Basic Calculus | 4th Quarter

Performance Task 0: Derivative of Functions

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Find the derivative of the function.

1. $F(x)=(5x^6+2x^3)^4$

   $\frac{dy}{dx}=(4)(5x^6+2x^3)^3(30x^5+6x^2)$ or $\frac{dy}{dx}=(120x^5+24x^2)(5x^6+2x^3)^3$

2. –
3. –
4. $h(t)=(t+1)^{\frac{2}{3}}(2t^2-1)^3$

   $=[(2t^2-1)^3(\frac{2}{3})(t+1)^{\frac{-1}{3}}(1)]+[(t+1)^{\frac{2}{3}}(3)(2t^2-1)^2(2t)]$

   $=(2t^2-1)^2(t+1)^{\frac{-1}{3}}[(\frac{2}{3})(2t^2-1)+(6t)(t+1)]$

   $=(2t^2-1)^2(t+1)^{\frac{-1}{3}}[\frac{4}{3}t^2-\frac{2}{3}+6t^2+6t]$

   $\frac{dy}{dx}=(2t^2-1)^2(t+1)^{\frac{-1}{3}}(\frac{22}{3}t^2+6t-\frac{2}{3})$

5. –
6. –
7. $y=\cos(sec\,4x)$

   $\frac{dy}{dx}=(-\sin(sec^2 x\tan x))$

8. $y=(\frac{1-\cos 2x}{1+\cos 2x})^4$

   $\frac{du}{dx}=\frac{(1+\cos 2x)(2\sin 2x)-(1-\cos 2x)(-2\sin 2x)}{(1+\cos 2x)^2}$

   $=\frac{(2\sin 2x+2\sin 2x\cos 2x)-(-2\sin 2x+2\sin 2x\cos 2x)}{(1+\cos 2x)^2}$

   $=\frac{2\sin 2x+2\sin 2x\cos 2x+2\sin 2x-2\sin 2x\cos 2x}{(1+\cos 2x)^2}$

   $\frac{du}{dx}=\frac{4\sin 2x}{(1+\cos 2x)^2}$

   $=(4)(\frac{1-\cos 2x}{1+\cos 2x})^3(\frac{4\sin 2x}{(1+\cos 2x)^2})$

   $\frac{dy}{dx}=(\frac{16\sin 2x}{(1+\cos 2x)^2})(\frac{1-\cos 2x}{1+\cos 2x})^3$

9. –

10. –
11. –
12. –
13. –
14. $y=\dfrac{x}{2-\tan x}$

$\dfrac{du}{dx}=1$
$\dfrac{dv}{dx}=-sec^2 x$

$=\dfrac{(2-\tan x)(1)-(x)(-sec^2 x)}{(2-\tan x)^2}$
$=\dfrac{2-\tan x-(-x\,sec^2 x)}{(2-\tan x)^2}$
$=\dfrac{2-\tan x+x\,sec^2 x}{(2-\tan x)^2}$

$\dfrac{dy}{dx}=\dfrac{2-\tan x+x\,sec^2 x}{(2-\tan x)^2}$

15. –
16. $f(x)=e^5$

$=e^5\cdot 0$
$\dfrac{dy}{dx}=0$

17. –
18. $f(x)=\dfrac{x^2 e^x}{x^2+e^x}$

$\dfrac{du}{dx}=(x^2)(e^x)+(e^x)(2x)$
$=x^2 e^x+2xe^x$
$\dfrac{du}{dx}=xe^x(x+2)$

$\dfrac{dv}{dx}=2x+e^x$

$=\dfrac{(x^2+e^x)(xe^x)(x+2)-(x^2 e^x)(2x+e^x)}{(x^2+e^x)^2}$
$=\dfrac{(x^3 e^x+xe^{2x})(x+2)-(2x^3 e^x+x^2 e^{2x})}{(x^2+e^x)^2}$
$=\dfrac{(x^4 e^x+x^2 e^{2x}+2x^3 e^x+2xe^{2x})-(2x^3 e^x+x^2 e^{2x})}{(x^2+e^x)^2}$

$$=\frac{x^4e^x+x^2e^{2x}+2x^3e^x+2xe^{2x}-2x^3e^x+x^2e^{2x}}{(x^2+e^x)^2}$$
$$=\frac{xe^x(x^3+xe^x+2x^2+2e^x-2x^2-xe^x)}{(x^2+e^x)^2}$$

$$\frac{dy}{dx}=\frac{xe^x(x^3+2e^x)}{(x^2+e^x)^2}$$

19. –
20. –
21. –
22. –
23. –
24. –
25. $f(x)=x^5+5^x$

$$\frac{dy}{dx}=5x^4+5^x\ln(5)$$

26. $y=\ln(e^{-x}+xe^{-x})$

$$\frac{du}{dx}=(e^{-x})(1)+(1+x)(-e^{-x})$$
$$=e^{-x}+(-e^{-x}-xe^{-x})$$
$$=e^{-x}-e^{-x}-xe^{-x}$$
$$\frac{du}{dx}=-xe^{-x}$$

$$=\frac{1}{x^{-x}+xe^{-x}}\cdot -xe^{-x}$$
$$=\frac{-xe^{-x}}{x^{-x}+xe^{-x}}$$

$$\frac{dy}{dx}=-\frac{x^x+xe^x}{xe^x}$$

27. $G(x)=4^{\frac{C}{x}}$

$$\frac{du}{dx}=\frac{(x)(0)-(C)(1)}{x^2}$$
$$=\frac{0-C}{x^2}$$
$$\frac{du}{dx}=\frac{-C}{x^2}$$

$$\frac{dy}{dx}=(4^{\frac{C}{x}})(\ln 4)(\frac{-C}{x^2})$$
$$=(4^{\frac{C}{x}}\ln 4)(\frac{-C}{x^2})$$

$$\frac{dy}{dx} = \frac{-C\,4^{\frac{C}{x}} \ln 4}{x^2}$$