the surface and makes a constant angle with the surface
 - The most convenient surface is one that is perpendicular to the field at all (or most) points
 - The field may be zero over a portion of the surface
 - The flux through that part of the surface is zero
 - The field may be parallel to some part of the surface
 - The flux through that part of the surface is zero

Problem Solving Strategy, final

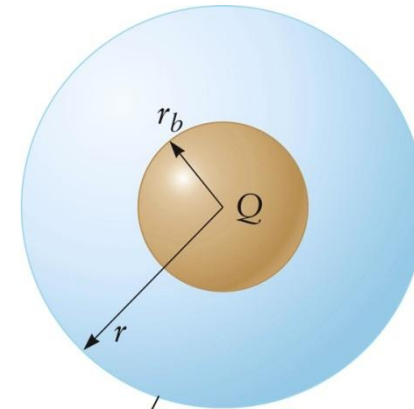
- **Solve**
 - Calculate the total electric flux through the entire Gaussian surface
 - Find the total electric charge inside the surface
 - Apply Gauss' Law to solve for the electric field
- **Check**
 - Consider what your answer means
 - Check that your answer makes sense

Electric Field from Spherical Charge

- Given a uniform spherical ball of charge
- Choose a sphere as a Gaussian surface

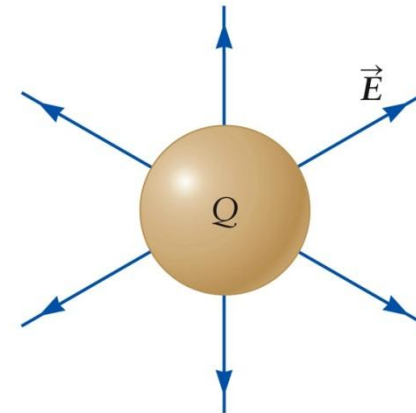
$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$$

- The electric field from any spherical distribution of charge is the same as the field from a point charge with the same total charge
- This applies only outside the ball of charge



Gaussian surface = sphere

A

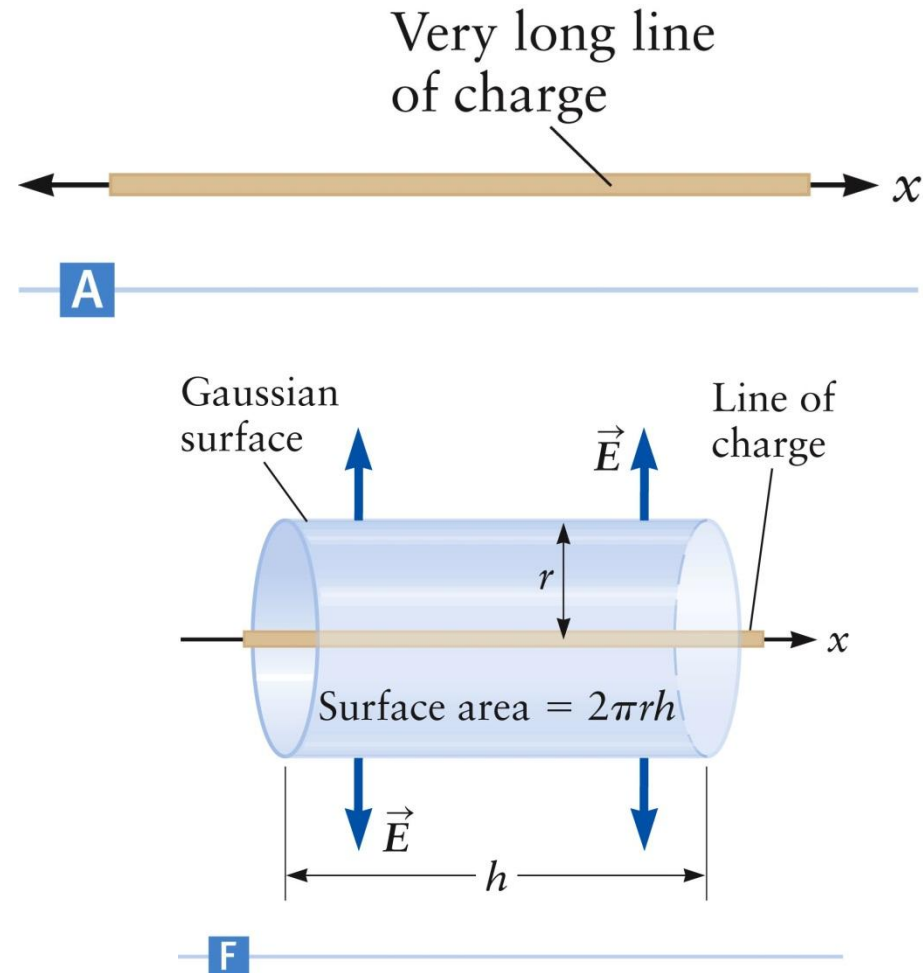


B

Electric Field from Line of Charge

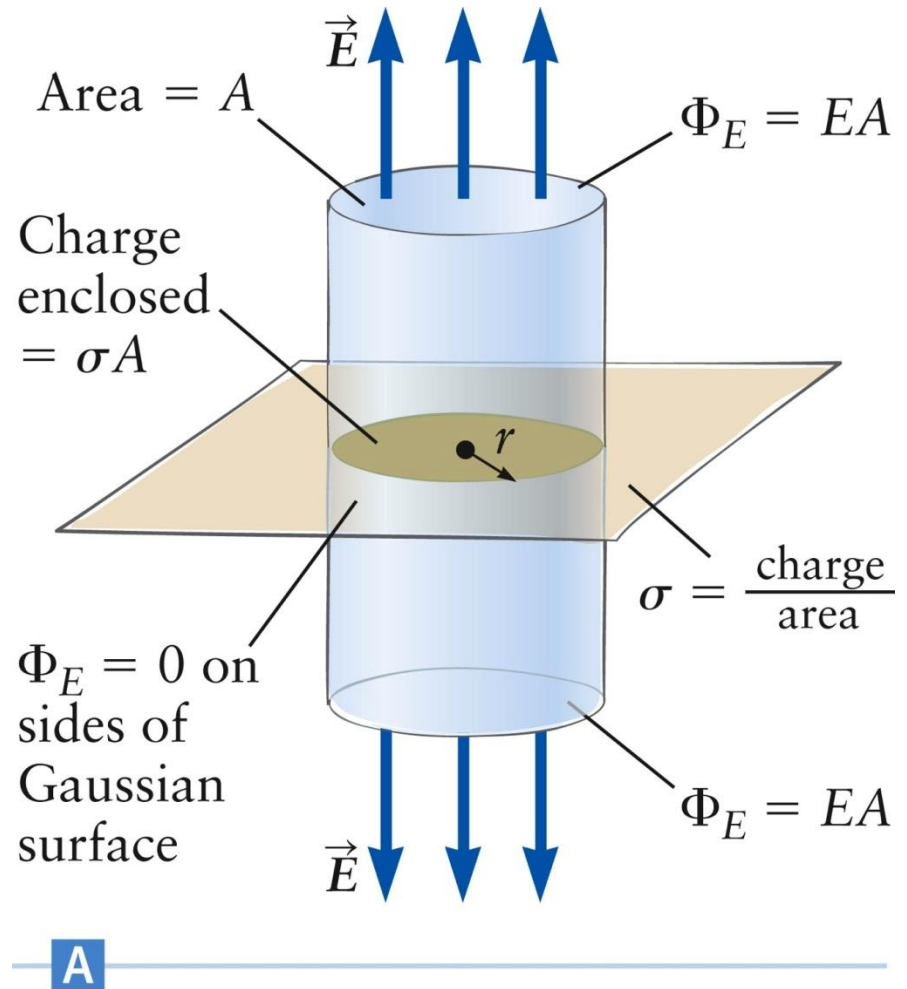
- The line of charge has a total length L and a total charge Q
- The electric field is perpendicular to the line
- Choose a cylinder of radius r for the Gaussian surface

$$E = \frac{Q}{4\pi\epsilon_0 Lr}$$



Electric Field: Flat Sheet of Charge

- The large, flat sheet of charge has a charge per unit area of σ
- Choose a cylinder as the Gaussian surface
- The field through the sides of the surface is zero
- Through each end of the surface, $\Phi_E = EA$
- Since there are two ends, $\Phi_E = 2EA$



Flat Sheet of Charge, cont.

- The total charge is equal to the charge / area multiplied by the cross-sectional area of the cylinder
 - $q = \sigma A$
- Therefore, the electric field is

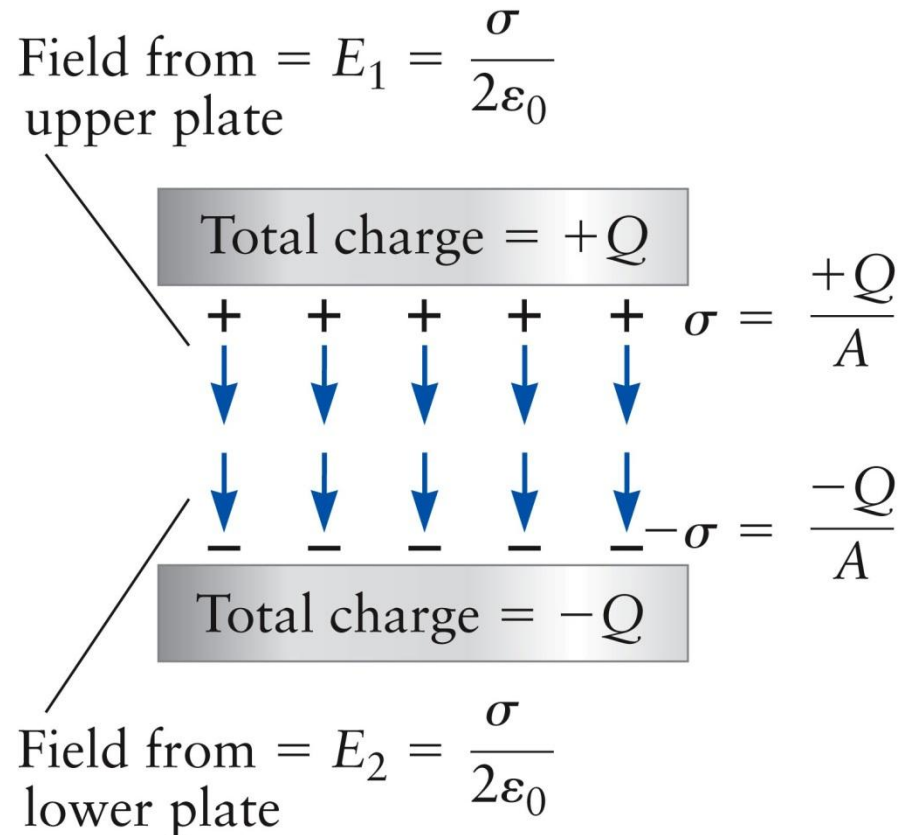
$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

- The electric field is constant and independent of the distance from the sheet of charge

Capacitor

- A capacitor consists of two parallel metal plates that carry charges of $+Q$ and $-Q$
- The excess charge on the two plates will attract each other and draw all the excess charge to the inner sides of the two plates



Capacitor, cont.

- Each plate has an area A and charge densities of $+\sigma = +Q/A$ and $-\sigma = -Q/A$
- The total field is the vector sum of the fields from the individual plates
 - From superposition
- The field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Notes on Gaussian Surfaces

- To make analysis easier, some simplifications have been made in the examples
 - Highly symmetric charge distributions
 - Gaussian surfaces on which the electric field was either constant or zero
- Gauss' Law applies to any surface and any charge distribution
 - Even with little or no symmetry
- The total electric flux through a closed surface depends only on the total charge enclosed