Chapter 2

Electric Potential

Electric Potential

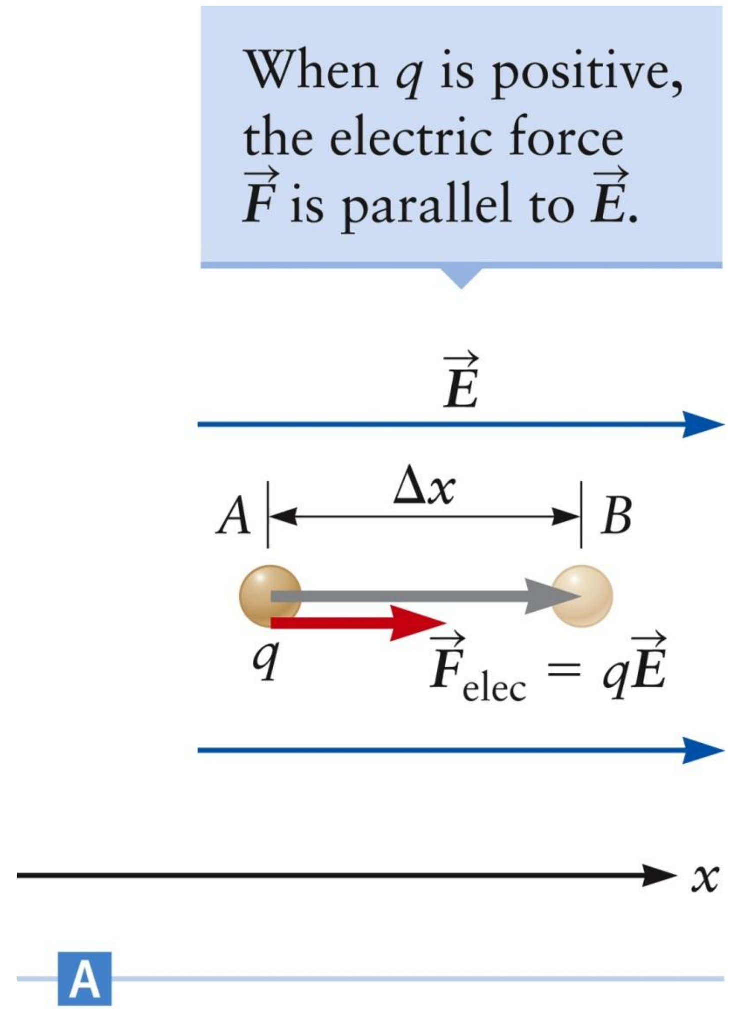
- Electric forces can do work on a charged object
- Electrical work is related to electric potential energy
  - Analogous in many ways to gravitational potential energy
- Electric potential is closely related to electric potential energy
- Conservation of energy will be revisited
- The ideas of forces, work, and energy will be extended to electric forces and systems

Electric Potential Energy

- A point charge in an electric field experiences a force: $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$
- Assume the charge moves a distance Δx
- The work done by the electric force on the charge is W = F Δx
- The electric force is conservative, so the work done is independent of the path
- If the electric force does an amount of work W on a charged particle, there is an accompanying change in electric potential energy

Electric Potential Energy, cont.

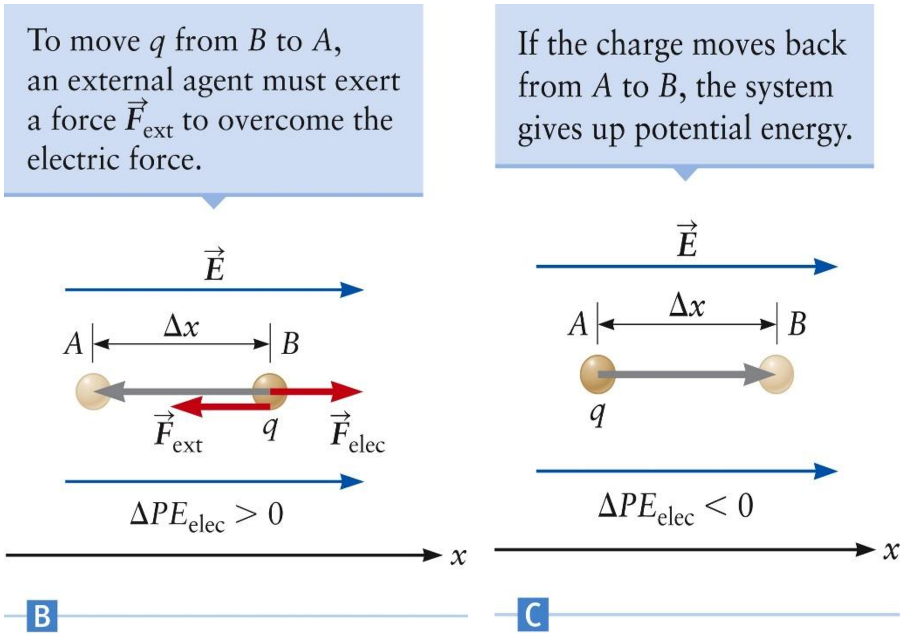
- The electric potential energy is denoted at PE_{elec}
- The change in electric potential energy is
- $\Delta PE_{elec} = - W = - F\,\Delta x = -q\,E\,\Delta x$
- The change in potential energy depends on the endpoints of the motion, but not on the path taken



DISCLAIMER: This document is a draft and the information contained herein is subject to change as this document is currently undergoing review. The final version of this teacher's resource manual will be published as soon as the review has been completed.

Potential Energy – Stored Energy

- A positive amount of energy can be stored in a system that is composed of the charge and the electric field
- Stored energy can be taken out of the system
  - This energy may show up as an increase in the kinetic energy of the particle

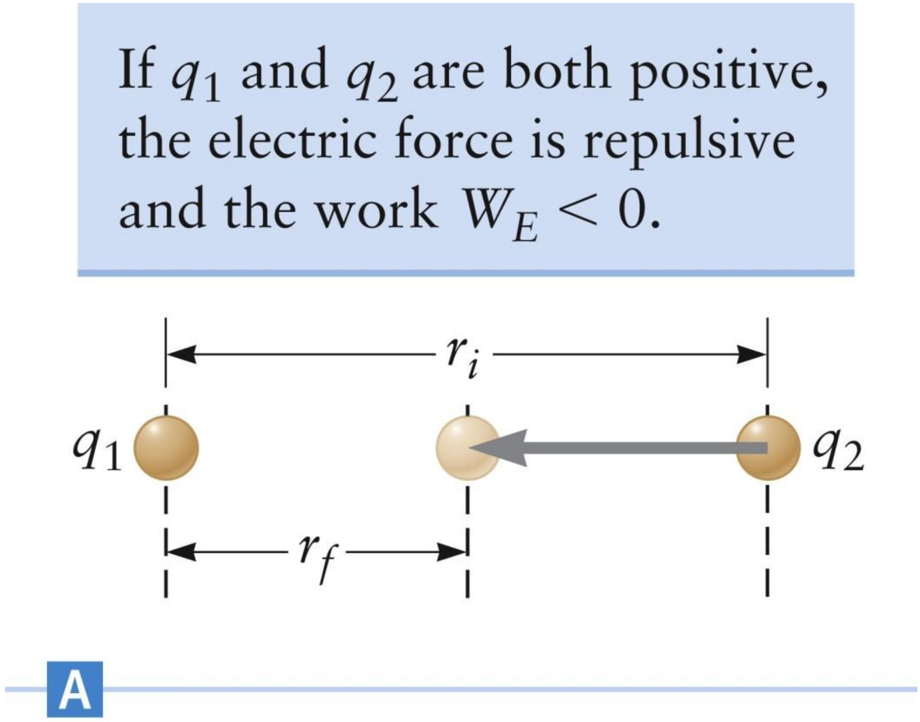


Potential Energy – Two Point Charges

- From Coulomb's Law:

$$F = \frac{kq_1q_2}{r^2}$$

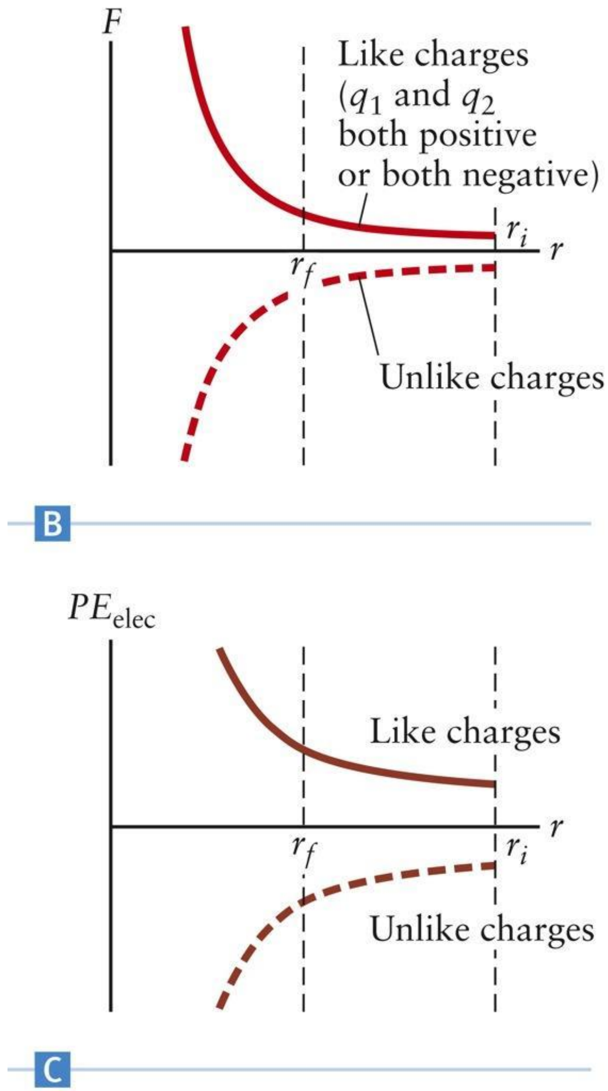
- If they are like charges, they will repel
  - Solid line on the graph
- If they are unlike charges, they will attract
  - Dotted line on the curve



Two Point Charges, cont.

- The electric potential energy is given by

$$PE_{elec} = \frac{kq_1q_2}{r} \text{ or } \frac{q_1q_2}{4\pi\varepsilon_o r}$$



Two Point Charges, final

- PE_{elec} approaches zero when the two charges are very far apart
  - r becomes infinitely large
- The electric force also approaches zero in this limit
- The changes in potential energy are important

$$\Delta PE_{elec} = PE_{elec,f} - PE_{elec,i} = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$

See Examples pages 44-45

Electric and Gravitational PEs

- Both vary as 1/r
- The electric potential energy falls to zero when the separation between two charges is infinite
- The gravitational potential energy falls to zero when the separation between two masses is infinite

Electric Potential: Voltage

- Electric potential energy can be treated in terms of a test charge
  - Similar to the treatment of the electric field produced by a charge
- A test charge is placed at a given location and its potential energy is measured

Electric Potential Defined

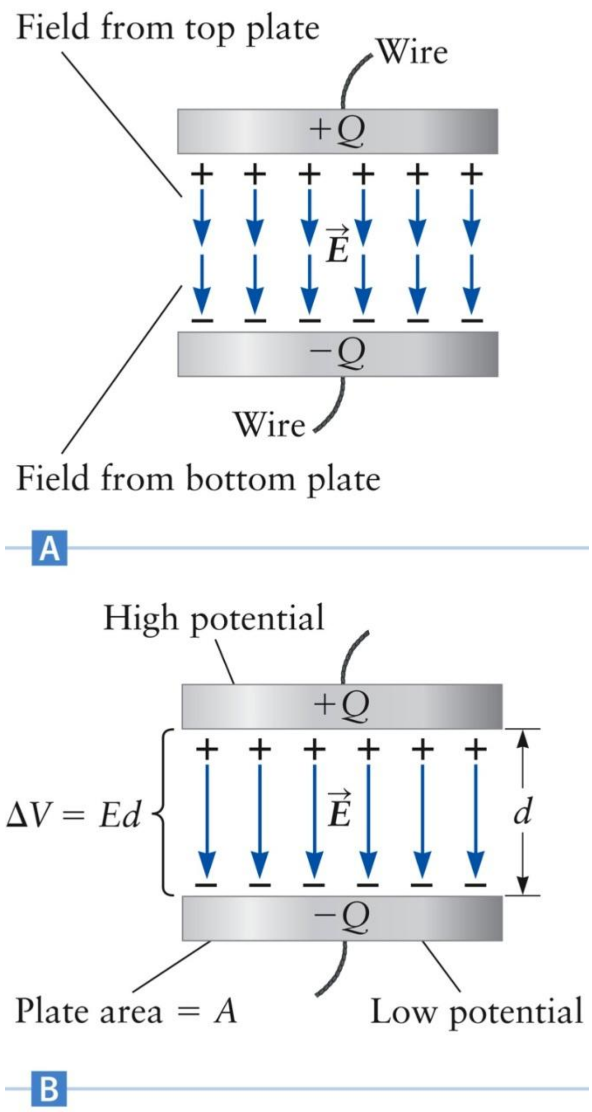
- The electric potential is defined as

$$V = \frac{PE_{elec}}{q}$$

  - Often referred as “the potential”
- SI unit is the Volt, V
  - Named in honor of Alessandro Volta
  - 1 V = 1 J/C = 1 N · m / C
- The units of the electric field can also be given in terms of the Volt: 1 V / m = 1 N / C

Capacitors

- A capacitor can be used to store charge and energy
- This example is a ***parallel-plate capacitor***
- Connect the two metal plates to wires that can carry charge on or off the plates



Capacitors, cont.

- Each plate produces a field $E = Q / (2\,\varepsilon_o\,A)$
- In the region between the plates, the fields from the two plates add, giving

$$E = \frac{Q}{\varepsilon_o A}$$

  - This is the electric field between the plates of a parallel-plate capacitor
- There is a potential difference across the plates
- $\Delta V = E\,d$ where d is the distance between the plates

Capacitance Defined

- From the equations for electric field and potential,

$$\Delta V = Ed = \frac{Qd}{\varepsilon_o A}$$

- Capacitance, C, is defined as

$$C = \frac{Q}{\Delta V} \; therefore \; \Delta V = \frac{Q}{C}$$

- In terms of C,

$$C = \frac{\varepsilon_o A}{d} \; (parallel - plate\ capacitor)$$

  - A is the area of a single plate and d is the plate separation

Capacitance, Notes

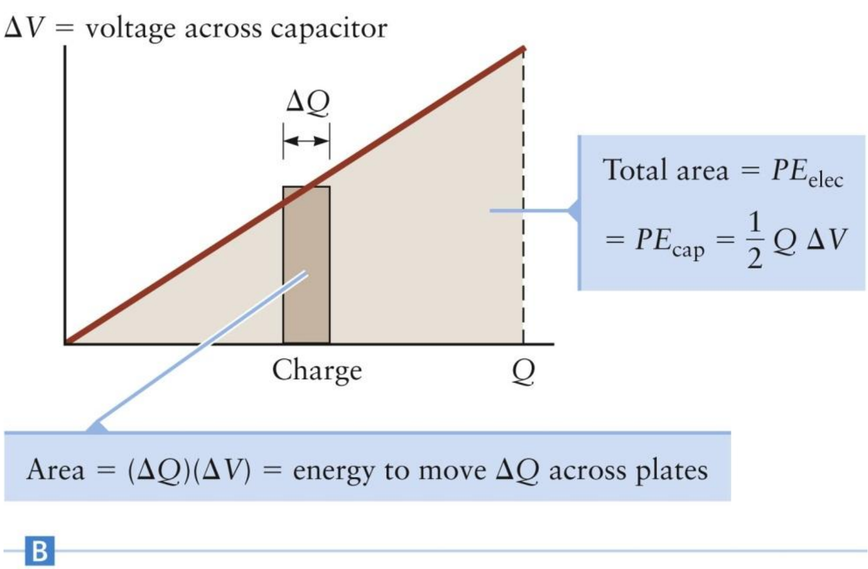
- Other configurations will have other specific equations
- All will employ two plates of some sort
- In all cases, the charge on the capacitor plates is proportional to the potential difference across the plates
- SI unit of capacitance is Coulombs / Volt and is called a Farad
  - 1 F = 1 C/V
  - The Farad is named in honor of Michael Faraday

Storing Energy in a Capacitor

- Applications using capacitors depend on the capacitor's ability to store energy and the relationship between charge and potential difference (voltage)
- When there is a nonzero potential difference between the two plates, energy is stored in the device
- The energy depends on the charge, voltage, and capacitance of the capacitor

Energy in a Capacitor, cont.

- To move a charge ΔQ through a potential difference ΔV requires energy
- The energy corresponds to the shaded area in the graph
- The total energy stored is equal to the energy required to move all the packets of charge from one plate to the other



Energy in a Capacitor, Final

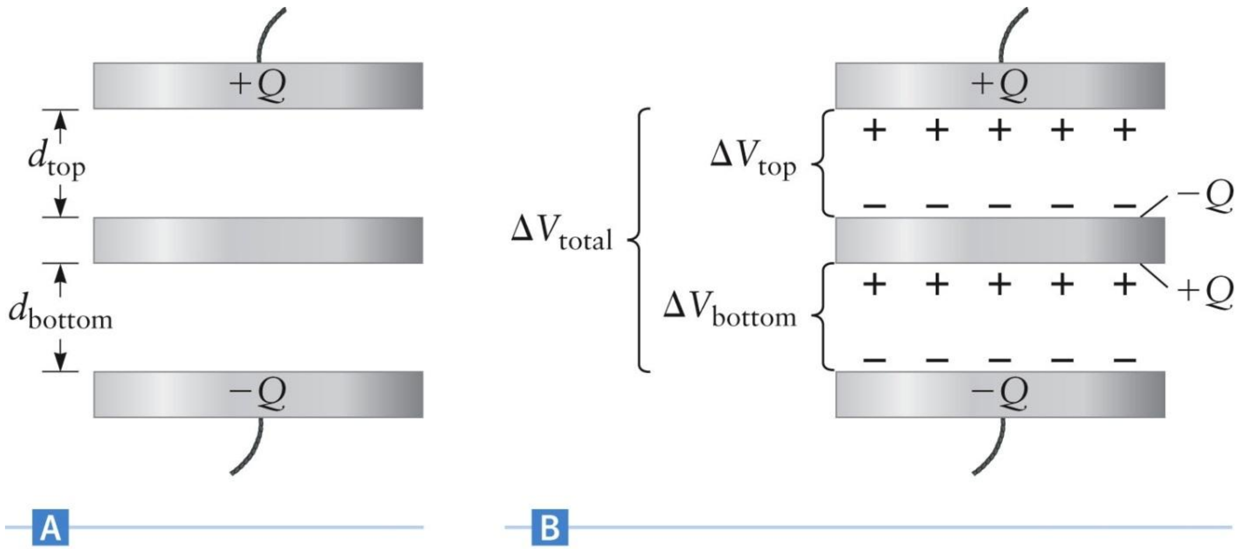
- The total energy corresponds to the area under the ΔV – Q graph
- Energy = Area = ½ Q ΔV = PE_{cap}
  - Q is the final charge
  - ΔV is the final potential difference
- From the definition of capacitance, the energy can be expressed in different forms

$$PE_{cap} = \frac{1}{2} Q\Delta V = \frac{1}{2} C\left(\Delta V\right)^2 = \frac{1}{2}\frac{Q^2}{C}$$

- These expressions are valid for all types of capacitors

Check example 2.8 page 64

Capacitors in Series



- When dealing with multiple capacitors, the equivalent capacitance is useful
- In series:
  - $\Delta V_{total} = \Delta V_{top} + \Delta V_{bottom}$

Capacitors in Series, cont.

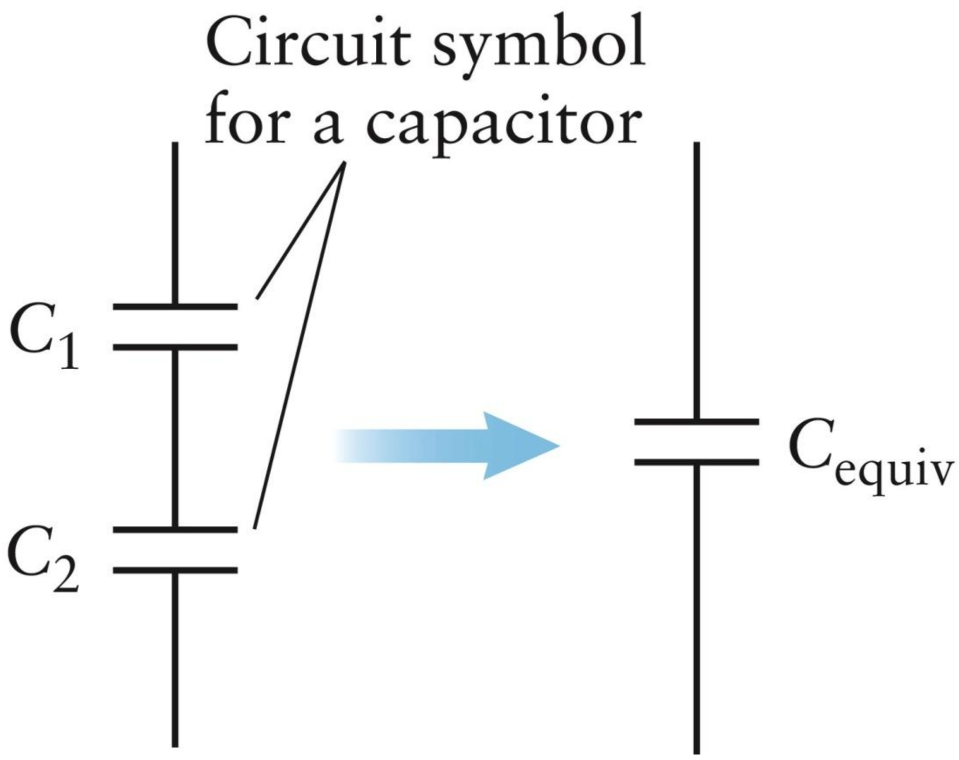
- Find an expression for C_{total}

$$\Delta V_{total} = \frac{Q}{C_{top}} + \frac{Q}{C_{bottom}}$$

$$and\ C_{total} = \frac{Q}{V_{total}}$$

Rearranging,

$$\frac{1}{C_{total}} = \frac{1}{C_{top}} + \frac{1}{C_{bottom}}$$

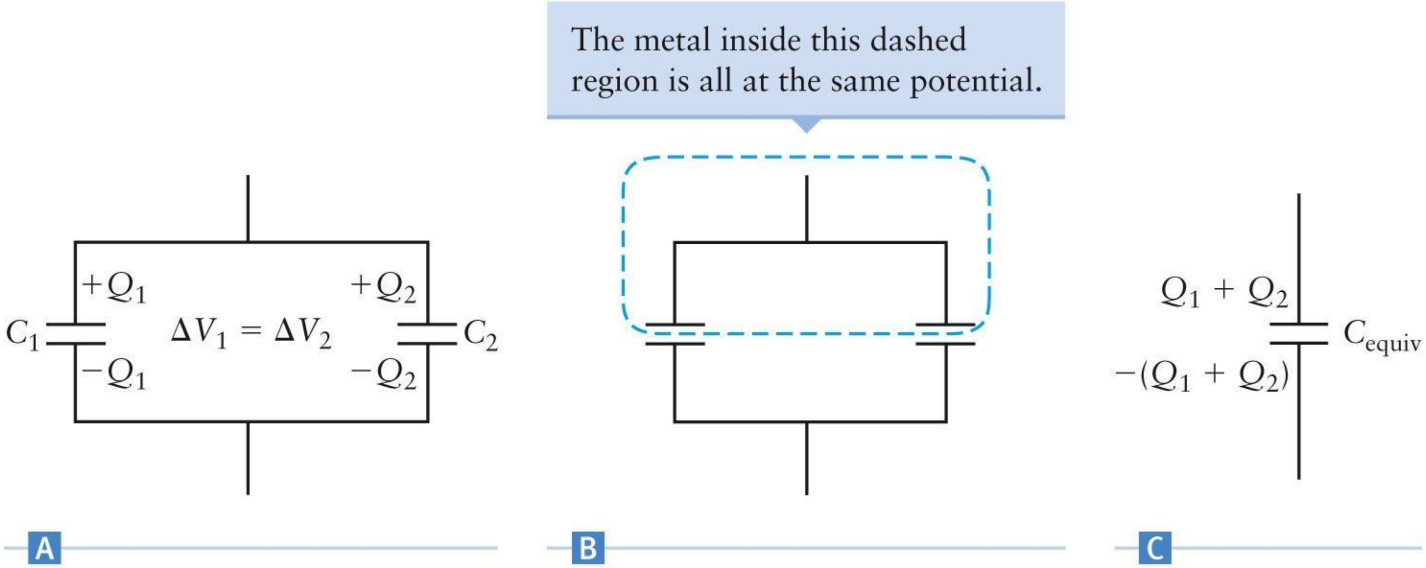


Capacitors in Series, final

- The two capacitors are ***equivalent*** to a single capacitor, C_{equiv}
- In general, this equivalent capacitance can be written as

$$\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in Parallel



- Capacitors can also be connected in parallel
- In parallel:
  - $Q_{total} = Q_1 + Q_2$; $V_1 = V_2$ and $C_{equiv} = C_1 + C_2$

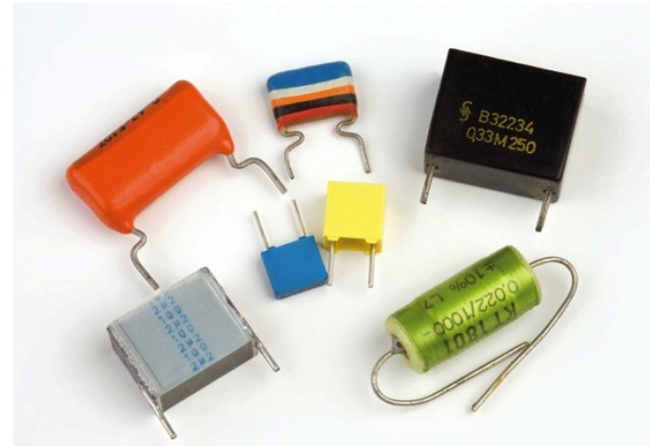
Combinations of Three or More Capacitors

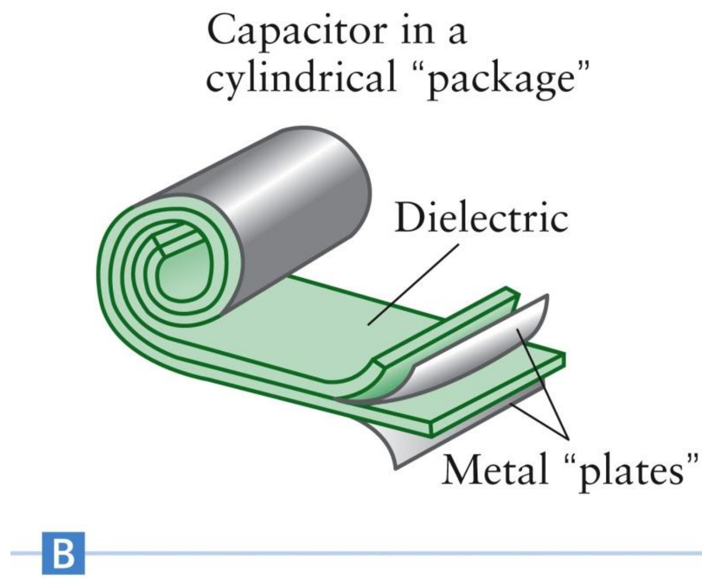
- For capacitors in parallel: $C_{equiv} = C_1 + C_2 + C_3 + \dots$
- For capacitors in series: $$\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$
- These results apply to all types of capacitors
- When a circuit contains capacitors in both series and parallel, the above rules apply to the appropriate combinations
- A single equivalent capacitance can be found

Check example 2.9 page 69

Dielectrics

- Most real capacitors contain two metal "plates" separated by a thin insulating region
  - Many times these plates are rolled into cylinders
- The region between the plates typically contains a material called a ***dielectric***

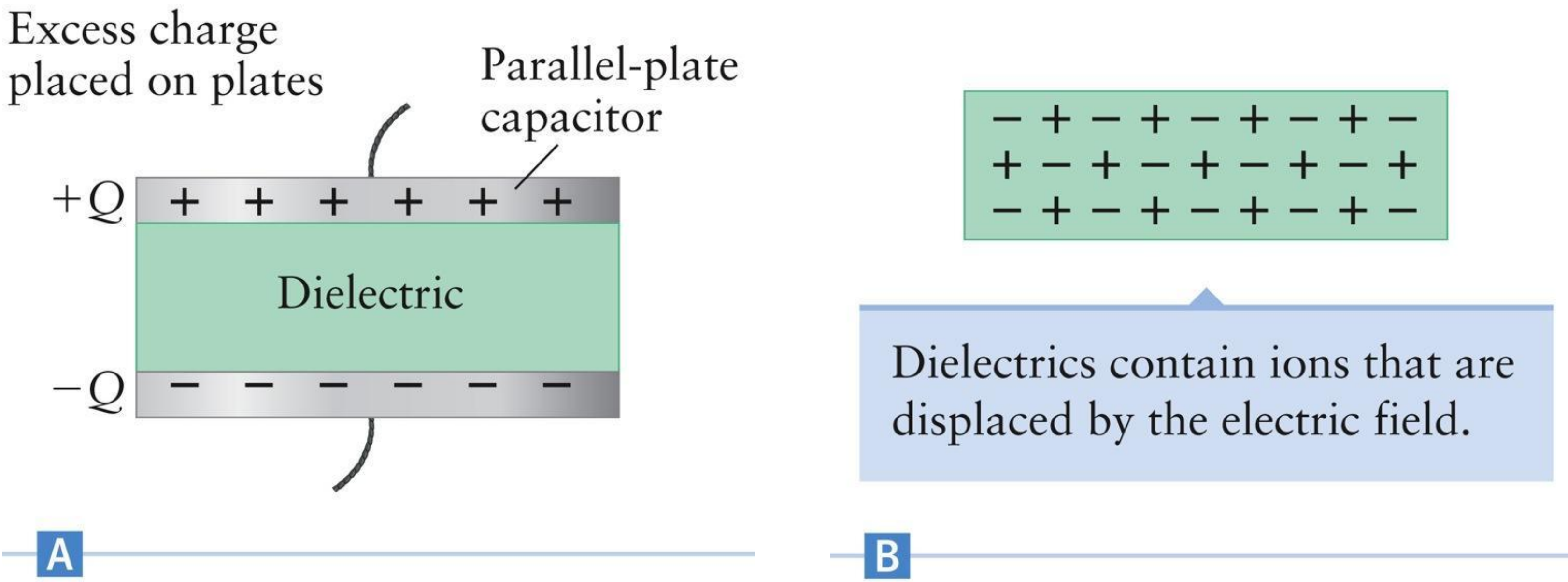




Dielectrics, cont.

- Inserting the dielectric material between the plates changes the value of the capacitance
- The change is proportional to the ***dielectric constant, κ***
- If C_{vac} is the capacitance without the dielectric and C_d is with the dielectric, then $C_d = \kappa C_{vac}$
- Generally, $\kappa > 1$, so inserting a dielectric *increases* the capacitance
- κ is a dimensionless factor

Dielectrics, final



- When the plates of a capacitor are charged, the electric field established extends into the dielectric material
- Most good dielectrics are highly ionic and lead to a slight change in the charge in the dielectric
- Since the field decreases, the potential difference decreases and the capacitance increases

Dielectric Summary

- The results of adding a dielectric to a capacitor apply to any type capacitor
- Adding a dielectric increases the capacitance by a factor κ
- Adding a dielectric reduces the electric field inside the capacitor by a factor κ
  - The actual value of the dielectric constant depends on the material
    - See table 2.1 for the value of κ for some materials

Dielectric Breakdown

- As more and more charge is added to a capacitor, the electric field increases
- For a capacitor containing a dielectric, the field can become so large that it rips the ions in the dielectric apart
  - This effect is called ***dielectric breakdown***
- The free ions are able to move through the material
  - They move rapidly toward the oppositely charged plate and destroy the capacitor
- The value of the field at which this occurs depends on the material
  - See table 2.1 for the values for various materials

Electric Potential Energy Revisited

- One way to view electric potential energy is that the potential energy is stored in the electric field itself
  - Whenever an electric field is present in a region of space, potential energy is located in that region
- The potential energy between the plates of a parallel plate capacitor can be determined in terms of the field between the plates:

$$PE_{elec} = \frac{1}{2}Q\Delta V = \frac{1}{2}QEd = \frac{1}{2}\varepsilon_o E^2 \left(Ad\right)$$

where Ad is the volume of the region

Electric Potential Energy, final

- The ***energy density*** of the electric field can be defined as the energy / volume:

$$u_{elec} = \frac{1}{2}\varepsilon_o E^2$$

- These results give the energy density for any arrangement of charges
- Potential energy is present whenever an electric field is present
  - Even in a region of space where no charges are present