Cálculo de Programas
Algebra of Programming

Lic./Mest.Int. em Engenharia Informática (3^{o} ano)
Lic. Ciências da Computação (2^{o} ano)
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1. No cálculo de programas, as definições condicionais do tipo

Conditional expressions of pattern

$$h\ x = \textbf{if}\ p\ x\ \textbf{then}\ f\ x\ \textbf{else}\ g\ x \tag{F1}$$

são escritas usando o combinador ternário

are expressed in the algebra of programming by the ternary combinator

$$p \rightarrow f, g$$

conhecido pelo nome de *condicional de McCarthy*, cuja definição

known as the McCarthy conditionald*, whose definition*

$$p \rightarrow f, g \quad = \quad [f\ , g] \cdot p? \tag{F2}$$

vem no formulário. Baseie-se em leis desse formulário para demonstrar a chamada 2^{a}-lei de fusão do condicional:

can be found in reference sheet. Use this reference sheet to prove the so-called 2nd fusion-law of conditionals:

$$(p \rightarrow f, g) \cdot h \quad = \quad (p \cdot h) \rightarrow (f \cdot h), (g \cdot h)$$

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2. Numa máquina paralela pode fazer sentido, em (F1), não esperar por $p\ x$ para avaliar ou $f\ x$ ou $g\ x$, mas sim correr tudo em paralelo,

On a parallel machine it might make sense, concerning (F1), not to wait for $p\ x$ to evaluate either $f\ x$ or $g\ x$, but rather to run everything in parallel,

$$parallel\ p\ f\ g = \langle \langle f, g \rangle, p \rangle$$

e depois fazer a escolha do resultado:

and the choose the outcome:

$$choose = \pi_2 \rightarrow \pi_1 \cdot \pi_1, \pi_2 \cdot \pi_1$$

Mostre que, de facto:

Show that, indeed:

$$choose \cdot parallel\ p\ f\ g \quad = \quad p \rightarrow f, g$$

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3. Sabendo que as igualdades

Assuming

$$
\begin{aligned}
p \to k, k &= k && \text{(F3)}\\
(p? + p?) \cdot p? &= (i_1 + i_2) \cdot p? && \text{(F4)}
\end{aligned}
$$

se verificam, demonstre as seguintes propriedades do mesmo combinador:

prove the following laws of the McCarthy conditional:

$$
\begin{aligned}
\langle (p \to f, h), (p \to g, i) \rangle &= p \to \langle f, g \rangle, \langle h, i \rangle && \text{(F5)}\\
\langle f, (p \to g, h) \rangle &= p \to \langle f, g \rangle, \langle f, h \rangle && \text{(F6)}\\
p \to (p \to a, b), (p \to c, d) &= p \to a, d && \text{(F7)}
\end{aligned}
$$

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4. Mostre que a propriedade de cancelamento da exponenciação

Show that the cancellation property

$$\mathsf{ap} \cdot (\overline{f} \times id) = f \qquad \text{(F8)}$$

corresponde à definição

is nothing but the definition

$$\mathsf{curry}\ f\ \ a\ \ b = f\ \ (a, b)$$

quando se escreve $\mathsf{curry}\ f$ em lugar de $\overline{f}$.

once $\mathsf{curry}\ f$ *is written instead of* $\overline{f}$.

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5. Mostre que a definição de $\mathsf{uncurry}$ se pode obter também de (F8) fazendo $f := \mathsf{uncurry}\ g$, introduzindo vaiáveis e simplificando.

Show that the definition of $\mathsf{uncurry}$ *can also be obtained from (F8) by instantiating* $f := \mathsf{uncurry}\ g$*, introducing variables and simplifying.*

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6. Prove a igualdade

Prove the equality

$$\overline{f \cdot (g \times h)} = \overline{\mathsf{ap} \cdot (id \times h)} \cdot \overline{f} \cdot g \qquad \text{(F9)}$$

usando as leis das exponenciais e dos produtos.

using the laws of products and exponentials.

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7. É dada a definição

Let flip *be defined by*

$$\mathsf{flip}\ f = \overline{\widehat{f} \cdot \mathsf{swap}} \qquad \text{(F10)}$$

de acordo com:

according to:

$$
\begin{array}{ccccccc}
(C^B)^A & \cong & C^{A \times B} & \cong & C^{B \times A} & \cong & (C^A)^B\\
f & \mapsto & \widehat{f} & \mapsto & \widehat{f}.\mathsf{swap} & \mapsto & \overline{\widehat{f} \cdot \mathsf{swap}} = \mathsf{flip}\ f
\end{array}
$$

Mostre que flip é um isomorfismo por ser a sua própria inversa:

Show that it is an isomorphism because it is its own inverse:

$$\mathsf{flip}\ (\mathsf{flip}\ f) = f \qquad \text{(F11)}$$

Mostre ainda que:

Furthermore show:

$$\mathsf{flip}\ f\ x\ y = f\ y\ x$$

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8. Mostre que

Show that

$$junc \cdot unjunc = id \qquad \text{(F12)}$$
$$unjunc \cdot junc = id \qquad \text{(F13)}$$

se verificam, onde

hold for

$$\begin{tikzcd} A^{B+C} \arrow[rr, bend left, "unjunc"] \arrow[rr, phantom, "\cong"] & & A^B \times A^C \arrow[ll, bend left, "junc"] \end{tikzcd} \qquad \left\{ \begin{array}{l} junc\ (f, g) = [f\ , g] \\ unjunc\ k = (k \cdot i_1, k \cdot i_2) \end{array} \right. \qquad \text{(F14)}$$

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9. Considere a seguinte sintaxe concreta em Haskell para um tipo que descreve pontos no espaço tridimensional:

Consider the following concrete syntax in Haskell for a type that describes 3D-points:

$$\textbf{data}\ Point\ a = Point\ \{x :: a, y :: a, z :: a\}\ \textbf{deriving}\ (Eq, Show)$$

Pelo GHCi apura-se:

GHCi tells:

$$Point :: a \to a \to a \to Point\ a$$

Raciocinando apenas em termos de tipos, conjecture a definição de in na seguinte conversão dessa sintaxe concreta para abstracta:

Reasoning only in terms of types, conjecture the definition of in *in the following conversion from concrete to abstract syntax:*

$$\begin{tikzcd} Point\ A \arrow[rr, bend left, "\mathsf{out} = \langle\langle x, y\rangle, z\rangle"] \arrow[rr, phantom, "\cong"] & & (A \times A) \times A \arrow[ll, bend left, "\mathsf{in} = \cdots"] \end{tikzcd}$$

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10. **Questão prática** — Este problema não irá ser abordado em sala de aula. Os alunos devem tentar resolvê-lo em casa e, querendo, publicarem a sua solução no canal **#geral** do Slack, com vista à sua discussão com colegas.
Dão-se a seguir os requisitos do problema.

***Open assignment** — This assignment will not be addressed in class. Students should try to solve it at home and, whishing so, publish their solutions in the **#geral** Slack channel, so as to trigger discussion among other colleagues.*
The requirements of the problem are given below.

***Problem requirements**: The solution given for a previous problem,*

$$store\ c = \mathsf{take}\ 10 \cdot nub \cdot (c{:}) \qquad \text{(F15)}$$

calls the standard function

$$nub :: (Eq\ a) \Rightarrow [\,a\,] \rightarrow [\,a\,]$$

available from the Data.List library in Haskell.

After inspecting the standard implementation of this function, define f *so that*

$$nub = [\mathsf{nil}\ , \mathsf{cons}] \cdot f.$$

is an alternative to the standard definition, where $\mathsf{nil}\ \_ = [\,]$ *and* $\mathsf{cons}\ (h, t) = h : t$. *Check that* $store\ c$ *(F15) works properly once the standard* nub *is replaced by yours.*

***Important**: Structure your solution across the* $f \cdot g$, $\langle f, g \rangle$, $f \times g$, $[f\ , g]$ *and* $f + g$ *combinators available from library Cp.hs. Use **diagrams** to plan your solution, in which you should avoid re-inventing functions over lists already available in the Haskell standard libraries.*

□