

Activity 2: Final Activity

April 1 2021 (Happy April Fools!)

Solve each problem. Show your solution. (5 pts each)

1. What is $f'''(x)$ if $f'(x) = x^2 - \frac{1}{3}x$.

$$f''(x) = 2x - \frac{1}{3}$$

$$f'''(x) = 2$$

2. Find the derivative of $\frac{2x-5y^2}{4y^3-x^2} = -x$ at $(1,1)$.

$$\frac{2-10y}{12y^2-2x} = -1$$

$$2-10y = -(12y^2-2x)$$

$$2-10y = 2x-12y^2$$

$$2-2x = 10y-12y^2$$

$$2-2x = 10y \frac{dy}{dx} - 12y^2 \frac{dy}{dx}$$

$$2-2x = \frac{dy}{dx} (10y-12y^2)$$

$$\frac{2-2x}{10y-12y^2} = \frac{dy}{dx}$$

$$\frac{2-2(1)}{10(1)-12(1)^2} = \frac{dy}{dx}$$

$$\frac{0}{10-12} = \frac{dy}{dx}$$

$$\text{slope} = \frac{dy}{dx} = 0$$

3. Assuming that one of the sides of a rectangular field is a river bank and no fencing is needed along the bank, what are the dimensions of a rectangular field with the maximum area that can be fenced off using 50m of fencing materials?

$$\text{Perimeter} = 2L + 2W$$

$$P = 2(L + W)$$

$$50 = 2(x + y)$$

$$\frac{50}{2} = \frac{2(x+y)}{2}$$

$$25 = x + y$$

$$y = 25 - x$$

$$\text{Area} = L \cdot W$$

$$A = x \cdot y$$

$$A = x(25 - x)$$

$$A = 25x - x^2$$

$$\frac{dy}{dx} = 25 - 2x$$

$$0 = 25 - 2x$$

$$\frac{2x}{2} = \frac{25}{2}$$

$$x = \frac{25}{2}$$

$$y = 25 - \left(\frac{25}{2}\right)$$

$$y = \frac{25}{2}$$

$$\frac{dy}{dx} = 25 - 2x$$

$$\frac{d^2y}{dx^2} = -2 \quad \text{where if } \frac{d^2y}{dx^2} < 0 \quad \text{then critical value is maximum.}$$

Therefore, $L = \frac{25}{2}$, $W = \frac{25}{2}$ are the maximum dimensions of a rectangular field that have a perimeter of 50m.