Frequency Domain Analysis of a Discrete-Time Pulse Train using FFT

Supriya Anand Mishra
Roll No: 24B1312

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1 Introduction

The Fast Fourier Transform (FFT) is an efficient algorithm used to compute the Discrete Fourier Transform (DFT) of a discrete-time signal. It allows analysis of a signal in the frequency domain by decomposing it into sinusoidal components of different frequencies, magnitudes, and phases.

In signal processing, FFT is widely used to identify the spectral content of periodic and non-periodic signals, study harmonic structures, and understand the relationship between time-domain behavior and frequency-domain representation.

2 Objective

The objective of this assignment is to:

- Generate a discrete-time periodic pulse train signal.
- Compute its FFT using a finite number of samples.
- Plot and analyze the magnitude and phase spectra.
- Understand the harmonic structure and phase behavior of a pulse train.

3 Theory

A periodic pulse train consists of rectangular pulses repeated at a fixed interval. Such a signal is periodic but non-sinusoidal, and hence its Fourier representation contains multiple harmonics.

For a pulse train with period N_0 samples and pulse width W samples:

- The frequency spectrum consists of discrete spectral lines at integer multiples of the fundamental frequency:
$$f_0 = \frac{f_s}{N_0}$$

- The magnitudes of these spectral components follow a sinc-shaped envelope.
- Phase discontinuities occur at frequencies where the magnitude approaches zero.

4 Parameters Used

The following parameters were used in this experiment:

- Sampling frequency: $f_s = 1024$ Hz
- FFT length: $N = 2048$
- Pulse period: $N_0 = 128$ samples
- Pulse width: $W = 32$ samples

5 Methodology

1. A discrete-time pulse train was generated using modular arithmetic.
2. The FFT of the signal was computed using a 2048-point FFT.
3. Only the positive frequency components were considered.
4. The magnitude and phase of the FFT coefficients were computed and plotted.

The implementation was carried out in Python using NumPy and Matplotlib.

6 Code Repository

Listing 1: FFT of a Discrete-Time Pulse Train

```
import numpy as np
import matplotlib.pyplot as plt

# Parameters
fs = 1024           # sampling frequency (samples/sec)
N = 2048            # samples
N0 = 128            # pulse period (samples)
W = 32              # pulse width (samples)

n = np.arange(N)

# Pulse train
x = ((n % N0) < W).astype(float)

# FFT
```

```
X = np.fft.fft(x, N)
X = X[:N//2]        # positive frequencies

f = np.linspace(0, fs/2, N//2)

magnitude = np.abs(X)
phase = np.angle(X)

# Plot
plt.figure(figsize=(10,6))

plt.subplot(2,1,1)
plt.stem(f, magnitude, basefmt=" ")
plt.title("Magnitude Spectrum of Pulse Train")
plt.ylabel("Magnitude")
plt.xlim(0, 300)
plt.grid(True)

plt.subplot(2,1,2)
plt.plot(f, phase)
plt.title("Phase Spectrum of Pulse Train")
plt.xlabel("Frequency (Hz)")
plt.ylabel("Phase (rad)")
plt.xlim(0, 300)
plt.grid(True)

plt.tight_layout()
plt.show()
```

7 Results

7.1 Magnitude Spectrum

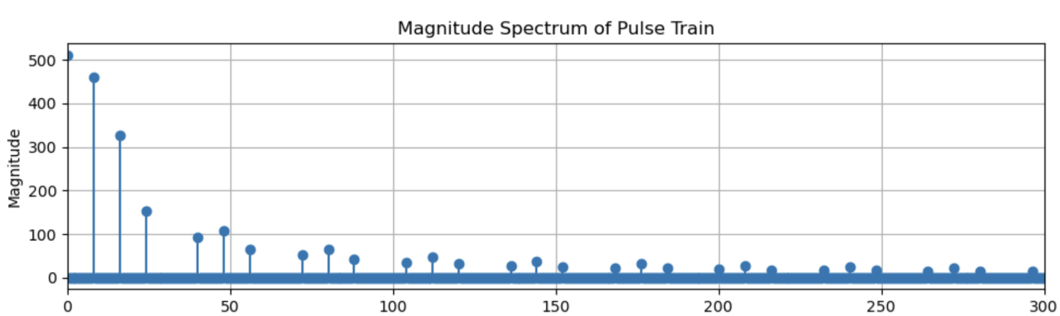

Figure 1: Magnitude Spectrum of the Discrete-Time Pulse Train

The magnitude spectrum exhibits discrete spectral lines at harmonics of the fundamental frequency. The amplitudes of these harmonics decay with frequency and follow a sinc-shaped envelope, which is characteristic of a rectangular pulse in the time domain.

7.2 Phase Spectrum

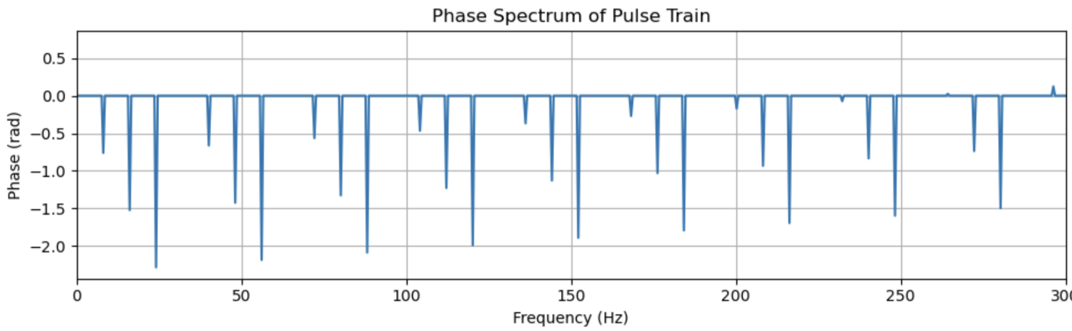

Figure 2: Phase Spectrum of the Discrete-Time Pulse Train

The phase spectrum shows piecewise-constant behavior with abrupt phase jumps. These discontinuities occur at frequencies where the magnitude spectrum approaches zero, leading to phase ambiguity. The structured nature of the phase confirms the periodicity and deterministic structure of the pulse train.

8 Discussion

The FFT results clearly demonstrate that a periodic pulse train contains multiple harmonics in its frequency spectrum. Unlike pure sinusoids, which produce isolated spectral lines, a pulse train distributes energy across many frequencies. The sinc-shaped envelope observed in the magnitude spectrum is directly related to the finite pulse width in the time domain.

The phase spectrum is meaningful only at frequencies where the magnitude is significant. The observed phase jumps are consistent with theoretical expectations and arise due to sign changes in the Fourier coefficients.

9 Conclusion

In this experiment, FFT was successfully used to analyze a discrete-time pulse train. The magnitude spectrum revealed the harmonic structure and sinc envelope, while the phase spectrum exhibited structured discontinuities. This study reinforces the relationship between time-domain signal shape and frequency-domain characteristics.