Lab Assignment 5: Karatsuba's Divide and Conquer Multiplication Algorithm

Let N_1 and N_2 be $2n$-digit numbers:

$N_1 = H_1 \times 10^n + L_1$,

$N_2 = H_2 \times 10^n + L_2$,

where H_1 is the number representing the high order n digits of N_1, and L_1 is the number representing the low order n digits of N_1. Similarly, H_2 is the number representing the high order n digits of N_2, and L_2 is the number representing the low order n digits of N_2.

We can write the product $N = N_1 \times N_2$ as follows:

$$N = N_1 \times N_2 = (H_1 \times 10^n + L_1) \times (H_2 \times 10^n + L_2)$$

$$= H_1 \times H_2 \times 10^n \times 10^n + (L_1 \times H_2 + H_1 \times L_2) \times 10^n + L_1 \times L_2$$

$$= B \times 10^n \times 10^n + (A - B - C) \times 10^n + C$$

where

$A = (H_1 + L_1) \times (H_2 + L_2)$,

$B = H_1 \times H_2$,

$C = L_1 \times L_2$.

Input: You will be given input in a single line as

N_1 X N_2

where N_2 and N_2 are numbers (not necessarily of the same length). Here "X" is the capital English letter X.

Procedure :

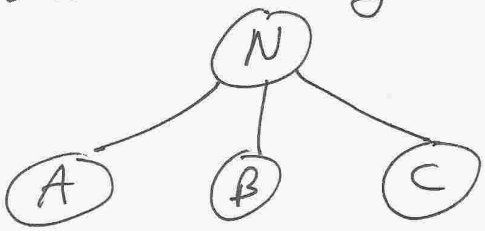
① Divide step (Top-Down Approach) : Make a Divide Tree that represents the problems and subproblems. The original problem $N = N_1 \times N_2$ is divided into three subproblems:

$A = (H_1 + L_1) \times (H_2 + L_2)$

$B = H_1 \times H_2$,

$C = L_1 \times L_2$.

We can represent this using tree as follows:



We recursively divide the subproblems A, B, and C into smaller subproblems and continue the process until we get a problem of multiplying two single digit numbers which will be the leaf nodes of the Divide Tree. For creating the Divide Tree, you will have to follow the level order traversal similar to lab assignment 3 (Game Tree Evaluation). You will have to make use of a queue. You will have to make use of a stack also. Whenever you delete a leaf node from the queue, you will have to push it on the stack.

(2) When the Divide Tree is ready, the queue will be empty and stack will have all the leaf nodes. Now pop the leaf nodes, and insert them into the queue.

(3) Conquer Step (Bottom-Up Approach): Now making use of the queue, perform the reverse level order traversal of the Divide Tree. Apply the Karatsuba's formula to solve the subproblems and print the problem together with its solution as follows (one problem and solution per line):

For leaf nodes:

$N_1 \times N_2 = N$

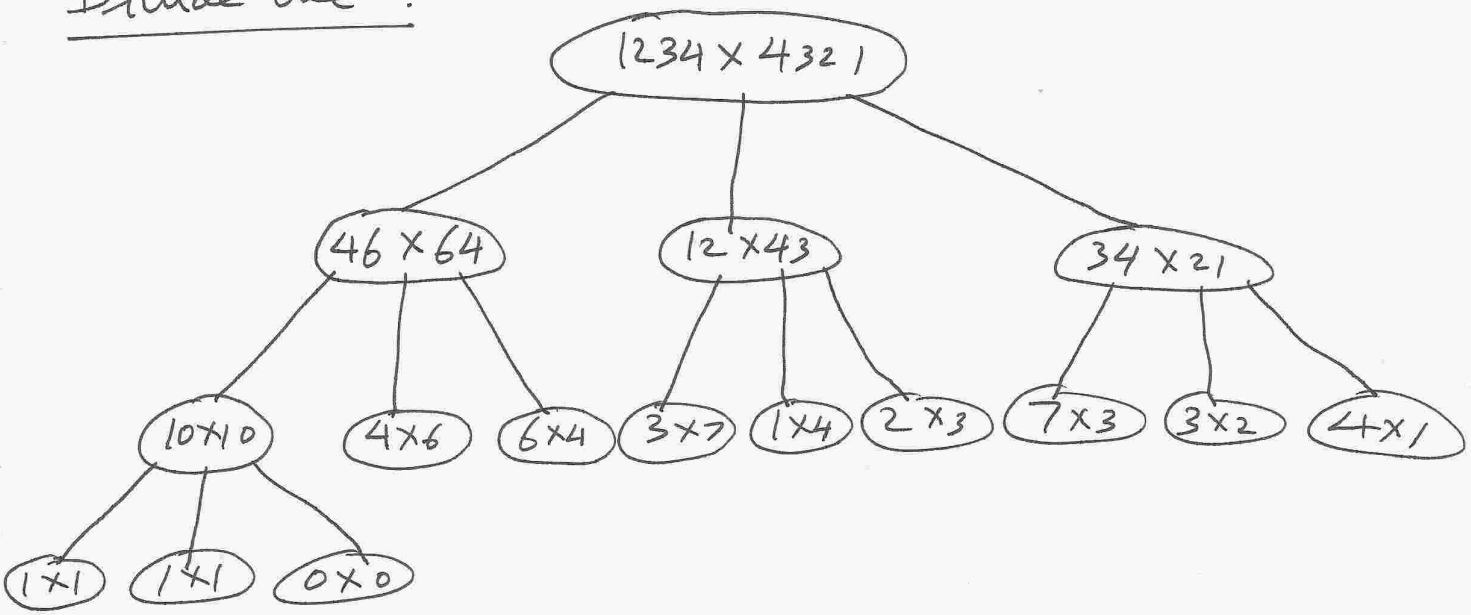
For non-leaf nodes:

$N_1 \times N_2 = B \times 10^n \times 10^n + (A-B-C) \times 10^n + C = N$

There will not be any space in between. "X" is English capital letter X.

Sample Input: 1234X4321

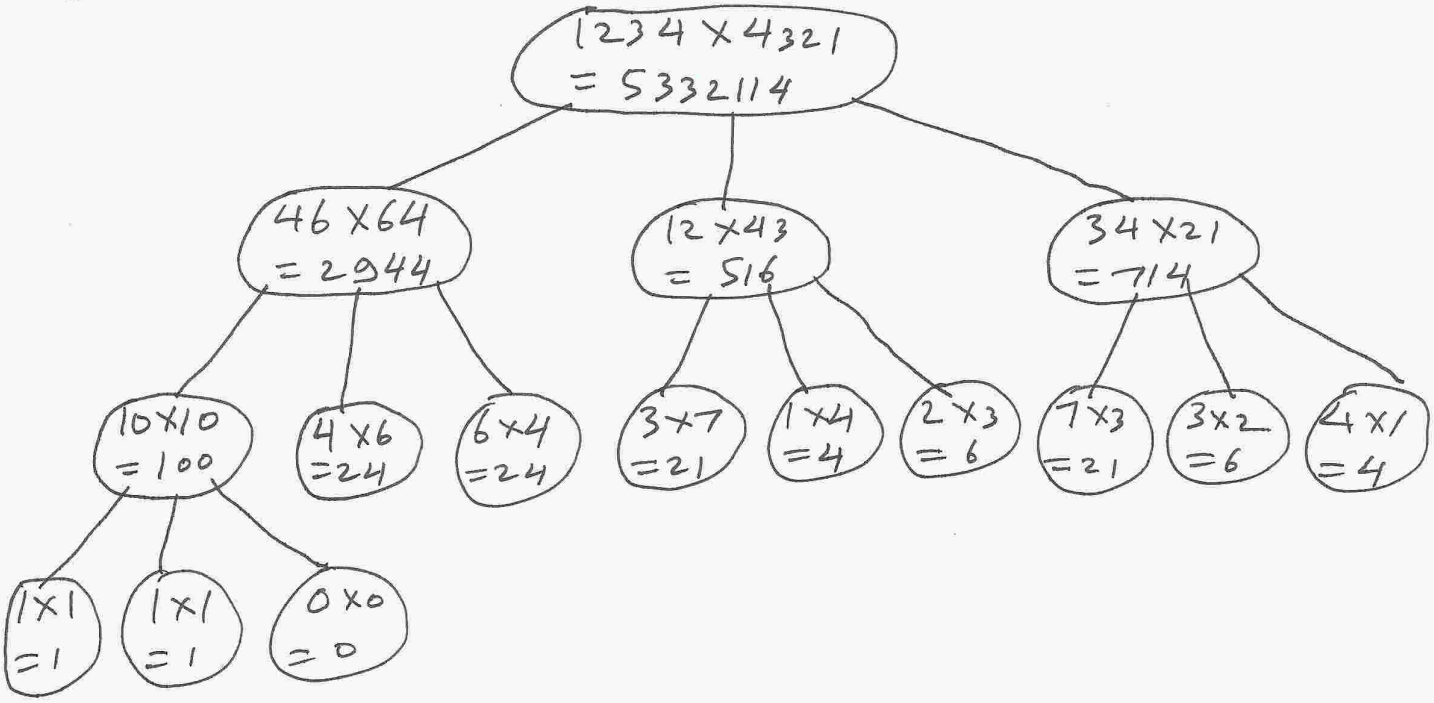
Divide Tree :



Stack : 0X0 1X1 1X1 4X1 3X2 7X3 2X3 1X4 3X7 6X4 4X6

top

Divide and Conquer Tree :



Sample Output :

0×0 = 0
1×1 = 1
1×1 = 1
4×1 = 4
3×2 = 6
7×3 = 21
2×3 = 6
1×4 = 4
3×7 = 21
6×4 = 24
4×6 = 24
10×10 = 1×10×10+(1−1−0)×10+0 = 100
34×21 = 6×10×10+(21−6−4)×10+4 = 714
12×43 = 4×10×10+(21−4−6)×10+6 = 516
46×64 = 24×10×10+(100−24−24)×10+24 = 2944
1234×4321 = 516×100×100+(2944−516−714)×100+714 = 5332114