Persistent Data Structures and Managed References

- **Speaker: Rich Hickey**
- **Conference: QCon - Oct 2009**
- **Video: http://www.infoq.com/presentations/Value-Identity-State-Rich-Hickey**

[Time 0:00:00]

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slide title: Persistent Data Structures and
                Managed References

 Clojure's approach to Identity and State

             Rich Hickey
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How many people program in a functional programming language? OK, so halfway preaching to the converted. And not in a functional programming language? A non-functional programming language? So still a lot of that.

I think this will be useful to both audiences. In particular if you are not in a functional programming language, in fact if you are not in Erlang, which I think has a complete story for how they do state, all the other functional programming languages have two aspects. They have this functional part, and then ... and then.

Haskell has this beautiful side where the type system keeps this part pure. And then there is the other part which is kind of imperative: do this, do that. And then they have a bunch of constructs to provide facilities for when you need state on that side.

Similarly there are a lot of "hybrid" functional languages, like Scala and F#, where I think there are questions to be asked about: OK here is the pure part, what is the story about the other part?

[Time 0:01:13]

```
slide title: Agenda

+ Functions and processes
+ Identity, State, and Values
+ Persistent Data Structures
+ Clojure's Managed References
+ Q&A
```

So what I want to do today is to talk about functions and processes, and to distinguish the two. In fact, the core concept in this talk is to try to parse out what we mean by identity, state, and values; try to separate those concepts. And see how programming with values, while a really important part of the functional part of your program, ends up being a critical part of the non-functional part of your program, the part that actually has to manage state and behave as if things are changing.

And there are two components to that. One is: how do you represent composite objects as values? A lot of people who are new to functional programming wonder about the efficiency and representation issues there, and I will talk about that. And finally I will talk about one approach to dealing with state and change in a program, the one that Clojure uses, which is compatible with a little bit of philosophy that I am going to start with.

[Time 0:02:11]

```
slide title: Clojure Fundamentals

+ Dynamic
+ Functional
  + emphasis on immutability
+ Supporting Concurrency
+ Hosted on the JVM
  + Compiles to JVM bytecode
+ Not Object-oriented
+ Ideas in this talk are not Clojure-specific
```

I am not really going to talk about Clojure very much. How many people were at my talk yesterday? OK, how many people who were not know something about Clojure? OK. This is not really a Clojure specific talk. There will be some code later. It should not be too threatening. I am just going to summarize quickly with this one slide what Clojure is about.

It is a dynamic programming language. It is dynamically typed. It is functional. In particular, it is functional in emphasizing immutability, not just in supporting higher order functions. All the data types in Clojure are immutable.

It supports concurrency. It is a two part story. One is: you have to have good support for immutability and pure functions. The other part is: you have to have a story for when multiple things are happening at a time and you are going to have some perceptible change, and Clojure does. In fact, I think it is an important part of a language that purports to be functional to have a story about the non-functional parts.

Clojure is not particularly object oriented. It may be clear after listening to this talk why not, because I think as currently implemented, a lot of object technologies have big problems when they face concurrency and functional programming. And as I said, from a conceptual standpoint, nothing about this is really Clojure specific.

[Time 0:03:32]

```
slide title: Functions

+ Function
  + Depends only on its arguments
  + Given the same arguments, always returns the same value
  + Has no effect on the world
  + Has no notion of time
```

So what do we mean by functions? I think that there is a really easy way to say, "Oh, a function is something that you call", and that is *not* what we are talking about here. We are talking about a very precise notion of a function, which is something that you call that takes values as arguments and produces a value as a return. When it is given the same arguments, it always produces the same value.

It does not depend on the outside world. It does not affect the rest of the world. So many methods in your classes are not functions by this definition, but in particular too, I want to highlight the fact that function, pure functions, have no notion of time. Time is going to be a critical notion through this talk.

[Time 0:04:12]

```
slide title: Functional Programming
```

```
+ Emphasizes functions
  + Tremendous benefits
+ But - most programs are not functions
  + Maybe compilers, theorem provers?
    + But - They execute on a machine
    + Observably consume compute resources
```

So what is functional programming? There are lots of answers to this question, and I think people who are into type systems will claim a stronger argument for what constitutes functional programming. But I am going to limit the definition here to programming that emphasizes programming with functions. So you want to try to write as much of your program as you can with pure functions. When you do that you get a ton of benefits. They have been talked about in other talks. It is not really the focus of this talk other than to say, even without concurrency, your program will be easier to understand, easier to reason about, easier to test, more modular, and so forth. That all falls out of programming with functions to as great an extent as possible.

On the other hand, when you step back and look at your entire program, very few programs, on the whole, are functions. You know, that take a single input, think about it, and produce a single output. Maybe some compilers or theorem provers work that way, but most real world programs that I have worked on, and I think most real world programs in the real world do not work that way.

In particular, even if you claimed your program was completely functional, if it is going to produce any output, it is not, because otherwise it will just warm up the machine. But even if it is mostly functional, there are still observable effects of a purely functional program running. It is running on a computer. As soon as it is running on a computer it is not math any more. It is a program running on a computer. It is consuming memory. It is consuming clock cycles. It is observably doing something over time. So *all* programs do things over time.

But most real programs, as I say, actually have observable behavior that is not just the fact that they are running on a computer, but that they are doing things. They are interacting with the outside world. They are talking over sockets. They are putting stuff on the screen. They are putting things in or out of the database.

In particular though, we will use one critical measure about how to define state, which is: if you ask the same question twice, and you get different answers, then there is state. I do not care where you put it. You can put it in a process, you put it in an agent, an atom, in a variable. It does not matter, in a database. If you ask the same question twice and get different answers at different times, you have state. So again, the word "time" just came up again there.

[Time 0:06:37]

slide title: Processes

```
+ Include some notion of change over time
+ Might have effects on the world
+ Might wait for external events
+ Might produce different answers at different times (i.e. have state)
+ Many real/interesting programs are processes
+ This talk is about one way to deal with state and time
  _in the local context_
```

So I think most programs are processes, which means we need to talk about the part of your program that cannot be purely functional, the part that is going to have to produce a different answer at

different times. How do you do that, and not make a complete mess out of what you created with the shiny pure part?

In particular, though, I want to highlight the fact that this talk is strictly about the notion of state and time in a local context. I am talking about in the same process. There are a completely different set of requirements and characteristics of distributed programs, where you cannot do the same things that you can do in the same process. So I am talking *only* about same-process concurrency and state.

[Time 0:07:35]

```
slide title: State

+ Value of an identity at a time
+ Sounds like a variable/field?
  + Name that takes on successive 'values'
+ Not quite:
  + i = 0
  + i = 42
  + j = i
  + j is 42? - depends
```

So I want to be a little bit more precise about what I mean when I say identity, state, and value, and these kinds of things. And in particular I want to talk about state and I will talk about it twice. One is just sort of a generic statement: state is the value of an identity at a time.

Maybe none of that makes sense. Maybe it sounds like a variable from a traditional programming language. Because I think if you ask somebody who is using a traditional programming language, "Do you have state?" They will be like, "Yeah I have some variables, and I change them". And that is not a good sound definition of what constitutes state.

So are variables state? Do they do this job? Do they manage the value of an identity over time? We can have a variable i, we can set it to zero, we can set it to 42, we can assign one variable to another. Is j 42? That depends. In a sequential program, probably, kinda, sorta.

In a program that had threads, what could go wrong? Well I did not say what order these things happened in, or what threads they happened in. For instances if you set j equals i in a separate thread, what bad thing could have happened to you? Would it say 42 necessarily? No, definitely not. Because that memory may not have been flushed through to the other thread's cache. It is not volatile, necessarily, "i".

What else could happen that is bad? Maybe i is a long, maybe setting a long is not atomic in your programming language. Bad.

[Time 0:09:05]

```
slide title: Variables

+ Variables (and fields) in traditional languages are predicated
  on a single thread of control, one timeline
+ Adding concurrency breaks them badly
  + Non-atomicity (e.g. of longs)
  + volatile, write visibility
  + Composite operations require locks
  + All workarounds for lack of a time model
```

4

So variables are not going to be good enough to do the job of managing state. They are predicated on a single thread of control. They actually do not work at all, otherwise. They are horribly broken by concurrency. The whole notion of, "there is this piece of memory", does not work. And our programs are built substantially on this, whether it is a variable sitting on the stack, or fields in your object, same problems. Pieces of memory are insufficient abstractions.

So we have the problem of non-atomicity of long writes. That is a problem in a lot of languages, that it is just not atomic. So you could get half of a number if you look at it from another thread. Write visibility and memory fences have to be accounted for once you have multiple threads of control on a true concurrent box.

If you have an object, and it has a bunch of these things collected together that constitute its state, now you have the problem of composing a composite operation. Because making it into another valid state requires touching several of these variable things, which now makes you impose locks or some sort of synchronization that say, "Stay away from me, so I can pretend there is only one thread of control, because that is what my language thought when they wrote it", or the language they copied thought when they wrote *it*.

All of these things are examples of the same problem. We are having to work around the lack of a model for time. Because there is no point to having variables if you do not have time. And just think about that for a second. If there is no time notion, why would you need a variable? If you cannot go back to it later and see something different, how is it a variable?

[Time 0:10:47]


```
slide title: Time

+ When things happen
  + Before/after
  + Later
  + At the same time (concurrency)
  + Now
+ Inherently relative
```


So if we want to be clearer about time, which we are not going to be in a non-physics lecture. We are just going to say, "What are some things you think of when you think of time?" You think of things being before or after other things. You think of something happening later. You think of something happening at the same time, two things happening at the same time. You think of something happening right now, which is sort of a self-relative perspective of time.

But all of these concepts are important in that they are *inherently* relative. When you think about time, there is not a lot about time that are hours, or this particular moment with a name on it. Most of our notions of time have to do with relative time, the ordering between two discrete things.

[Time 0:11:36]


```
slide title: Value

+ An immutable magnitude, quantity, number ...
  _or composite thereof_
+ 42 - easy to understand as value
+ But traditional OO tends to make us think of composites
  as something other than values
  + Big mistake
```

```
        + aDate.setMonth("January") - ugh!
    + Dates, collections etc are all values
```

What do we mean by values? Again, here is an area where there is just so much ambiguity and loose thinking, that we cannot write correct program until we sort of nail this down.

So the core characteristic of a value is that it is immutable. Some values are obvious. Numbers. We all are comfortable with that concept that it is a value. But I will contend that until you start thinking about composites of these things, like numbers, as values, you are doomed in the future. You may not be doomed right now, but at some point it is going to be a problem for your programs.

So what went wrong? We all think 42 is indivisible. Of course we saw that if we store it in a long, it may not be, depending on our language. But the idea of 42 we consider to be an atomic concept.

But we have a *big* problem in the way we think about composite objects. Some of that falls out of our languages I think. They have date libraries where you can set the month is a crazy concept. There is this date and there is that date. There is not setting the month of a date, and it is another date. Right away you have that problem: I set the month of a date and it is another date. If it is another date, you have two dates. You do not have one settable date. And our class libraries have destroyed our brains in this area.

Also the default behavior of our languages: you create a new class in most languages and everything is variable. And instantly you have this stateful mess that maybe you have to clean up with a lot of discipline on your part. So I am going to contend that dates, sets, maps, everything is a value, and should be treated like a value. And you should separate your concept of value from your concept of change.

[Time 0:13:24]


```
slide title: Identity

+ A logical entity we associate with a series of causally
  related values (states) over time
+ Not a name, but can be named
  + I call my mom 'Mom', but you wouldn't
+ Can be composite - the NY Yankees
+ Programs that are processes need identity
```

So one more concept in the philosophy portion of the talk, which is the concept of identity. This is probably the most nebulous of these things, but it is an important thing.

What happens in the real world? What happens in the real world when we talk about today, or mom, or Joe Amstrong? Is that a single unchanging thing? Or one way to think about it is: we have a logical entity, that we associate with different values over time. In other words, at any particular moment, everything is frozen. And the next moment we look, we see something different. Is that the same thing? Well, if some force has acted on this thing to produce that next thing, I consider it to be the same thing. Otherwise, they are unrelated things. Two things can pass through the same space. They are not the same thing because they are in the same space.

So a set of values over time, where the values are causally related, is something we need to name. These are different values. They may be in different spaces. I can walk over here. I am still Rich. So what is happening?

What is happening is easy to understand if you have three notions. There is a state: I am standing right here. [moves a step sideways] There is a state: I am standing over here. They are both values.

If you could stop time for a second, nothing about me would be changing. And it is me, because I am using my legs to move myself over here, and I am still talking, and you see a set of causal connections between me being here and being there. So you say: that is all Rich. That is not two people doing this.

Identities are not the same things as names. I just want to make that clear. I have a mother. You now have that concept in your head, this identity. But I call her mom, and you would call here Mrs. Hickey, I hope.

These identities can be composite. We can talk about the New York Yankees, or Americans, no problem. Those are sets, but they are also identities. They change over time, but at any particular point in time they have a value. These are the guys who are on the Yankees right now. Any program that is a process needs to have some mechanism for identity. This all goes together.

[Time 0:15:52]

```
slide title: State

+ Value of an identity at a time
+ Why not use variables for state?
  + Variable might not refer to a proper value
  + Sets of variables/fields never constitute a proper composite value
  + No state transition management
    + i.e., no time coordination model
```

So I will go back and talk about state. We have some terms that hopefully mean something. Now we can say: a state is a value of an identity at a time. Hopefully that makes sense. The identity is a logical thing, it is not necessarily a place, it is not a piece of memory. A value is something that never changes. And time is something that is relative.

Now it is easy to see, I think, why we cannot use variables for state. In particular, that variable may not refer to something that is immutable, so already [makes buzzer sound] that is a problem. If you make a variable refer to a variable, you are building on sand. The key concept is: variable – or whatever we are going to do to manage time – has got to refer to values.

Sets of variables, as we traditionally have them, can never constitute a value, because they are not atomically composite. Because we are saying a value is something that is immutable. If you can change the parts independently, then it is not immutable, because there is going to be a moment when one part is halfway there and another part is not there. That is not a valid value now. Something is happening in the middle.

And more globally you can say about variables, their problem is they have no state transition management. That is the management of time, a coordination model for time. How do you go from: I am in this state, now I am in that state, both states being immutable values.

[Time 0:17:20]

```
slide title: Philosophy

+ Things don't change in place
+ Becomes obvious once you incorporate time as a dimension
  + Place includes time
+ The future is a function of the past, and doesn't change it
+ Co-located entities can observe each other without cooperation
+ Coordination is desirable in local context
```

So this is the summary of the philosophy portion. A key concept I think is: things do not change in place. We think that they do, but they do not. The way you can see that this is the case is to incorporate time as a dimension. Once time is a dimension, once you have x, y, z, time, guess what? That is over here. If something is happening here, this is no different. Things do not change in place. Time proceeds. Functions of the past create the future. But both things are values.

There are a couple of aspects, I think, to the design of the things I am going to show you that I think are important when you try to model time in the local context. These are things I do not want to give up. These are things I know I can achieve by brute force in Java, and I cannot sell my language if I cannot achieve them in Clojure, for instance. Which is: co-located entities can see each other without cooperation. There are a lot of messaging models that require cooperation. If I want to see what you are about, I have to ask you a question. You have to be ready to be asked that question. You have to be willing to answer my question.

But that is not really the way things are when you are co-located. I do not know what is happening in the next building, but I can see all of you, and I can certainly look at the back of your head without asking you permission.

The other thing that I think is really important in a local context – it really should be written off as impossible in a distributed context – is: you can do things in a coordinated manner with co-located entities in the same process. You can say: let us all work together, and do this all right now. As soon as you are distributed, you cannot do that. So the models I am going to show you support visibility of co-located entities and coordination.

[Time 0:19:15]

```
slide title: Race-walker foul detector

+ Get left foot position
  + off the ground
+ Get right foot position
  + off the ground
+ Must be a foul, right?
```

So let us take a little example. A little race-walker foul detector. Race walkers, they have to walk, they cannot run. They have to walk step step step, heel toe, and they cannot have both feet off the ground at the same time. That is a foul, and then you get kicked out of the race.

So how do we do this? Well we go and we get the left foot position, and we see it is off the ground. We go and we get the right foot position and we say: oh, it is off the ground. So they are running, right?

It sounds funny, but everybody writes programs that do exactly this all the time, *exactly* this. And you wonder: why didn't it work? We cannot work that way. We cannot have time and values all munged together where things are changing while we are trying to look at them. That does not work. We cannot make decisions. We do not make decisions as human beings this way.

[Time 0:20:09]

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slide:

[ photo of competitive race-walk ]

+ Snapshots are critical to perception and decision making
+ Can't stop the runner/race (locking)
```

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+ Not a problem if we can get runner's value
+ Similarly don't want to stop sales in order to calculate
  bonuses or sales report
```

Snapshots, and the ability to consider something as a value at a point in time, are *critical* to perception and decision making. And they are as critical in programs as they are to us as human beings. If you look at our sensory systems, they are completely oriented on creating momentary glimpses of a world that we would otherwise just perceive to be completely fungible.

Now how do we achieve this, programmatically? Well one of the things I think we have to advocate if we want to write programs that can work on multiple cores, and benefit from being on multiple cores, is: we cannot stop the race. We cannot stop the runner. We cannot say, "Whoa! Could you just hang on a second, because I just want to see if you are running."

Also we cannot expect the runner to cooperate. Could you just tell me if you are running? But if we could consider the runner to be a value, like this guy on the right here, it is kind of nice that we can look at him as if he was a value. There is a point in time that was captured by this photograph, right? It is a single value. I do not have to independently look at the left and right while *time proceeds*. I have got a value in hand. It was captured at a point in time. The race kept going, but I can see that guy has got a foul on the right. He has got both his feet off the ground. That is easy.

That is the kind of easiness you want to have in the logic of your applications. You want to be working with values. You do not want to be working with things that are running away from you as you are trying to examine them.

So it is not a problem to do this work if we can get the runner's value. In a similar way, we do not want to stop people from conducting sales so we can give them bonuses or do sales reports. We need to move to a world in which time can proceed *and* we can do our logic, and we do not need to *stop everybody* so we can do our logic. The two things have to be independent.

[Time 0:22:00]

```
slide title: Approach

+ Programming with values is critical
+ By eschewing morphing in place, we just need to manage the
  succession of values (states) of an identity
+ A timeline coordination problem
  + Several semantics possible
+ Managed references
  + Variable-like cells with coordination semantics
```

So, how does it work? Well, the first thing is: we have to program with values. We have to use values to represent not just numbers, and even small things like dates, but pretty much everything: collections, sets. Things that you would have modeled as classes should be values.

I am not saying there could not be an object-oriented system that worked this way. I do not know of one that does. But you should start looking at your entire object as if it was a value. It should never be in pieces that you could twiddle independently. You want a new state of that object, you make an entire new value.

So then, what is the problem with time? It becomes a much smaller problem. All we need to do is get some language constructs, or some way, to manage the succession of values. An identity is going to take on a succession of values over time. We just need a way to model that. Because we have

pure functions, we know how to create new values from old values. We only need to model the time coordination problem.

What is nice about this is: when you separate the two things, when you have not unified values with pieces of memory, you end up with multiple options for the time semantics. You have a bunch of different ways to look at it. There is message passing, and there is transactional. But because it is now a separate problem, you can take different options. You can even have multiple options in the same program.

So I am going to say that there is a two-pronged approach. One part is programming with values, the other part is – in this example that I am going to be talking about today, in Clojure's example – is the concept called "managed references", which you can think of as, they are kind of like variables, except it fixes all of the problems with variables. In other words, they are variables that have coordination semantics, so they are pretty easy to understand. They are just variables that are not broken.

[Time 0:23:49]

```
slide title: Persistent Data Structures

+ Composite values - immutable
+ 'Change' is merely a function, takes one value and returns another,
  'changed' value
+ Collection maintains its performance guarantees
  + Therefore new versions are not full copies
+ Old version of the collection is still available after 'changes',
  with same performance
+ Example - hash map/set and vector based upon array mapped
  hash tries (Bagwell)
```

So there are two parts. We are going to talk now about the values. One of the things that people cringe at initially, if they have not used functional programming languages before, is "that sounds expensive". If I have to copy the whole runner every time he moves his foot, there is just no way I am going to do this.

And in particular, when you start talking about collections and things like that, people get extremely paranoid. Because what they know are sort of the very bad collection class libraries they have, which either have *no* capabilities in this case, or some very primitive things. Sometimes there are copy-on-write collections, where every time you write to it, the entire thing gets copied.

But there is a technology, which is not complicated, and it has a fancy name. It is called a persistent data structure. That has nothing to do with databases or disks. But it is a way to efficiently represent a composite object as a value, as an immutable thing, and to make changes to that in an inexpensive way.

So change for one of these persistent data structures is really just in quotes. They do not change. It is a function that takes an existing instance of the collection or composite, and returns another one that has the change enacted.

But there is a very particular meaning to Persistent Data Structures, which is that, in order to make these changes, the data structure and the change operation have to meet the performance guarantees you expect from the collection. So if it is a big O log N collection, or a collection that has constant time access, or near constant time access, those behavioral characteristics have to be met by this changing operation. It means you cannot conduct the change on something that you expect to have log N behavior and copy the entire thing, because copying the entire thing is linear behavior, right? So that is the critical thing.

The other critical aspect of persistent data structure is: sometimes you will see libraries try to cheat. They will make the very most recent version this good immutable value, but on the way they ruin the old version. That also is not persistent. That is another key aspect of the word persistent. In a persistent data structure, when you make the new version, the old version is perfectly fine. It is immutable. It is intact. It has the same performance guarantees. It is not decaying as you produce new values.

Every functional programming language tries to cheat this side, and eventually says "forget this, we are going all immutable and we are going to pay whatever the performance costs are, because the logical cost of having old versions decay, or have some bizarre behavior either from a multi-threading perspective or performance perspective, is too high". So what I am going to show you are legitimate persistent collections where the old values have the same performance characteristics.

And the particular example I am going to show you is the one that is in Clojure. It is derived from some work that Phil Bagwell did on these ideal, he called him ideal hash tries. And there are bit mapped hash tries that have really good performance. His versions were not persistent, and so what I did for Clojure was I made them persistent.

[Time 0:26:57]

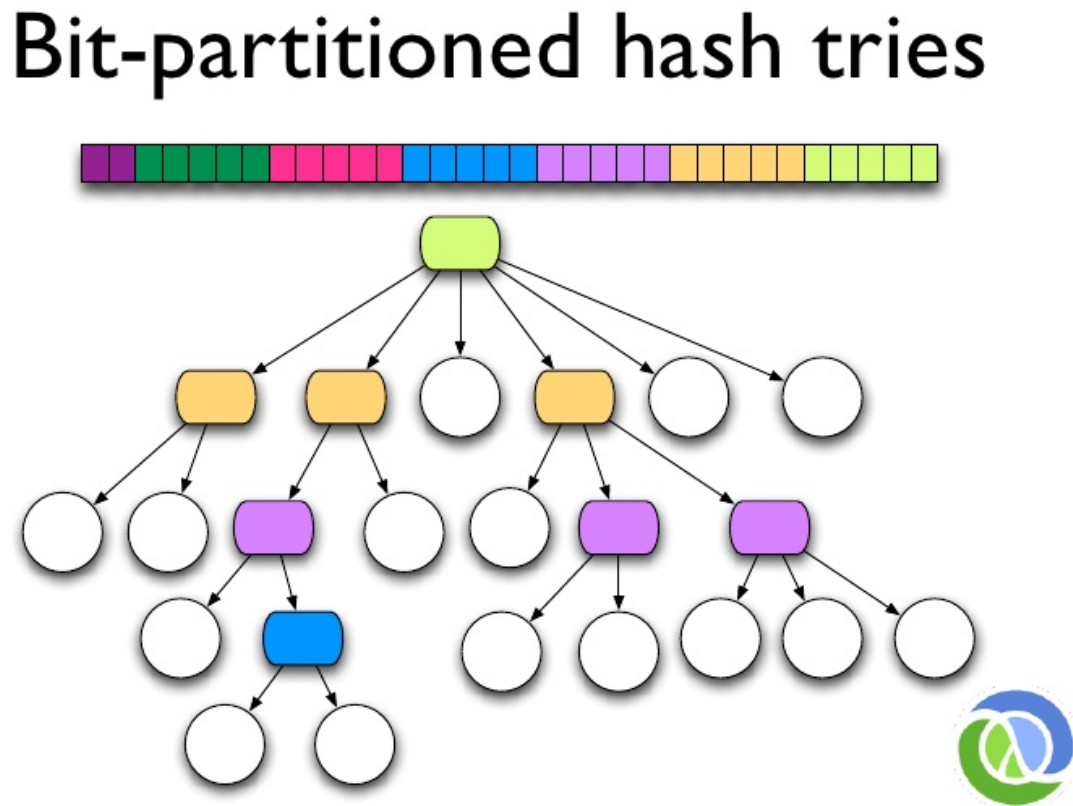slide title: Bit-partitioned hash tries

Figure 1: 00.26.57 PersistentDataStructure

The secret to all persistent data structures is that they are tries. There you go. Now you know. There are lots of different recipes and I think people are very familiar with B-trees and red-black trees, or maybe you know, Erlang uses some generalized balanced trees, I think, which are interesting. And there are trees that use randomization techniques for balancing and other things. These are different. In particular, they are different because they are tries with the `i`, some people call them `tries` [rhymes with fries]. Tries [rhymes with flees].

The idea behind that is that are not going to have a fixed path down to a leaf. You are going to use only as much of a path as you need to produce a unique leaf position. You usually see these things in string search things, or maybe I think they are also used in Internet routing tables and stuff like that.

Here the model is very simple. We want – at least I wanted for Clojure – something equivalent to a hash table. I know I cannot sell Clojure to Java programmers if it does not have something equivalent to a hash table. They do not want to hear about a red-black tree. They known that it is OK, but it is not as good as the hash tables they are used to. They need something faster than that. And these are.

The way they work is that you hash the value you want to put into the collection. You end up with a 32 bit hash. You are going to use the first 5 bits of that hash to see if there is a unique position in the first layer of your trie. So effectively what is happening is you have a 32-way branching going on in this trie. In addition, there is some fancy bit twiddling going on in each node so that those nodes are sparse. They are not fully populated, so you are not wasting the space of not fully populated nodes.

You end up using a combination of population, bit pop, and some algorithms you copy out of "Hacker's Delight". Buy that. Or you can just use Clojure's. In fact, Clojure's vectors, which is the same kind of technology, has been ported to Factor and Scala already, which is fine by me.

So if it is unique in the first 5 bits we are done. We put it in the first level of the trie. If it is not, we are going to look at the next 5 bits and walk down one more level to the trie, until we find some unique position, and then we are done. We are going to put the value there.

The key thing about this is: how deep can this trie get? So this one is the root so, down one two three four five six if you had, whatever, 4 billion things. So it branches extremely fast, and you can get a million items in depth three. It is very very shallow. So the combination of it being very shallow, and using this bit twiddling to walk through the sparsely populated nodes in the intermediate levels, makes it really fast. So that is the representation, now we only need to talk about: how do we make a changed version efficiently?

[Time 0:30:06]

slide title: Structural Sharing

+ Key to efficient 'copies' and therefore persistence
+ Everything is immutable so no chance of interference
+ Thread safe
+ Iteration safe

And the key there, as is true for all of these things, is structural sharing. All functional data structures are essentially recursively defined, structurally recursively defined. Which means that you can make a new version that shares substantial structure with the version you just had. And that is the key to making efficient copies. You are not copying everything. You are copying a very little bit. I will show you in a picture in a second how little a bit you use.

Since everything is immutable, sharing structure is no problem. Nothing is going to change about the structure that you are sharing, which means it is safe for multi-threaded use. It is safe for iteration. You get none of this mutated while iterating nonsense.

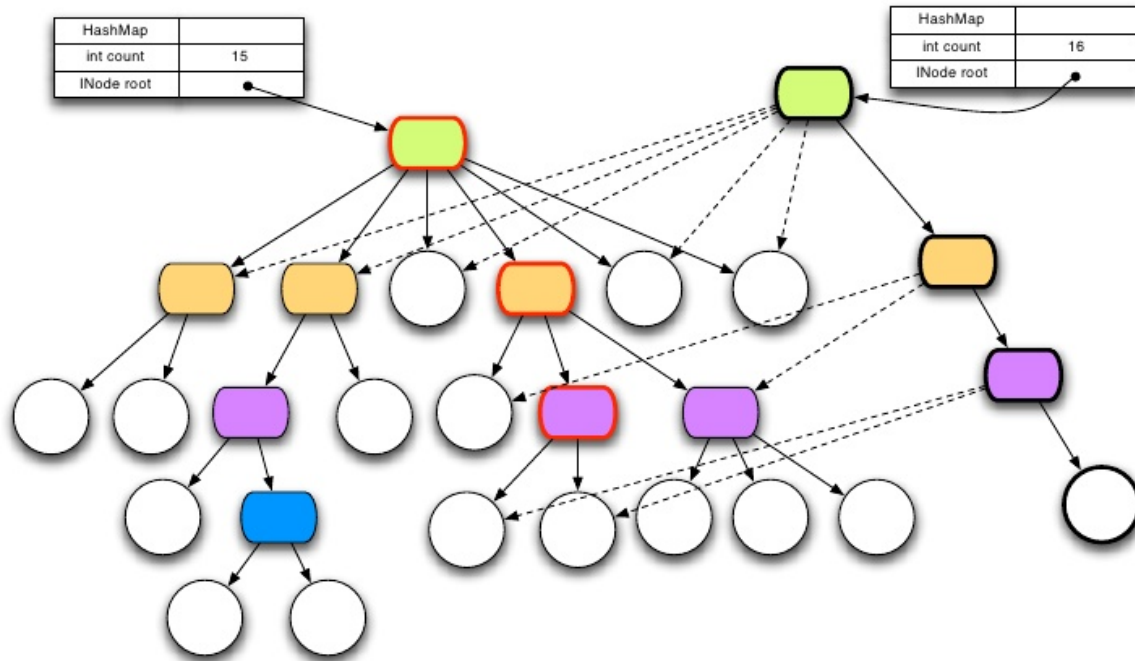`slide title: Path Copying`

Path Copying

Figure 2: 00.30.48 PersistentDataStructure

So how do we share structure? We use a technology called path copying. Again, this is true for all trie data structures. They all work exactly the same way, which is: if we ignore the right hand path here, that is the trie I showed you before. It has 15 leaves. We want to add one under that red outlined purple guy at the bottom. We want to add a new node. Add a 16th guy.

So what needs to happen is: we need to make a copy of that node, obviously, because we are going to be giving him a new child. A copy of his parent. And finally, the root. This copy gets one additional child, and the rest of the structure of the old version was shared.

So I said 3 levels could hold a million items, right? 32 times 32 times 32, did I get that right? No, that is 32,000. A lot.

[Audience laughter]

Well, 3 levels down. If you count the root, it is 4 levels. However populated this last level was, making a new node here is only ever going to copy 4 items.

How is that old tree? Looking good, still fine. We did not touch it. And this is the path we need to copy to make the new one, which looks like a new tree with this extra item. If we are no longer

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referring to that root, it will get garbage collected, as will the things it was referring to that are no longer referenced.

So it is kind of basic, but I want to show it because a lot of people just are not aware that this is a possible thing. This is the kind of data structure I think you should be using all the time unless you have some emergency reason. And that is why Clojure works this way. All of the data structure work like this, by default. You have to go through extraordinary efforts to pick something else.

[Time 0:32:40]

```
slide title: Coordination Methods

+ Conventional way:
  + Direct references to mutable objects
  + Lock and worry (manual/convention)
+ Clojure way:
  + Indirect references to immutable persistent data structures
    (inspired by SML's ref)
  + Concurrency semantics for references
    + Automatic/enforced
    + _No locks in user code!_
```

So that is the way to efficiently represent composite objects as values. We got one part of the problem solved. Now we need to talk about coordination methods.

The conventional way is not really a method. The conventional way is: it is your problem. We saw in the Scala talk, there was a var. It did not have volatile semantics, but it happened to be the case that the actors library in Java conducts some synchronization thing which causes a happens-before, happens-after memory fence effect, in order to make sure the contents of that var was valid in another thread. In your own program, that is going to be your worry. It is nice that the actors library takes care of vars in actors. But vars in your program otherwise are your problem.

And typically if we are trying to do composite objects we have to use locks. And everybody knows the problems with locks, I think. Everybody know the problem with locks? Everybody know the pain of locks? Locks are . . . Experts can build programs that work with locks, but most people do not have the time or energy to do that well, and maintaining it is really really difficult. It is *extremely* difficult.

So in Clojure what we are going to just do is just add a level of indirection. Instead of directly referring memory, those variables, we are going to use indirection. And then we are going to add concurrency semantics to these references. If you watched me talk yesterday I said that, but I will show you some more details today.
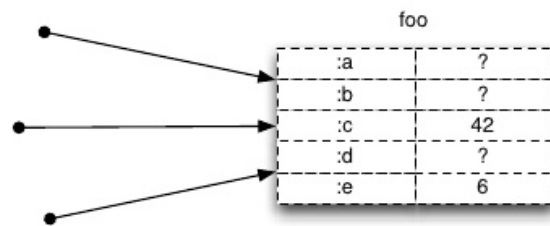
[Time 0:34:05]

```
slide title: Typical OO - Direct references to Mutable Objects

[ figure ]

+ Unifies identity and value
+ Anything can change at any time
+ Consistency is a user problem
+ Encapsulation doesn't solve concurrency problems
```

So this, quickly, is the picture of the current state of the world in a lot of object-oriented languages. You have a lot of references to the same chunk of memory. Basically it is a free for all. They do not

Typical OO - Direct references to Mutable Objects

- Unifies identity and value
- Anything can change at any time
- Consistency is a user problem
- Encapsulation doesn't solve concurrency problems

Figure 3: 00.34.05 PersistentDataStructure

know they are going to see a consistent object, that all the parts are related to each other, that no one is twiddling with anything, unless they can somehow stop the world.

But the core problem here now that we have lingo for it is: this unifies identity and value, right? The only place to put this value for this identity is in the same piece of memory. That is a problem. We just looked at how to do new values. It is great. So what do we need to do? We need to create some new memory to represent that new value. If we say all values of foo have to end up in the same chunk of memory space, we can never do a good job. So that has a lot of problems.
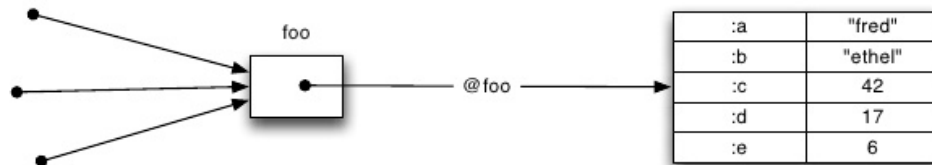
[Time 0:34:53]


```
slide title: Clojure - Indirect references to Immutable Objects

[ figure ]

+ Separates identity and value
  + Obtaining value requires explicit dereference
+ Values can never change
  + Never an inconsistent value
+ Encapsulation is orthogonal
```


Figure 4: 00.34.53 PersistentDataStructure

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How do you solve this? You just use indirection. It is the solution to all computer science problems, right? One level of indirection, and now we have options. Because this guy now can be immutable. We have separated the value, which is now immutable, and the identity, which we are going to model with these little boxes.

Values never change. If you want to see the current state of an identity, you have to dereference it. You have to say "give me your state". What you get out of that is a value that cannot change. You can spend all day looking at it, just like you can spend all day looking at the photo to try to see if the runner was fouled.

I want to emphasize, if you think your object oriented programming language's encapsulation techniques are a solution to this problem, that is *not* true. If you have a variable or a field inside your object and you write three methods that can change that field, and people can call those methods from different threads, you have not encapsulated *anything* from a concurrency standpoint. You have just spread the problem and hid it behind something.

[Time 0:35:57]

```
slide title: Clojure References

+ The only things that mutate are references themselves,
  in a controlled way
+ 4 types of mutable references, with different semantics:
  + Refs - shared/synchronous/coordinated
  + Agents - shared/asynchronous/autonomous
  + Atoms - shared/synchronous/autonomous
  + Vars - Isolated changes within threads
```

So I am going to call those boxes references. We have too many overloaded terms. I cannot think of any new words. It is a reference because it refers to something else. So identities are references that refer to their values.

But the critical thing is: in Clojure these are the only things that you can mutate, unless you drop to Java and use Java stuff. Of course there are still classes and arrays and all that. But if you want to follow the Clojure model, you are going to have these references. They are the only things that can change. And what they do is just manage time. In other words, you can atomically move from one value which is immutable, to another value which is immutable, and each of the reference type provides different semantics for time.

So what are the characteristics of these semantics? One of them is: "Can other people see these changes I am making? Is it shared?" Because there is one way to manage time, which is the Star Trek alternate universe model, where there is a bad Kirk in one universe timeline, and a good Kirk in another, and they will never meet. Of course the problem with that is that occasionally they do meet. But one way is isolation. So we will see the last model is isolation. But in general most of these models are around making changes that other parts of your program can see, so sharing.

The second part is synchronicity, and here we mean synchronicity in the sense of now. What now means to the caller. In other words, from a self-relative standpoint, is the change I am asking for going to happen now, or at some other time, relative to me? Is it independent? And we are going to call those differences synchronous. If it happens now relative to me it is synchronous. If it does not happen now relative to me it is asynchronous. It just happens at some point in the future. We cannot say exactly when.

[Time 0:37:50]

And the final characteristic of these references, where again you get different choices and options, is whether or not the change is coordinated. I can be an independent runner and run all by myself, and I am completely fine. But a lot of times you need to move something from one collection to another collection. You do not ever want it to be in both. You do not ever want it to be in neither. That requires coordination. That is impossible to do with independent autonomous entities. You need coordination.

And it ends up that in the local case you can do coordination. Distributing coordination like this is a fool's errand probably, but people keep trying. I do not think that there is ever going to be distributed coherent coordinated change, but people are already recognizing the fact that if you are willing to delay consistency, you can sort of have coordination. But in the local model, it is perfectly possible to get coordinated change.

Otherwise, change is autonomous. I change by myself. I do not care what you are doing, and no two of us can do something together.

So now we have these 3 characteristics. Clojure has 4 types of references that make different choices in these 3 areas. Refs are shared, people can see the changes, they are synchronous, they change right now. They change in a transaction, which means that you can change more than one reference in the same transaction and those changes will be coordinated. Sort of the hardest problem is that coordinated change problem.

Agents are autonomous. They will feel a lot more like actors in an actor model. They are shared, people can see them. They are asynchronous, so you ask for a change, it is going to happen at some point in the future, but you are going to immediately return. And they are autonomous. There is no coordinating the activities of agents.

[Time 0:39:42]

Atoms are shared. People can see the changes. They are synchronous. They happen right now, so that is the difference between them and agents. And they are also autonomous. You cannot change more than one atom in a single unit of work.

And finally, Clojure has something called vars. They isolate changes with the good Kirk bad Kirk alternate universe model. For any identity there is a unique value in every thread. You cannot possibly see the changes in different threads. I am not going to talk too much about that, but it is kind of a special purpose construct derived from Lisps.

[Time 0:40:17]

```
slide title: Uniform state transition model

+ ('change-state' reference function [args*])
+ function will be passed current state of the reference (plus any args)
+ Return value of function will be the next state of the reference
+ Snapshot of 'current' state always available with deref
+ No user locking, no deadlocks
```

One of the things that is nice about the way these references work is: they have a uniform state transition model. All of them have different functions that change the state, that say: move from one state to another state. And they use different names because they have different semantics. I do not want people to get all confused about: is this happening synchronously or asynchronously, or do I need a transaction?

But the model is always the same. You are going to call one of the changing functions. You are going to pass the reference, the box, and you are going to say, "Please use this function". So you are going

to pass a function, maybe with these arguments, apply it to the current state of the box, and use its return value as the new state. So the function will be passed the current state, under some constraints, either atomically, within a transaction, some way. It will be passed the current state. It can calculate a new state. Again it is a pure function you are passing. That new state becomes the new value of the reference.

In Clojure's references, you can always see the current state of a reference by dereferencing it. In other words, that is the local visibility, because it is completely free to do. And it yields much more efficient programs, to be able to do that. If you have to ask for permission to see collections every time you want to see them, it does not work in the local context.

In addition, one of the other shared attributes of these things is that there is no user locking, you do not have any locking to do this work, and none of these constructs can deadlock.

[Time 0:41:45]

```
slide title: Persistent 'Edit'

[ figure ]

+ New value is function of old
+ Shares immutable structure
+ Doesn't impede readers
+ Not impeded by readers
```

So what does it mean to edit something in this new world? You are going to have a reference to a value. We can make a new value a la carte, on the side. We are going to call a function and create this new value, which we intend to become the new state of foo.

The new values are a function of the old. They can share structure. We just saw that. Doing this does not impede anybody who is reading foo. They are completely free to keep reading. They do not have to stop while we figure out the new version of foo.

In addition, it is not impeded by people reading. We do not have to wait for people to stop reading so we can start making a new version. This is the kind of thing you are going to need for high-throughput concurrency.

[Time 0:42:25]

```
slide title: Atomic State Transition

[ figure ]

+ Always coordinated
  + Multiple semantics
+ Next dereference sees new value
+ Consumers of values unaffected
```

And then going to a new state is just an atomic swapping of this box to look at the new immutable value. That is always coordinated. There are always rules for how that happens. I just showed you the multiple semantics. Any time somebody dereferences this after – more time words – after this happens, they will see the new value.

Consumers are unaffected. If I was looking at the old value, I do not get disturbed by this happening. I am just looking at an old value. It is like I am looking at a picture of the runner to see. I know the race is over. That is OK. We *need* to behave that way.
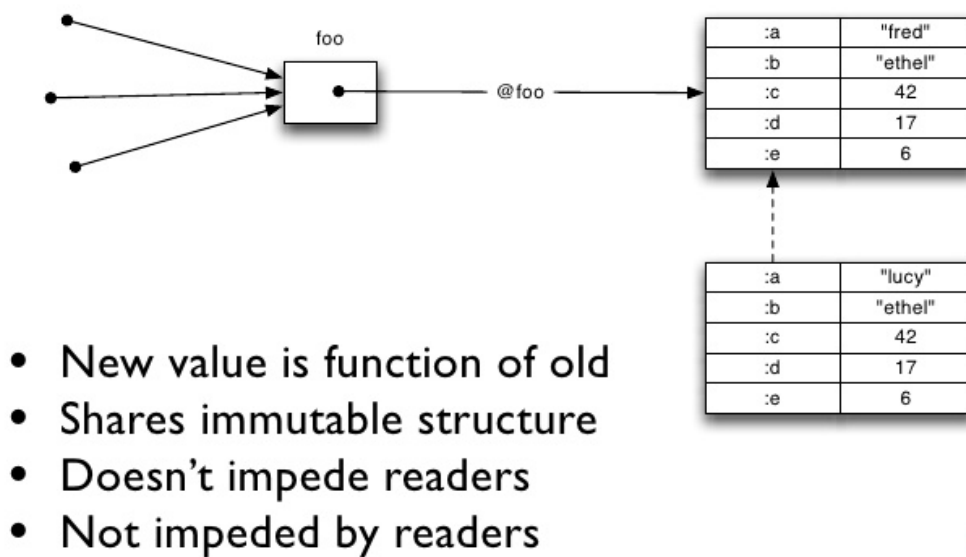
Persistent 'Edit'

:a	"fred"
:b	"ethel"
:c	42
:d	17
:e	6

foo → @foo →

:a	"lucy"
:b	"ethel"
:c	42
:d	17
:e	6

- New value is function of old
- Shares immutable structure
- Doesn't impede readers
- Not impeded by readers

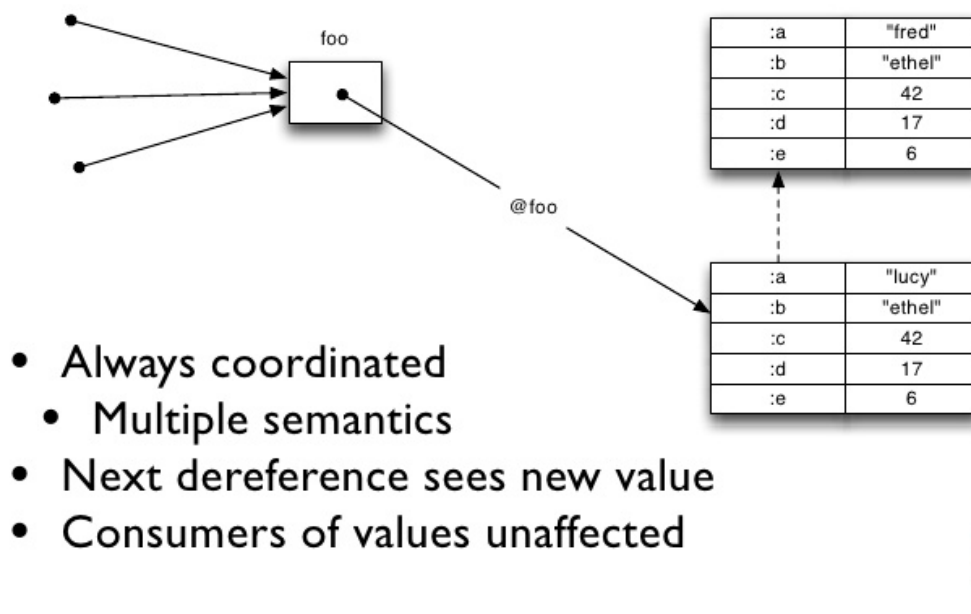Figure 5: 00.41.45 PersistentDataStructure

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Figure 6: 00.42.25 PersistentDataStructure

If you have been programming for so long as I have, it is really hard to break from, "I own the world. And I stop the world. The world goes when I say go." We have to just break from that. That is the future. We have to understand that we are going to be working with data that is not necessarily the very latest data. That is just the future for us.

[Time 0:43:18]

```
slide title: Refs and Transactions

+ Software transactional memory system (STM)
+ Refs can only be changed within a transaction
+ All changes are Atomic and Isolated
  + Every change to Refs made within a transaction
    occurs or none do
  + No transaction sees the effects of any other transaction
    while it is running
+ Transactions are speculative
  + Will be retried automatically if conflict
  + Must avoid side-effects!
```

OK, so the hard reference, as I said, are the transactional ones. Clojure has a software transactional memory system. I almost hate using this term because people like to criticize STM as if it was one thing. There are a whole bunch of different STMs. They have radically different characteristics. Clojure's is radically different from the other ones.

But they all share some things, which is basically a model that feels a lot like a database model. You can only change them within a transaction. All the changes you make to an entire set of references, refs, inside a transaction, happen together, or none of them happen. That is atomicity. You do not see the effects of any other transactions while you are running. They do not see your effects. It is the normal things.

The one unique thing about STM transactions is that they are speculative. You may not win. Somebody else may win. You will automatically retry up to a certain limit, which means that your transactions cannot contain side effects.

[Time 0:44:15]

```
slide title: The Clojure STM

[ photo of rowing team in a racing boat on the water ]

+ Surround code with (dosync ...), state changes through
  _alter/commute_, using ordinary function (state => new-state)
+ Uses Multiversion Concurrency Control (MVCC)
+ All reads of Refs will see a consistent snapshot of the
  'Ref world' as of the starting point of the transaction, + any
  changes it has made.
+ All changes made to Refs during a transaction will appear to
  occur at a single point in the timeline.
```

This is the way you do coordination. You cannot really do coordination without some technique like this. You cannot build a system on independent entities and do this kind of work.

So in practice what do you do? You just wrap your code with `dosync`, which just means this is a transaction. There are two functions alter and commute, which work like I described. They take a reference and a function and some args and say, "apply this to the reference in the transaction and make the return value the new state."

Internally Clojure uses multiversion concurrency control (MVCC) which I also think is a very critical component to doing STM in a way that is going to work in the real world. A lot of STM designs are: you just write your app in the terrible way you were, with your object oriented language, banging on fields, and STM is magically going to make that better.

I do not believe in that at all. Clojure's STM is not designed for that kind of work. If you make every part of your object a ref, it is not going to work and I am not going to feel bad for you, because I just explained how to do it. You make your object the value and atomically switch that value, and everything is better.

But you do have this issue of, again, people would criticize STMs universally, because *most* STMs do something called "read tracking". In order to make sure that nothing bad happened while your transaction was going on, they track every *read* that you do, in addition to all the writes that you do. I also believe that that is not going to work. So Clojure does no read tracking.

The way it accomplishes that is with a technique called Multiversion Concurrency Control, which is the way Oracle and PostgreSQL work as databases, where essentially old values can be kept around in order to provide a snapshot of the world for transactions, while other transactions that are writing can continue. That ends up being extremely effective.

But it falls out of this necessity to be using references to *values*. It has got to be cheap for me to keep an old value around for you. I just showed you how it is cheap, if you are using persistent data structures. All these things go together. If you do not do all of this stuff together, you do not have an answer to this problem, in my opinion. But when you do this, it is really nice. So MVCC STM does not do read tracking.

[Time 0:46:24]

```
slide title: Refs in action

(def foo (ref {:a "fred" :b "ethel" :c 42 :d 17 :e 6}))

@foo -> {:d 17, :a "fred", :b "ethel", :c 42, :e 6}

(assoc @foo :a "lucy")
-> {:d 17, :a "lucy", :b "ethel", :c 42, :e 6}

@foo -> {:d 17, :a "fred", :b "ethel", :c 42, :e 6}

(commute foo  assoc :a "lucy")
-> IllegalStateException: No transaction running

(dosync (commute foo  assoc :a "lucy"))
@foo -> {:d 17, :a "lucy", :b "ethel", :c 42, :e 6}
```

So what does it looks like in practice? We define foo to be a ref. That is a transactional box to that map. We can dereference foo, and we see what is in there. Unfortunately the name order changes because they are hash maps, so they do not guarantee any order of iteration.

We can go and manipulate the value inside foo. We can say, "give me the map that is inside foo, and associate the `a` key with `lucy`". That returns a new value. Nothing about that impacted the reference.

I took the value out. I made another value. We can do all kinds of calculations completely outside of the transactional system. It is still a functional programming language. Get the value out and write functional programs. So that did not have any effect on foo.

We can go and we can use that `commute` function, which actually says: take a reference, commute a reference with a function `assoc`, which adds a value to a map, the key `a` and the value `lucy`. And that fails, because there is a semantics to those refs which is that you can only do this for refs inside a transaction. So you get an error.

If however you put that same work inside a transaction, it succeeds. And when the transaction is complete, that is the value of foo.

[Time 0:47:40]

```
slide title: Implementation - STM

+ _Not_ a lock-free spinning optimistic design
+ Uses locks, wait/notify to avoid churn
+ Deadlock detection + barging
+ One timestamp CAS is only global resource
+ No read tracking
+ Coarse-grained orientation
  + Refs + persistent data structures
+ Readers don't impede writers/readers, writers don't impede readers,
  supports _commute_
```

I do not have a lot of time to talk about the implementation details. But again, do not think that STM is one thing. If you have read one paper on STM, you know nothing about STM. If you have read all the papers about STM, you know a little bit more than nothing about STM. None of us knows anything about STM. This is still a research topic.

But I do know this: this works, and it works really well, and it makes it easy to write programs that do not use locks. I think all the programs that I have written in my career, I could have used this any time I needed coordinated change, and it would have been fine. People can bang on it and try to push the scalability issues and whatnot. From a correctness standpoint, this is a godsend.

However, inside, unlike some STMs, Clojure's STM is not spinning optimistic. It does use locks. It uses wait notify, it does not churn. Processes will wait for other processes. It has got deadlock detection. It has got age-based barging. There is extreme minimum – in fact, I think what is actually the minimum amount – of sharing in a transactional system, which is one CAS, which is for the timestamps.

People have demonstrated you can hammer on one CAS continuously with 80 threads, and that is about the limit of scalability. But when you actually have some work in your transactions, it is no problem. I have run stuff on an Azul box with 600 cores, and that CAS is not going to be the problem.

As I said there is no read tracking. It is important that this STM is designed for coarse-grained orientation. It is not one of these snake-oil STMs that you can do what you were doing. You have to do this new thing. You have to use references to immutable values, then you can use my STM. It is not going to make your old programs good.

And readers do not then get impeded by writers, and vice versa. It also supports commute, which I do not really have time to explain right now.

[Time 0:49:33]

```
slide title: Agents
```

```
+ Manage independent state
+ State changes through actions, which are ordinary functions
  (state => new-state)
+ Actions are dispatched using _send_ or _send-off_, which
  return immediately
+ Actions occur _asynchronously_ on thread-pool threads
+ Only one action per agent happens at a time
```

I do want to show you one other model, because it is very different, And it is nice in that it is very different, yet very much the same. Because we have sort of isolated change from values, you can take a completely different approach to time.

So in an agent, which is another kind of these reference cells, each agent is completely independent. They have their own state and they cannot be coordinated with any other. State change is through actions, which are essentially just ordinary functions that you are going to send to the agent with a function called send or send-off. That function is going to return immediately. You are going to send this function and some data, say, "at some point in the future, apply this function to the current value of the agent with these arguments, and make the return value of the function the new state of the agent."

That happens asynchronously on a thread from a thread-pool. Only one action per agent happens at a time, so agents essentially have sort of an input mailbox queue. So they also do all of their work serially. This is another promise of the semantics of an agent.

[Time 0:50:38]

```
slide title: Agents

+ Agent state always accessible, via _deref/@_, but may not
  reflect all actions
+ Any dispatches made during an action are held until _after_
  the state of the agent has changed
+ Agents coordinate with transactions – any dispatches made
  during a transaction are held until it commits
+ Agents are not Actors (Erlang/Scala)
```

Again as with the other reference types, you can just dereference it and see what is in there. If you do successive actions to agents inside the same action, they are held until the action completes, so they can see the new state.

The agents do coordinate with transactions, which is kind of nice. So one of the problems is you saw, "no side effects in transactions". So you are wondering, how do I let somebody know I completed this transaction successfully? Do I need to send them a message, or do something side effect-y? It ends up that if you send an agent action during a transaction, that is held until the transaction commits. So if the transaction gets retried, those messages do not go out until the transaction actually succeeds. So that coordination is a really nice feature. These two things work together.

They are not quite actors. The difference with an actor model is: that is a distributed model. You do not have direct access to the state in an actor model, because you cannot distribute that. Since I am not doing distribution, I can let you access the state directly, which means it is a suitable place to put something that you actually may need to share a lot, without necessarily serializing activity.

[Time 0:51:40]

```
slide title: Agents in Action

(def foo (agent {:a "fred" :b "ethel" :c 42 :d 17 :e 6}))

@foo -> {:d 17, :a "fred", :b "ethel", :c 42, :e 6}

(send foo assoc :a "lucy")

@foo -> {:d 17, :a "fred", :b "ethel", :c 42, :e 6}

... time passes ...

@foo -> {:d 17, :a "lucy", :b "ethel", :c 42, :e 6}
```

So what does this look like to use? I say `def foo` to be an **agent** referring to a map. I dereference it. I see the contents of the map. I send that reference the same function **associate :a** with `lucy`. I look at it right away. It may not be there yet. Some amount of time will pass. I cannot promise you what. And then it will be different.

This is a different way of thinking about things, but people who program in Erlang completely do amazing things with thinking about things this way. Things can be asynchronous. You cannot keep programming your computer as if it was your old Apple and there was only you and your assembly language and you were king of the universe. Things happen at the same time now.

[Time 0:52:27]

```
slide title: Atoms

+ Manage independent state
+ State changes through _swap!_, using ordinary function
  (state => new-state)
+ Change occurs _synchronously_ on caller thread
+ Models compare-and-set (CAS) spin swap
+ Function may be called more than once!
  + Guaranteed atomic transition
  + Must avoid side-effects!
```

Atoms, a very similar story to agents. They are independent. You cannot coordinate change to atoms. There is a different name for the state change function. It is called `swap`. Again it takes an ordinary function of the old state to the new state. The change happens *synchronously* now, so that is the difference between atoms and agents. It happens right now.

This is a model for compare and swap. Compare and swap, or compare and set, is a primitive that is going to let you look at a piece of compare and set memory and say, "I want it to turn in to this." And it will turn into this only if it is still that. So you look at it. You see it is that. You want to turn it into this. If it is still that, inside atomically it will say, OK I will make it this.

The problem with CAS by itself is: you usually want to read the value, do something with it, and then put it back. So you get this interval between when you looked at it, and when you try to do the CAS. And of course when you do that, and somebody else has done something, that CAS is going to fail. And then what do you do? Typically, a well written CAS thing, where CAS is a suitable data structure, will have a little spin loop. You are going to spin in a value into a CAS. Atoms do the spinning for you. As a result, the function may be called more than once.

Again, we are in this world where you should be programming with these side effect free functions, because they need to be called more than once, both in transactions and in atoms. So you have to avoid side effects, but the value you get out of this is that when you succeed, you know the function you applied was applied to the value the function was passed, and the result that got put in had no intervening activity occur on that atom. That is a powerful construct you need to have.

[Time 0:54:08]

```
slide title: Atoms in Action

(def foo (atom {:a "fred" :b "ethel" :c 42 :d 17 :e 6}))

@foo -> {:d 17, :a "fred", :b "ethel", :c 42, :e 6}

(swap! foo assoc :a "lucy")

@foo -> {:d 17, :a "lucy", :b "ethel", :c 42, :e 6}
```

And look at these. It looks a lot like the other ones. Define `foo` to be an `atom` that refers to that map. Dereference it, it is there. We swap. Immediately we get the new value.

[Time 0:54:22]

```
slide title: Uniform state transition

; refs
(dosync
  (commute foo  assoc :a "lucy"))

; agents
(send foo assoc :a "lucy")

; atoms
(swap! foo assoc :a "lucy")
```

So this is the uniform state transition model. That is what refs look like. Start a transaction, commute or alter. Your ref, passing a function and some arguments. The result of the function is your new value.

Agents, same thing, except completely different time semantics. It happens asynchronously in a thread-pool, some time later. You return immediately.

Atoms, happen right now, but are independent from the others.

You need all of these things to write a real multi-threaded program, especially in the local model. These are all things that I have needed to do in my career writing concurrent programs in the local part of the program, and I do not think you can do without them. So here they are, but it is a uniform way to go.

[Time 0:54:57]

```
slide title: Summary

+ Immutable values, a feature of the functional parts of our programs,
```

```
    are a critical component of the parts that deal with time

+ Persistent data structures provide efficient immutable
  composite values

+ Once you accept immutability, you can separate time management,
  and swap in various concurrency semantics

+ Managed references provide easy to use and understand time
  coordination
```

So, in summary, immutable values are critical for functional programming. But it ends up they are also critical for state. We cannot really manage time and state without immutable values. If you you are going to let two things change, time and value, you cannot do anything that is reliable.

Persistent data structures let you represent composite objects efficiently, immutably. Once you are able to accept this constraint of immutability on your values, you have all of these options. I am working on a fifth reference type with slightly different semantics. It is easy to do, because I have separated time management from value management.

Finally, I think this is pretty easy to use. If you have seen some other models, this is a lot like variables that work.

[Time 0:55:51]

```
slide title: Thanks for listening!

      [ Clojure logo ]

    http://clojure.org

        Questions?
```

So, thank you!

[Time 0:55:52]