Mid - Boot camp Assignment

(A) Forward Pricing & Arbitrage Bounds

Market Data

~~Nifty~~ Nifty spot index: 19800
Risk free rate (annual): 6.5 %
Dividend yield: 1.8 %
Time to maturity: 0.25 yrs.

(1) Theoretical forward price using no-arbitrage pricing :-

$$F = S_0 e^{(r-q)T}$$

where $r = 0.065$
$q = 0.018$
$S_0 = 19800$

$$\therefore F = 19800\, e^{(0.065 - 0.018)(0.25)}$$

$$= 19800\, e^{0.01175}$$

$$\simeq \underline{\underline{20,034}}$$

② Observed markets future price is 20,150. ②

Arbitrage opportunity existence?

Arbitrage Profit / contract.

→ Theoretical Price from prev ≈ 20,034.

market price > Theoretical Price. → overpriced. Contract.

→ my strategy:- Borrowing 19800 at 6.5% interest & then buying the Nifty index so that I own it & collect 1.8% in dividend & then sell (short) in the future at 20,150

∴ Profit = Profit per Contract
20,150 − 20,034.
≈ 116 per unit.

③ Sensitivity Analysis:-

- Interest Rates (±100 bps) 1%
- Dividend yield (±50 bps) 0.5%

Interest Rate → +100 bps.
Sensitivity (r). Fair price of the future must go up to compensate the person holding the stocks.

Dividend yield sensitivity (q) → +50 bps.
The future contract (which doesn't get dividends) are less valuable. Hence fair price goes down.

~~New inter.~~
Now,

Base price = 20~~,034.102~~ ≈ 20,034.

Variable	Change	New Price	Direction	logic.
① Interest Rate. (r)	+1%.	≃ 20,084	increase	higher future price
② r	−1%.	19,984.	decrease	lower future price
③ Dividend rate. (q)	+0.5%.	20,009	decrease	cheaper future
④ q	−0.5%.	20,059.	increase.	more expensive future

Calculation ① r = 6.5 + 1 = 7.5%. ∴ r = 0.075

$q = 0.018$

$$\Rightarrow 19800\, e^{(0.075 - 0.018)(0.25)}$$

$$= 19800\, e^{(0.057 \times 0.25)}.$$

$$\simeq \underline{20,084}$$

② $r = \frac{5.5}{100}\% = 0.055$.

$q = 0.018$

$$\Rightarrow 19800\, e^{(0.055 - 0.018)(0.25)}.$$

$$= 19800\, e^{0.037 \times 0.25}.$$

$$\simeq \underline{19,984}$$

(3) $q = \frac{1.8\cancel{0} + 0.5\cancel{0}}{100} = \underline{\underline{0.023}}$

$r = 0.065 \therefore$

$$19800\, e^{(0.065 - 0.023) \times 0.25}$$

$$= 19800\, e^{0.042 \times 0.25}$$

$$\simeq \underline{\underline{20{,}009}}$$

(4) $q = \frac{1.8\cancel{0} - 0.5\cancel{0}}{100} = \underline{\underline{0.013}}$

$r = \underline{\underline{0.065}}$

$$\Rightarrow 19800\, e^{(0.065 - 0.013) \times 0.25}$$

$$= 19800\, e^{(0.052 \times 0.25)}$$

$$\simeq \underline{\underline{20{,}059}}$$

<u>Note:</u>

$\Delta P_1 = 20{,}084 - \cancel{20{,}000}\ 20{,}034 = \cancel{100} \simeq \underline{\underline{+50}}$

$\Delta P_2 = 19\,984 - \cancel{18\,800}\ 20{,}034 \simeq \underline{\underline{-50}}$

$\Delta P_3 \simeq 20\,009 - 20{,}034 \simeq \underline{\underline{-25}}$

$\Delta P_4 \simeq 20\,059 - 20{,}034 \simeq \underline{\underline{+25}}$

$\uparrow$

Base Case Ans

Interpretation :-

Forward price is tuely correlated with interest rates and negatively correlated with dividend yields. (holding the spot becomes more attractive, lowering the forward premium)

[B] Option Pricing & Greeks :- (Black-Scholes modelling for pricing.)

Market data :-

① Spot price : 19,800 $S = 19800$

② Strike price : 20,000 $K = 20{,}000$

③ Time till expiry : 0.25 yrs. $T = 0.25$ yrs

④ Risk free rate : 6.5 %. $r = 0.065$

⑤ Implied Volatility : 22 %. $\sigma = 0.22$

Now, finding d_1 & d_2 firstly :-

$$d_1 = \frac{\ln\left(\frac{19800}{20{,}000}\right) + \left[0.065 + 0.5(0.22)^2\right]\left[0.25\right]}{0.22\sqrt{0.25}}$$

$$= \frac{-0.01005 + (0.065 + 0.0242) \cdot 0.25}{0.11} = \frac{-0.01005 + 0.0223}{0.11}$$

$$\approx \underline{0.1114}$$

$$\therefore d_2 = d_1 - (0.22\sqrt{0.25}) = 0.1114 - 0.11$$

$$= \underline{0.0014}$$

Now, $N(d_1) = 0.5443$
$N(d_2) = 0.5006$.

$e^{-0.065 \times 0.25} \simeq \underline{0.9839}$

Task 1 European Call price $= C = S \cdot N(d_1) - K e^{-rT} N(d_2)$

$= (19800)(0.5443) - 20{,}000(0.9839)(0.5006)$

$\simeq 10{,}777 - 9851$

$= \underline{926}$

European Put Price $= P = K e^{-rT} N(-d_2) - S N(-d_1)$.

$= 20{,}000 (0.9839)(0.4994) - 19800 (0.4557)$

$\simeq 9827 - 9023$

$\simeq \underline{804}$

Task 2 Estimation of Greeks numerically using finite differences

Greek	Poking
Delta	Spot Price (S)
Vega	Volatility (σ)
Theta.	Time (T)

$\rightarrow$ Shift inputs by '1'

1 Delta

S: 19800 → 19801

Now, calculating again with S = 19,801

that is calculate

$$d_1 = \frac{\ln\left(\frac{19801}{20,000}\right) + \left(0.065 + 0.5(0.22^2)\right) 0.25}{0.22\sqrt{0.25}}$$

$$\approx \frac{-0.0100005 + 0.0223}{0.11} \approx 0.11181$$

$$d_2 = d_1 - 0.11 = \underline{0.0081}$$

$N(d_1) = 0.5445$.

$N(d_2) = 0.5007$.

$$C = (19801)(0.5445) - 20,000)(0.9839)(0.5007)$$

$$\approx 10,781.64 - 9851.05$$

i.e $\underline{\underline{930.59}}$

$$\therefore \Delta = \frac{C_{new} - C_{old}}{S_{new} - S_{old}} \approx \frac{930.59 - 926}{1} \approx \underline{\underline{4.59}}$$

$\therefore$ ~~If 100 is moved the~~

② Vega

volatility $\sigma \rightarrow$ 0.22 to 0.23 ($\because$ 22% $\rightarrow$ 23%)

$$d_1 = \frac{\ln\left(\frac{19800}{20,000}\right) + (0.065 + 0.5(0.23^2))\, 0.25}{0.23\sqrt{0.25}}$$

$$= \frac{-0.01005 + (0.065 + 0.02645)(0.25)}{0.115}$$

$$\approx \underline{\underline{0.1114}}$$

$$d_2 = 0.1114 - 0.115 = \underline{\underline{-0.0036}}$$

~~N(d~~ $N(d_1) = 0.5443$

$N(d_2) = 0.4986$

$$C_{new} = 19800(0.5443) - 20,000(0.9839)(0.4986)$$
$$\approx 965.91$$

$$\therefore \text{Vega} = \frac{C_{new} - C_{base}}{\text{Vol Change } (1\%)} = 965.91 - 926 = \underline{39.91}$$

(8) Theta

T → remove 1 day $\therefore$ T = 0.25 to 0.2472

↓

Find new call price ~918.5 (same as above (calculations shown for Delta & Vega)

$$\theta = \frac{P_{new} - P_{base}}{\Delta t}$$

$$= 918.5 - 926$$

ie -6.5 (per day)

~~Delta~~ :-

— XXXXX —