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Master's Thesis

Emergent Inflation-Deflation Cycles from Minimalistic Wage Dynamics

Author:
Tobias H. B. Holm

Supervisor:
Professor Kim Sneppen

Programme:
MSc Computational Physics, University of Copenhagen

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Abstract

Modern macroeconomic models often rely on complex equations, dozens of parameters, and representative agents to describe inherently unstable systems. In contrast, this thesis builds a deliberately simple agent-based model where companies compete through wages, and workers choose companies probabilistically in response. The model avoids commonly thought key features such as debt, unemployment, and rigid wages, yet still generates realistic macroeconomic patterns such as inflation-deflation cycles, clustered bankruptcies, and cyclic, but non-periodic recessions. These dynamics emerge not from forced equilibrium but from feedback loops driven by inflationary wage growth and endogenous shocks. The model is compared against real-world data, including inflation, company lifespans, and company sizes, achieving qualitative agreement in most cases. The results question the necessity of complexity in macroeconomic modelling and suggest that instability and cyclicality may be natural outcomes of basic interaction rules. This supports a central claim in econophysics: that complex macroeconomic patterns can emerge from simple, rule-based interactions without the need for detailed microeconomic realism.

Contents

Chapter 1

Introduction

The economy is fundamentally unstable. Periods of prosperity and optimism are often followed by downturns, crises, and recovery, a pattern so persistent that it has earned the name "the business cycle". Understanding the mechanisms behind these recurrent booms and busts remains a central economic challenge. In particular, macroeconomic downturns such as the Great Depression and the 2008 financial crisis have repeatedly revealed the limitations of existing economic models to anticipate, explain, or mitigate crises. Mainstream macroeconomic approaches, especially those based on Dynamic Stochastic General Equilibrium models, rely heavily on assumptions of rational agents, equilibrium conditions, and aggregate representative behaviour. While useful in some contexts, these assumptions are poorly suited to capturing non-equilibrium dynamics, heterogeneity, and systemic fragility, all central to the crises of our world. As an alternative, the field of econophysics applies tools and methods from physics, particularly statistical mechanics, to economic systems, favouring minimal models and interaction-driven dynamics over many-parameter analytical frameworks. This thesis contributes to that tradition by introducing a minimal agent-based model where companies compete for workers by adjusting wages, and workers probabilistically choose employers based on offered wages. All companies sell products whose price is set by aggregate wages, linking income, demand, and capital dynamics.

The primary goal of this thesis is to investigate to what degree such a simple model can reproduce macroeconomic phenomena, particularly business cycles and inflation dynamics, and how these features depend on model parameters such as wage sensitivity and wage update rates. To do so, the model's output is compared against empirical benchmarks, including company size and lifespan distributions, and recession durations and periodicity.

The structure of the thesis is as follows. Chapter 2 introduces the necessary economic background and related literature. Chapter 3 presents the model and its assumptions. Chapter 4 explores the results to understand the model dynamics and validate its output. Chapter 5 discusses the implications, limitations, and possible extensions of the model, and Chapter 6 concludes with a summary. The appendix contains a paper on the same topic as the thesis, and, while not containing any new information, can be useful as a summary. The supplementary material for the paper is not finished, but will contain no content not covered in the results chapter of the thesis.

All code will be made available upon request.

Chapter 2

Background

In this section, the economic phenomena necessary for understanding the later results will be explained.

2.1 The business cycle

We feel and hear about the economy in one way or the other every single day, especially these days where it has been demonstrated how the actions of a single person can alter the global economy, going from a healthy economy to talks of a depression. But what does it mean that an economy is “healthy” or “depressed”? Economists measure the state of the economy through different variables, and while there is no agreed-upon set, typically you will hear of: gross domestic product (GDP; roughly speaking, the total sales of a country), inflation (how much less your money is worth), unemployment, and industrial production. When things like GDP are high and unemployment is low, the economy is considered healthy and vice versa. The economy changes between highs and lows, and this cycle is called the business cycle, described by the National Bureau of Economic Research (NBER) [1]:

> ”[...] a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; [...] they are not divisible into shorter cycles of similar character with amplitudes approximating their own.”

Today, the four epochs of the cycle are generally called “expansion”, “peak”, “recession”, and “trough”. “Depression” remains a colloquial term in the eyes of NBER [2], but a depression is generally thought of as a severe recession.

GDP

Formally, the gross (in economics, ”gross” means total) domestic product is defined as

$$\text{GDP} = C + I + G + (X - M), \tag{2.1}$$

where C is consumption (household spending), I investment (company spending), G government spending, X is exports and M imports.

Later, we will approximate the GDP to only include consumption only GDP $\approx C$, as the model does not include the rest of the variables.

Inflation

In a healthy economy, money becomes worth less over time, or in other words, things get more expensive. This is expressed through the inflation rate, and most federal banks aim to have an inflation rate of 2% [3]. This may seem surprising, but consider the opposite (deflation): Your money becomes worth more. Sounds great, but it also includes all your debt, and then it is no longer fun. In the business cycle, inflation rises in the expansion and falls in the recession, often explained through the Phillips curve [4], which states that inflation and unemployment are inversely proportional. A fall in inflation is called "disinflation", and it is first when the inflation rate becomes negative that it is called deflation.

When something is called "real", it means adjusted for inflation, and "nominal" is the opposite. For instance, if the bank sets a 10% interest rate on a loan (the nominal interest rate) and the inflation rate is 5%, then the effective rate at which the bank earns interest, i.e., the real interest rate, is only $10\% - 5\%$ because the money the bank earns from interest has its worth reduced by inflation.

Inflation is usually measured by looking at the change in price of a selection of household goods. There are different ways of averaging the selection, for example, excluding products with high variability or outliers. The Consumer Price Index (CPI) and Personal Consumption Expenditures Price Index (PCEPI) are two common measures of prices used to calculate inflation. Once the price has been found, there are then different time scales used to find the inflation, depending on the desired outcome. The simplest type of inflation is the "monthly inflation", which, as the name suggests, is the change in prices from one month to the next. "Annualized inflation" takes the inflation over a period and interpolates it to a whole year. So, annualized monthly inflation would take the inflation in one month and raise it to the power of 12. This type of inflation is often used as a measure of inflation momentum. The inflation most commonly shown in the news is the "year-over-year inflation", which is found by comparing the prices in a month and the same month the next year. This is useful as it considers seasonal trends (e.g., your heat bill will be larger in Winter than in the summer, not because of inflation but because of the temperature).

Fisher's Debt-Deflation Theory

Following the Great Depression of 1929, Irving Fisher wrote his seminal 1933 paper, *The Debt-Deflation Theory of Great Depressions* [5], which marked a departure from equilibrium-centric views of economic crises. Fisher argued that financial crises, such as the Great Depression, could be understood as a positive feedback loop between over-indebtedness and deflation. As companies attempt to reduce debt by rapidly selling assets, they lower the amount of money in circulation, causing deflation. During depressions, the increase in real debt burden from deflation is greater than the decrease from paying off the loans, repeating the cycle and ultimately leading to widespread bankruptcies and economic contraction.

Minsky's Financial Instability Hypothesis

Hyman Minsky's Financial Instability Hypothesis [6] builds on the idea that financial markets are inherently cyclical due to endogenous risk-taking behavior. In contrast to models that treat crises as exogenous shocks, Minsky proposed that prolonged periods of stability encourage companies and investors to take on increasing levels of financial risk. As optimism grows, economic units (companies, households, countries, etc.) transition from "hedge" positions (able to meet all obligations from cash flow), to “speculative,” and eventually to “Ponzi” positions that rely on selling off assets or paying off loans with new loans to avoid default. This shift makes the system increasingly fragile and prone to crisis.

2.2 So, how do we model the economy?

The economy is an extremely complex entity, and the general answer among economists has been to deploy highly complex modelling (for examples, see *The evolution of macro models at the Federal Reserve Board* [7], *Nominal rigidities and the dynamic effects of a shock to monetary policy* [8], and *Shocks and frictions in US business cycles: A Bayesian DSGE approach* [9]). But, as the financial crisis of 2008 suggests, this approach has proven inadequate for explaining and predicting economic downturns [10]. Economist Joseph Stiglitz writes in his critique [11] of the widespread modelling method Dynamic Stochastic General Equilibrium (DSGE), that:

> The resulting complexity often makes it even more difficult to interpret what is going on. And with so many parameters, macro-econometrics becomes little more than an exercise in curve fitting [...]

So while there is a disagreement in economics on the value of complexity in modeling, in physics, modeling complexity is widely regarded as a sin. Physicists aim to describe systems with as few parameters and assumptions as possible, following the principle of Occam’s razor. The idea of econophysics is then to use both the tools of physics (historically, primarily statistical mechanics) and the minimalistic approach to modelling, and apply them to economic problems. Popular examples include modeling the exchange of money as kinetic particles exchanging energy [12] and the modeling of traders shifting strategies according to a Boltzmann distribution [13].

Following this line of thinking, we go back to first principles and deconstruct the economy to its most fundamental elements, focusing on the dynamics of interactions. Broadly speaking, the economy can be viewed as a collection of markets, each organizing the exchange of goods, services, labor, or capital. These markets facilitate transactions between agents (individuals, households, companies), and transactions can be seen as the elementary interactions that make up economic activity. A physics analogy would be that just as a charged particle creates an electric field in isolation, an economic agent possesses preferences and resources independently. However, system dynamics require interaction, such as the repulsive force a second electron would create, and in the case of economics, through transactions between agents in markets.

This thesis will treat workers and companies as the two fundamental elements. There already exist economic works on the same topic, such as Dosi et. al in [14], modelling worker and company interactions, but with two types of companies: R&D companies who sell machines to consumption companies, who in turn sell products to workers. Lengnick [15] takes inspiration from the Dosi paper [14] and, just like the model in this thesis, only has workers and companies, however, workers are part of households and are treated in more detail. Further, companies and households are in a network dictating employment and which companies households buy from. Knowledge of prices is contained within the network, meaning companies do not know the price that other companies offer, only what their customers are willing to pay. However, while proposing an alternative to the too complex DSGE models, both papers mentioned employ close to 20 parameters, and many decisions need to be calculated at each time step.

2.3 Agent-Based Models in Econophysics

One way of achieving this first principles approach is to use agent-based models, where macroeconomic behavior emerges from the interactions of simple, rule-based agents. These models typically avoid representative agents (treating *all*, e.g., households as one, average agent) or equilibrium assumptions and instead simulate heterogeneous agents who interact locally through simplified market rules. In the abovementioned model by Lux and Marchesi [13], traders switch between optimistic and pessimistic strategies based on herd behavior, generating realistic asset price fluctuations and fat-tailed return distributions. These works demonstrate that even minimal micro-level assumptions can yield rich macro-level phenomena, an idea that this thesis follows.

What this thesis then strives to achieve is to apply the econophysic approach to develop models similar to those of Dosi [14] and Lengnick [15].

2.4 Price values and units

The price values of my model are not of particular insight; instead, price changes are typically more interesting. I will be using the dollar sign $ as the symbol representing the unit for prices [1], though I stress that it is not to be confused with the U.S. dollar or any other real-world currency.

[1] There exists a generic currency sign, the "scarab" ¤, but I find it hideous and everybody recognises $.

Chapter 3

The Model

3.1 Overview of the Model

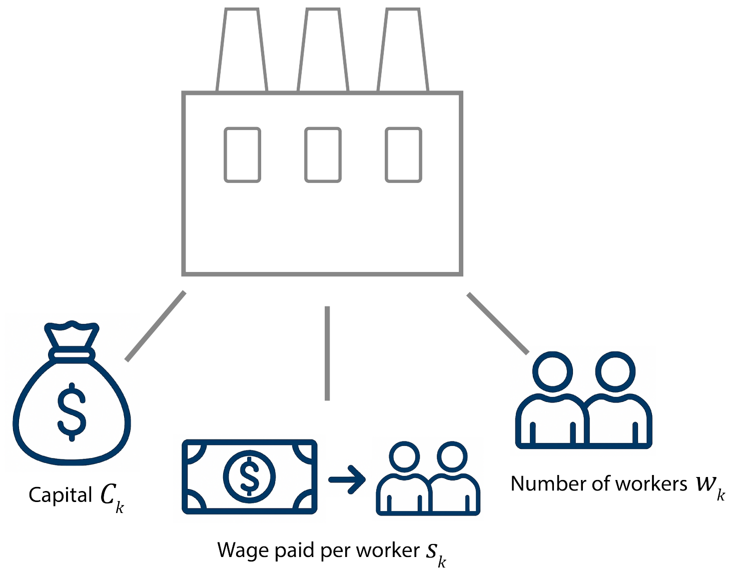

Figure 3.1: A company consists of three variables: Its capital $C_k(t)$ (effectively money), the wage paid per worker $s_k(t)$ (often abbreviated to just "wage"), and the number of workers w_k, which is the company size.

Our system consists of N companies and a fixed workforce of W workers. Each company k is characterized by three state variables: the number of workers w_k, the wage paid s_k, and the capital C_k. The system evolves in discrete time steps. At each step, a randomly selected set of companies performs transactions and updates their wage, and subsequently, workers relocate to new companies based on the wages offered. Companies whose capital falls below zero are declared bankrupt and replaced by new entrants. The anatomy of a company is seen in Fig. 3.1, and the transaction and worker relocation phases are illustrated in Fig. 3.2.

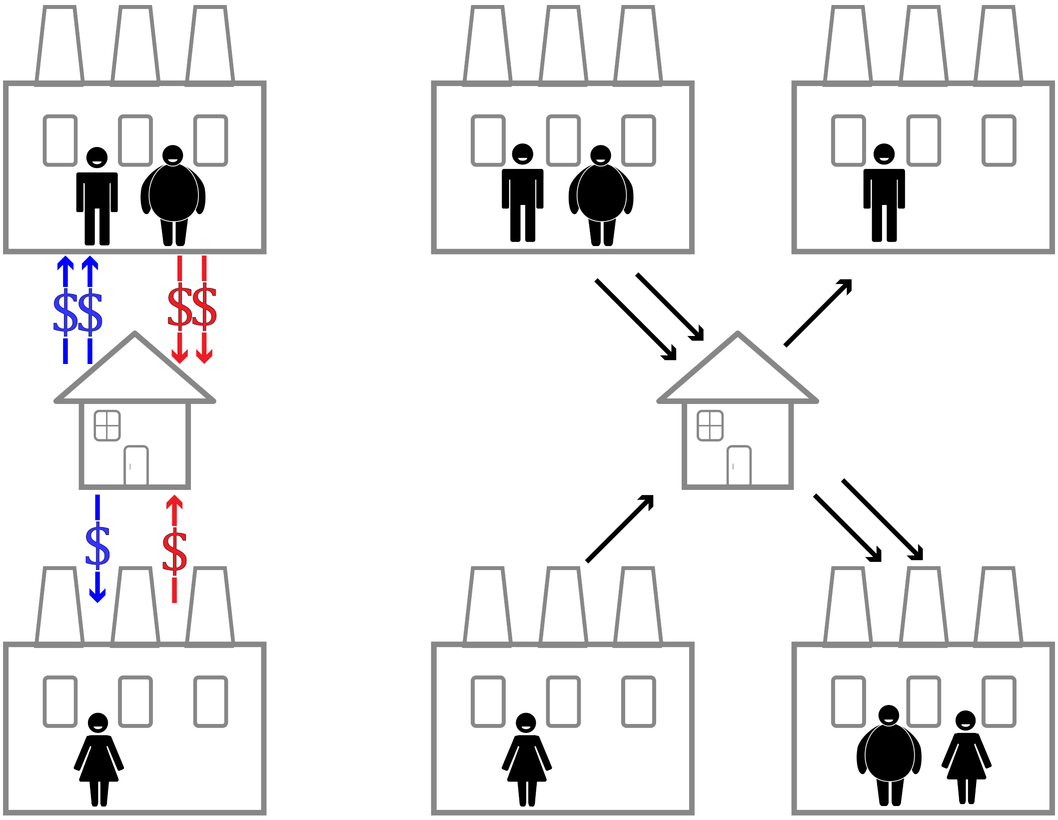

Figure 3.2: Model with its two central phases. At any time, a company has a wage, workers, and an accumulated capital. The left panel shows that each company sells and pays wages proportionally to the number of workers. The right panel illustrates worker relocation , where each worker independently selects a new company. The choice depends on the company wage s_i to an exponent α that quantifies competition. The model includes rules for how wages and company capital change.

3.2 Dynamics

Companies and Transactions

At each time step, $N - N_{w=0}$ companies are selected uniformly at random with replacement to perform a transaction, where $N_{w=0}$ is the number of companies with zero workers. In a transaction, workers each produce one unit of output, which is sold to increase capital. Assuming workers spend everything they earned in the previous time step, the average consumer will pay $\mu(t-1)/W$ per unit of output, where $\mu(t)$ is the sum of wages paid by all companies at time t. Simultaneously, the company pays each worker a wage of s_k. Thus, the net change in the company's capital is

$$\Delta C_k = w_k \frac{\mu(t-1)}{W} - w_k s_k(t-1), \tag{3.1}$$

where

$$\mu(t) = \sum_{\text{selected } i} w_i(t) s_i(t), \tag{3.2}$$

is the sum of wages paid by the companies selected to transact.

Wage Update

Companies that made a profit ($\Delta C_k > 0$) or had zero workers ($w_k = 0$) now increase their wage by a fixed percentage $\Delta s/s$ of their current wage, and companies that lost capital ($\Delta C_k < 0$) reduced their wage by the same percentage but adjusted for the asymmetry between percentage gains and losses.

$$s_k \leftarrow \begin{cases} s_k\,(1+\Delta s/s), & \text{if } \Delta C_k > 0 \text{ or } w_k = 0, \\ s_k\left(1-\frac{\Delta s/s}{1+\Delta s/s}\right), & \text{if } \Delta C_k < 0. \end{cases} \tag{3.3}$$

Companies with no capital change and non-zero workers do not update their wage.

Worker Reallocation

After the wage updates, all W workers reselect their employer. The probability that a worker chooses company k is given by

$$P(\text{worker chooses } k) = \frac{s_k^\alpha}{\sum_{i=1}^{N} s_i^\alpha}, \tag{3.4}$$

where α is a parameter describing workers' sensitivity to wage differences.

Bankruptcies and Startups

A company is declared bankrupt if its capital falls below zero. Upon bankruptcy, the company is replaced by a startup company with $w_k = 0$ and $C_k = 0$. The startup chooses its initial wage by sampling from the set of companies that made a profit in the current time step, with an added small random perturbation in the range $[-m, m]$, after which a minimal wage $s_{\min}$ is enforced. If no companies made a profit, the new wage is instead chosen randomly among the top 50% companies that lost the least.

Summary of the Model Dynamics

For T time steps:

1. A set of $N - N_{w=0}$ companies is randomly selected to make transactions.
2. In each transaction, companies sell and pay wages proportionally to the number of workers.
3. Companies update their wage based on whether they have made a profit or loss.
4. All workers are redistributed among companies, with selection probabilities determined by wages.
5. Companies with negative capital are declared bankrupt and replaced by startups whose initial wage is chosen from the profitable companies, with a small noise perturbation.

3.3 Model Parameters and their Roles

In total, this gives us the following parameters: The system size parameters N and W are the number of companies and total workers, respectively; the time scale parameter $\Delta s/s$ which determines the magnitude of wage updates; the workers' sensitivity to wage differences α, the mutation magnitude m added to startup's chosen wage, and finally the minimum wage $s_{\min}$.

3.4 Simplifications

The model has several simplifications at the micro level. First, I assume that all companies produce products sold at the same price. All workers are identical in productivity and mobility: they can be employed by any company and contribute equally wherever they work. Another key simplification is that companies only pay wages when transactions occur. Workers are thus effectively paid on a per-task basis, creating a scenario more akin to a freelance economy than a conventional employment system. While clearly unrealistic in most modern labor markets, these assumptions may be reasonable in certain contexts, such as untrained manual labor like food delivery or freelance driving (e.g., Uber), where task-specific skills and company-dependent productivity differences are minimal.

Furthermore, every worker is reallocated at each time step based on wage competitiveness. This eliminates the notion of long-term employment contracts or hiring frictions. Clearly, no real-world firm lays off all its employees each period only to hire new ones again. This assumption is most problematic when considering individual-level trajectories: tracking a single worker across time yields unrealistic, even nonsensical, behavior. However, the model behaves more reasonably when aggregated at the firm level. The number of workers at a given company, although highly volatile (as we will see), follows patterns that are more consistent with observed firm dynamics.

In addition, we do not allow companies to take on debt. A company may continue to operate only as long as its capital remains non-negative, after which it is declared bankrupt and replaced by a startup. While this assumption rules out an important real-world mechanism for smoothing temporary losses, it helps isolate the role of wage dynamics in generating macroeconomic fluctuations.

Another important deviation from real labor markets concerns wage flexibility. In reality, nominal wages are typically "downwards sticky"; they rarely decrease even in the face of worsening economic conditions. The model does not include this friction, and wages can be lowered directly when companies incur capital losses, rather than indirectly through layoffs or inflationary erosion of real wages. While the net effect (reduced wage burden) may be similar, the underlying microdynamics differ significantly.

I also simplify pricing by setting the price of products equal to μ/W, where μ is the total wage expenditure among transacting firms. This formulation implies that all wages earned in one time step are spent in the next. While convenient, this assumption allows μ to take on several overlapping economic interpretations: it may resemble GDP (since we only have consumption), the Consumer Price Index (it is the only price for all goods), or the total income of companies. When divided by the number of transacting workers, μ becomes the realized average wage. However, these interpretations should be used with caution. The overloading of μ with multiple economic meanings is a limitation rather than a strength, and I stress the need to be careful not to overstate what it represents. To clarify these distinctions, two different average wages are defined: the average company wage, denoted $\bar{s}_{\text{comp}}$, and the realized average wage, $\bar{s}_{\text{real}}$. The former is simply the mean of all company wages:

$$\bar{s}_{\text{comp}} = \frac{1}{N}\sum_{i=1}^{N} s_i, \tag{3.5}$$

and is used to estimate the average company profit per worker:

$$\bar{C} = \frac{\mu}{W} - \bar{s}_{\text{comp}}. \tag{3.6}$$

Here, the first term represents average income per worker from transactions, while the second denotes the average wage paid per worker. In contrast, the realized average wage,

$$\bar{s}_{\text{real}} = \frac{\mu}{\sum_{\text{transacting } i} w_i}, \tag{3.7}$$

measures the actual average wage paid by companies selected to transact and is used to define inflation in our model via its change over time.

3.5 Profit condition

A company makes a profit at time t when $\Delta C_k(t) > 0$, so from Eq. (3.1) the condition for making a profit is then

$$s_k(t-1) < \frac{\mu(t-1)}{W}. \tag{3.8}$$

So, roughly speaking, a company makes money when it pays a lower wage than the average. $\mu(t) = \sum_{\text{selected } i} w_i(t) s_i(t-1)$ can be seen as a weighted sum of wages. Since higher wage companies attract more workers, μ/W will generally be greater than $\bar{s}_{\text{comp}}$, allowing most companies to make a profit. Yet the same reason why μ/W is generally larger than $\bar{s}_{\text{comp}}$ is also the reason why μ/W may dramatically drop below $\bar{s}_{\text{comp}}$, namely, the reliance on large companies. Since high wage, large companies are the most important terms in μ, if they are not chosen to transact, μ will suffer much more than when small companies are not transacting. Thus, when the diversity D is low and a few companies hold most workers, the system becomes vulnerable to shocks. A shock in this context then becomes large, high-wage companies not transacting and therefore reducing μ/W, hurting future transactions.

At first, it may seem that companies aim for $\Delta C_k = 0$ because companies lower their wage when $\Delta C_k < 0$ and vice versa. However, they increase their wages in the hope of other companies doing the same, increasing μ/W and thus the likelihood of making a profit.

3.6 Simulation setup

There is no obvious termination criterion, but there is a warmup period with a length depending on the system parameters (especially $\Delta s/s$). When looking at time series, the simulation was typically run for around $T = 8000$ time steps, with the first 2000 to 3000 steps being warmup (and thus that part ignored). Then it is a matter of selecting a time range that highlights the interesting results.

When analyzing macro features, such as the distribution of recession durations, the simulation was run for $T = 250000$ time steps.

Chapter 4

Results

To avoid getting overwhelmed by the many variations the parameters allow, I will first present and interpret the results of a single set of standard parameters and then explore how changing the parameters influences the system dynamics in comparison to these parameters. The standard parameters are: $\alpha = 4$, $W = 20N$ (motivated by [16]), $N = 1000$, $\Delta s/s = 0.1$, $s_{\min} = 0.1m$ and $m = 0.1$.

Also, I will be using diversity D of company sizes, defined as the reciprocal Simpson index [17]:

$$\begin{aligned} D &= \frac{\left(\sum_j^N w_j\right)^2}{\sum_j^N w_j^2} \\ &= \frac{W^2}{\sum_j^N w_j^2}, \end{aligned} \tag{4.1}$$

to measure the spread of workers, i.e., company sizes, in the market. From Eq. (4.1), if one company employs all workers, $D = 1$, and if workers are equally distributed among all companies, $w_k = W/N$, then $D = N$.

4.1 Inflation Cycles

The model economy exhibits robust cyclic but non-periodic dynamics, similar to the traditional business cycle (expansion, peak, recession, trough) [1, 18] and the recent publication [19]. Fig. 4.1 measures the market by the distribution of wages over time in a), the distribution of capital over time in b), and c) shows the average profit per worker, and the company diversity defined in Eq. (4.1), again, over time.

In Fig. 4.1, I have identified four epochs that the system goes through in a cycle:

- Starting just after a trough, companies exhibit coherent growth, with relatively high D and wages that are relatively narrowly distributed around a mean. The average profit per worker (estimated as $\mu/W - \bar{s}_{\text{comp}}$ is positive but declining.

- At its peak, wage growth is no longer sustainable, and companies suffer capital losses until a mass bankruptcy is triggered.

- Bankruptcies create space for startup companies with low wages and, as a result, the surviving companies attract a large fraction of the workers, reflected in few large and

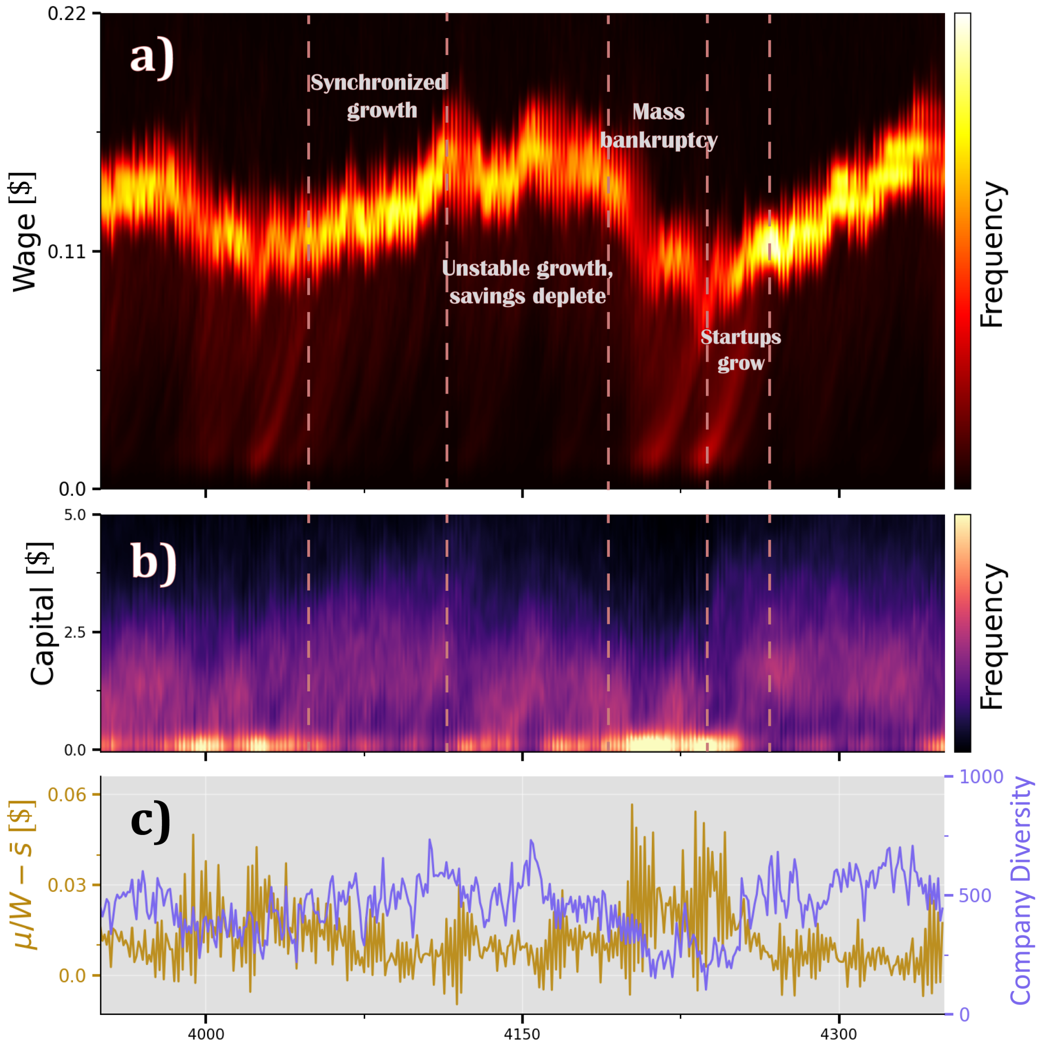

Figure 4.1: a) illustrates the wages in the system over time, for standard parameters. The colour shows the number of companies with the corresponding wage. One sees a cyclic pattern, with relatively sharp declines where some companies have low wages and others still maintain high wages. The different epochs are marked with text explaining the main characteristics. b) shows the company capital of the different companies, highlighting the short period with very low capital concurrent with wage collapse (for visual clarity, I have not differentiated between the top 0.5% of values). c) shows the average company profit and company size diversity defined in Eq. (4.1).

many small companies and thus a low D.

- With their low wages, startup companies are highly likely to make profits (explained later in this chapter) and continue to grow their wage until they meet the surviving companies' declining wages. After the convergence of wages, the economy again enters the epoch of coherent growth.

This cyclic dynamic resembles inflation-deflation cycles, where periods of synchronized growth and rising wages correspond to periods of inflation, driven by increasing consumer purchasing power (captured by μ/W). The high wages leave larger companies vulnerable due to higher capital volatility, such that occasional shocks (reduced number of transacting workers) can cause periods with wage reductions and bankruptcies, reflecting deflationary spirals where reduced purchasing power and capital losses feed back negatively on wages and consumer spending.

4.2 Comparison to Empirical Data

Fig. 4.2 examines six empirical data sets from the perspective of the model. These are asset return, company lifespan, time between recessions, duration of recessions, company size distribution, and inflation. The first column is the model, and the second is the data. The results only reproduce the historical financial data qualitatively, but allow for quantitative comparisons.

The first row shows the asset return $r(t;\tau)$. In the model, the closest to such a profit measure is the relative change in capital:

$$r_k(t;\tau) = \frac{C_k(t+\tau) - C_k(t)}{C_k(t)} \tag{4.2}$$

$$\approx \ln(C_k(t+\tau)) - \ln(C_k(t)). \tag{4.3}$$

The plot only includes $C_k \geq m$ values to prevent divergence near $C_k = 0$. I compare the model for $\tau = 1$ time step with the 503 companies currently on the S&P 500 index (data from [20]). The earliest possible date used was 2020-01-01, and the latest was 2025-04-01. The asset return is measured over a time interval of two working weeks $r(t, \tau = 10 \text{ days})$ (inspired by Fig. 1 and 2 in [24]). Note that the S&P 500 has an inherent survivorship bias, as delisted (dead) companies are not included. Despite this, the two distributions share three important characteristics: the peak is slightly positive, the tails are fat, and the left tail declines more slowly than the right tail.

The second row displays the lifespan of individual companies k, measured from their birth to $C_k < 0$. The data is read from Fig. 2c in [22] (process explained in Section 7.2). The data show an exponential decline, except for very low values, where startups with zero capital are particularly likely to go bankrupt the first time they transact, especially if they choose a too high wage (this is seen later in Fig. 4.3). The data also shows exponential decay, with a half-life of about 8 years, where the model has a half-life of 30 time steps. It seems that there are two timescales for the lifespan, one fast for companies dying almost instantaneously, and the slow, exponential decay. The short time scale can be seen in the insert in Fig. 4.2 c). Notably, the only way for companies to die in the model is due to bankruptcy, a fate that occurs only in 4.5% of real company terminations [22]. Thus, one cannot easily translate the timescale between models and financial data, though as explained in Section 4.5, the model has *more* deaths than expected.

Fig. 4.2 e) and f), compare the time between recessions for the model and data collected by

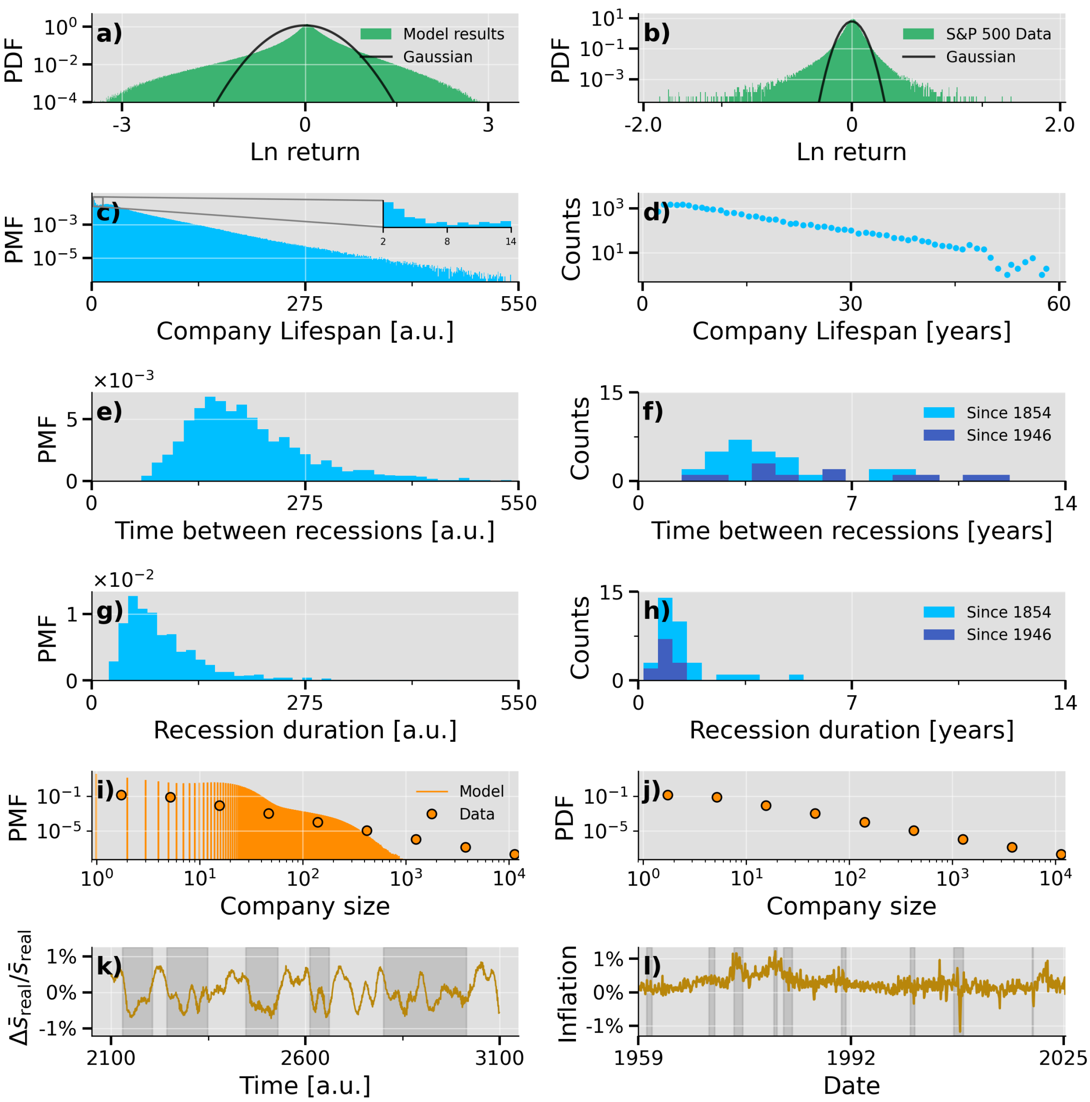

Figure 4.2: Comparing the model to empirical data. The first column shows results from the model, and the second empirical data. Qualitatively, the model roughly agrees with the chosen benchmarks, except for the distribution of company sizes. Standard parameters used with $T =$ 250.000. Asset return data is from Yahoo Finance [20], fetched using the Python library Yfinance [21]. Company lifespan data is read from Fig. 2c in [22], both recession datasets are from NBER [23], and the company size data is from [16]. Inflation is calculated from PCEPI data produced by BEA [3].

the National Bureau of Economic Research (NBER) [23]. NBER defines recessions as starting "in the month after a peak in the business cycle and ending in the month of the trough". The "peak" and "trough" are found over multiple variables like GDP and unemployment, while I identify extrema solely from μ in the model. The process and difficulties of finding peaks are detailed in Section 4.6.3. The NBER data start at 1854, and recessions since 1946 are in a darker color to highlight how the modern economy may differ. The spread in durations is roughly 25% larger in the data than the model for as seen in Table 4.1 alongside the spread over mean for other α values.

Panels g) and h) show the duration of each recession, with a median recession length being about 1/3 of the interval between recessions. In comparison, the recessions in the model are only about 1/2 of the interval between recessions. The spread over mean is again seen in Table 4.1, this time the data is roughly 40% larger than what the model suggests.

Panels i) and j) compare the distribution of company sizes (number of workers) to that of US company sizes in 2021 [25]. The data have been retabulated following [16], which also shows that company sizes are distributed by power law $p(w) \propto\approx 1/w^2$. The predicted distribution for the model is not scale invariant but peaks around the mean number of people allocated per company, $W/N = 20$. This distribution reflects the total relocation of workers at each time step, which only allows big companies to emerge when their wages are unusually large. The model company sizes between 50 and 500 are more broadly distributed and less in contrast to the reported power law. Noticeably, the distribution becomes more power-law-like for higher α (later explained, but see Fig. 4.6). However, in any case, the power law of real company sizes likely reflects the large diversity in types of production lines in analogy to the toolbox scenario for biological networks [26].

The final row of Fig. 4.2 compares inflation. Inflation is measured as the change in the real average wage paid $\Delta\bar{s}_{\mathrm{real}}/\bar{s}_{\mathrm{real}}(t-1)$ per time step. This is compared to the monthly inflation using Personal Consumption Expenditures Price Index (PCEPI) [3], shown from 1959 to 2025. I compare my model against the monthly inflation, as opposed to the more commonly used year-over-year inflation, as the monthly inflation more closely resembles my time-step inflation. The areas shaded in gray are recessions, in the case of the model found in μ, and the recessions in the data are from NBER [23]. The model does not include productivity growth, so its inflation fluctuates around 0%, whereas the empirical data exhibits a small positive trend, roughly equal to 0.1% (in year-to-year inflation it fluctuates around the 2% that many federal banks aim for [27]). Despite this offset, the two series display similar qualitative features. Both series show irregular but recurring inflation-deflation cycles, with visual similarities in the burstiness and jagged structure of the time series. The model also captures the mean-reverting nature of inflation, and from the shaded recession areas, it is seen how generally the recessions occur during disinflation/deflation and vice versa, just like in the data. However, the model inflation changes rapidly and reaches extreme values much more often than the empirical data.

4.3 Single company trajectory

To better grasp what happens at the micro level, the time series for company size, capital, and wage paid, $w_k(t), C_k(t), s_k(t)$, is seen in Fig. 4.3. The wage graph in a) shows how companies generally either pick a high wage when going bankrupt and soon after go bankrupt again, or pick a wage near $s_{\min}$ and exponentially increase their wage until they enter the synchronized state, and then their capital runs out and they go bankrupt. Generally, the company size is around

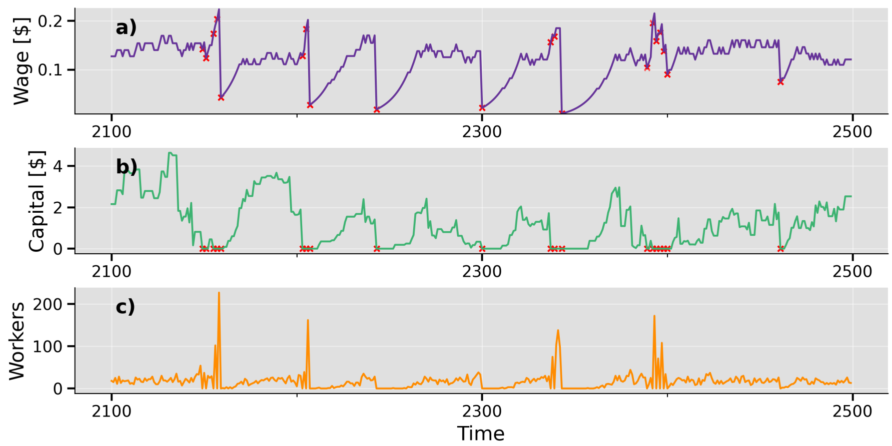

Figure 4.3: The wage paid, capital, and number of workers in a), b), and c), respectively, for a single company. The red crosses in a) and b) indicate points of bankruptcy.

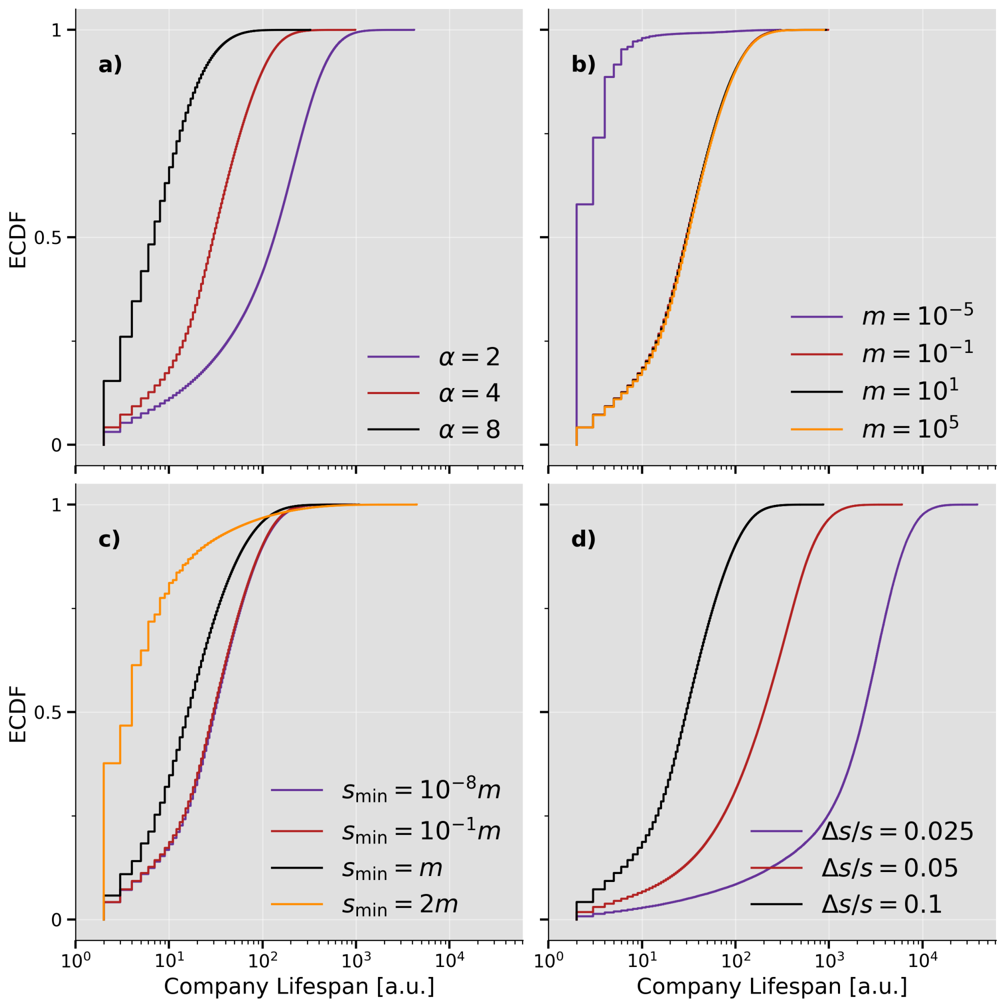

Figure 4.4: Empirical Cumulative Distribution Function (ECDF) of company lifespan for different parameter variations. a) is α, b) shows m, c) the minimum wage, and d) wage update size $\Delta s/s$. The lowest lifespan is 2, as companies do not transact the step after bankruptcy.

the mean value of $W/N = 20$, but sometimes it spikes in size.

4.4 Parameter Analysis

The role and importance of each parameter will now be explored. The general approach will be to compare wage densities and lifespan distributions, except for N and W. The cumulative lifespan distribution function for different varying parameter values can be seen in Fig. 4.4, and the content of it will be discussed in each parameter's respective section.

4.4.1 Wage sensitivity

Because, as will soon be revealed, α significantly impacts the system, it will be investigated most thoroughly. To see both how α and changing multiple parameters at the same time affect the dynamics, I show the system for $\alpha = 2, 4$ and $N = 100, 1000$ in Fig. 4.5, the first column with $\alpha = 2$, and the second column with $\alpha = 4$. Comparing the systems $N = 100$ to $N = 1000$ between the top and bottom panels, I see that the smaller systems are more irregular. The diversity of company sizes is similarly less predictable, with more erratic collapses into a state with few companies.

For $\alpha = 2$, wages are tightly clustered around the mean, average profits per worker are close to zero, and diversity remains consistently high. The larger system remains nearly unchanged over time. In contrast, for $\alpha = 4$, small waves of bankruptcies occur in addition to the larger ones, visible as "trails" of subpar wages converging toward the mean. Although similar trails exist in the $\alpha = 2$ case, they are rarer and involve fewer companies, making them less noticeable. These dynamics are also reflected in panels e) and j), which show the fractions of profitable and passive companies. At most time steps, more companies raise wages than reduce them. This wage-increasing tendency (seen in the green and blue curves vs. the red) represents an over-optimistic behavior tempered by bankruptcies.

α's influence on the fragility of companies is analyzed by looking at the cumulative company lifetime distribution, seen in Fig. 4.4 a). To get a rough idea of the impact on aggregate behaviour, I compare the coefficient of variation (standard deviation divided by mean) of the recession duration and the time between recessions, seen in Table 4.1. The cumulative lifespan distributions in Fig. 4.4 a) behave as expected: Increased sensitivity to wage means larger companies, and larger companies make the system more vulnerable to shocks, and thus more bankruptcies.

Table 4.1: Coefficient of variation for recession duration and time between recessions for different α. Data is highlighted in blue.

	Data	$\alpha = 2$	$\alpha = 4$	$\alpha = 6$	$\alpha = 8$
Duration	0.71	0.79	0.57	0.62	0.71
Time Between	0.53	0.55	0.39	0.45	0.54

From Table 4.1, it would seem that $\alpha = 8$ is the best match to the economic data of the α's investigated, so to validate this, I compare the $\alpha = 8$ results to the data in Fig 4.6. Comparing c) and e), an immediate problem is seen: On average, companies die many times between recessions, as the median company lifespan is much shorter than the median time between recessions. On the other hand, the distribution of company sizes is much closer to being power-law distributed. In the end, $\alpha = 8$ is not a good choice because of the even higher fragility, while $\alpha = 2$ has too damped cycles to be of much interest either.

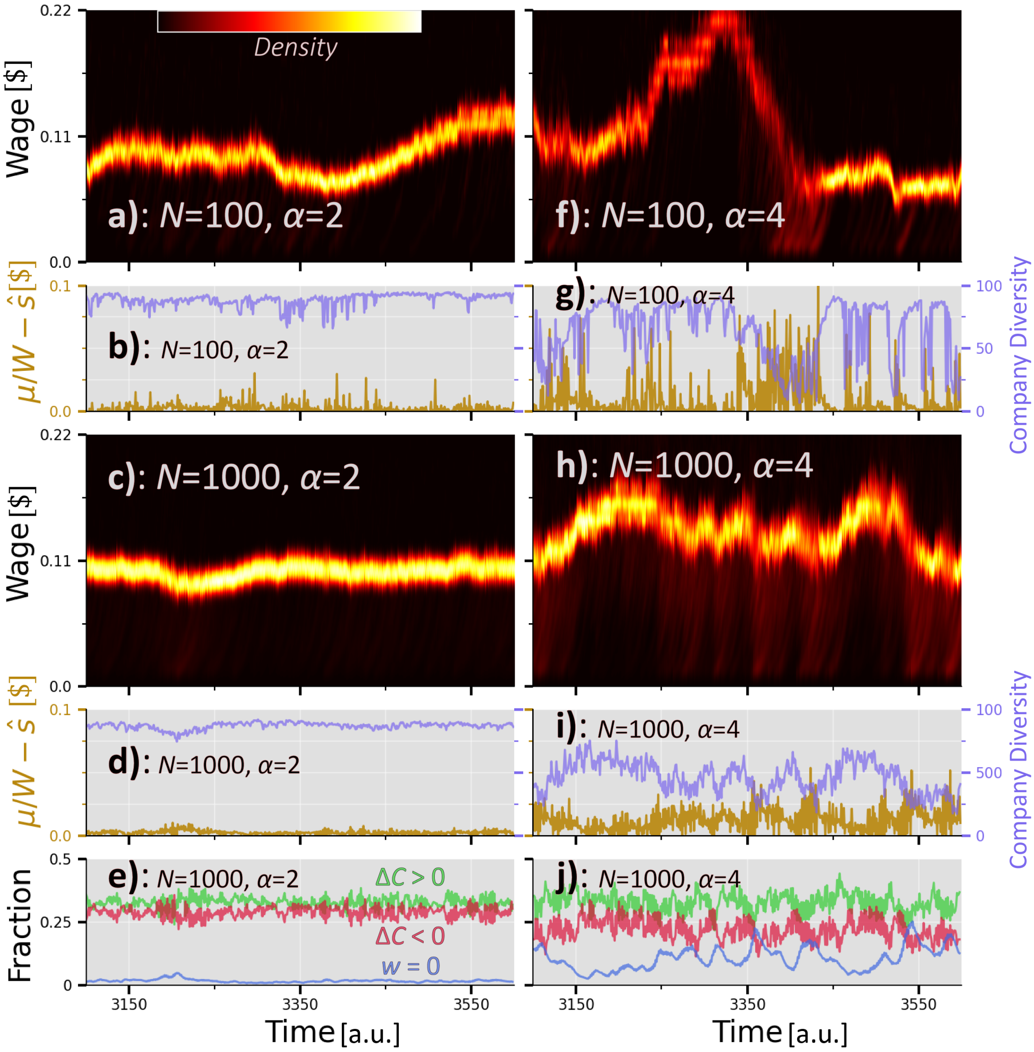

Figure 4.5: Changing multiple parameters and their influence on system behaviour. In the first column $\alpha = 2$, while the second column uses $\alpha = 4$. In the upper four panels, $N = 100$, while the lower six consider $N = 1000$. a), c), f), and h) illustrate the wages and their probability distribution as a function of time. Panels b), d), g), and i) show the average profit per worker and company size diversity D; e) and j) show the fraction of companies with a profit (green), with capital loss (red), and with zero workers (blue).

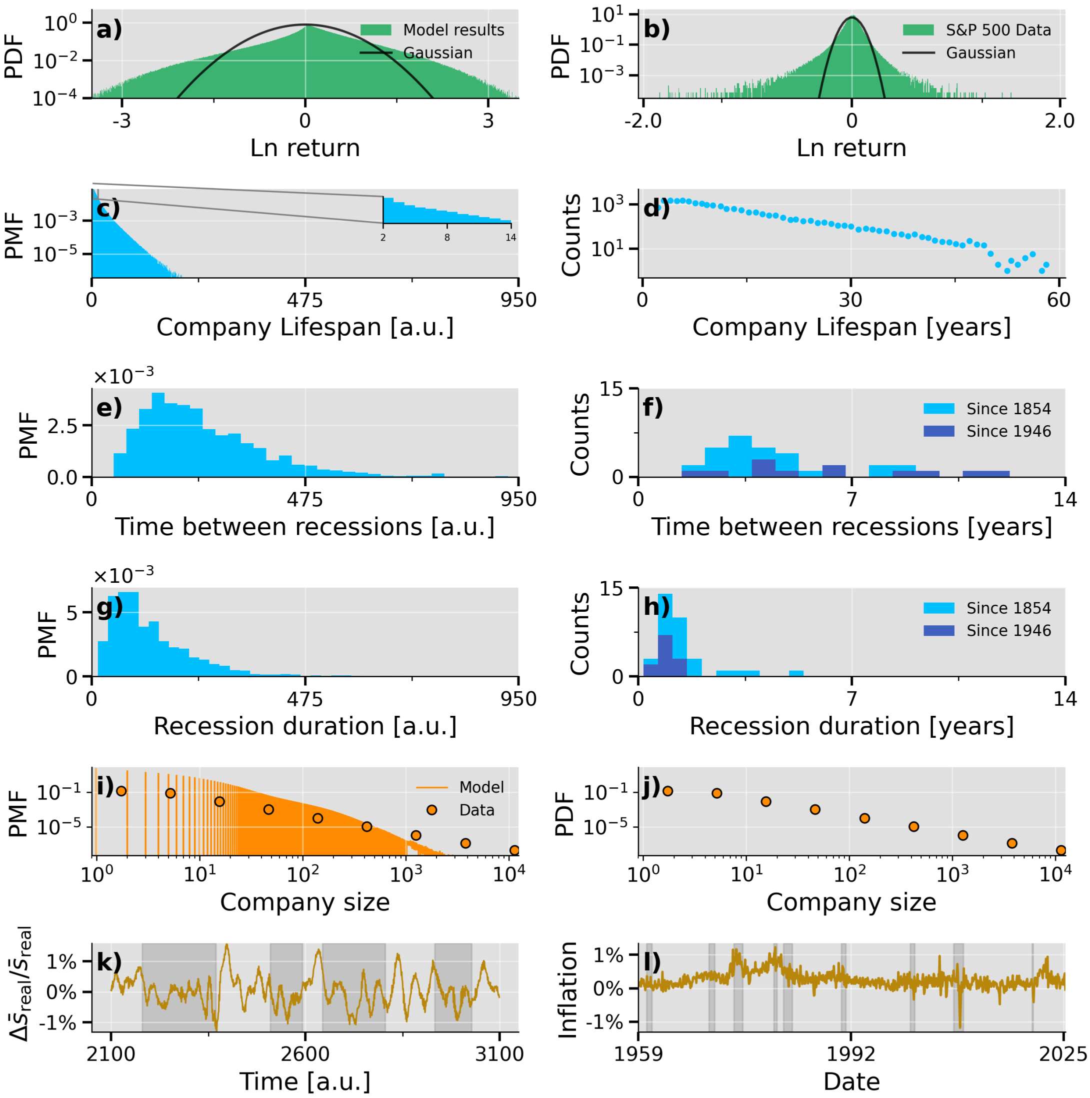

Figure 4.6: Comparing $\alpha = 8$ results to empirical data, similarly to Fig. 4.2. The first column shows results from the model with $\alpha = 8$, and the second is the same empirical data as in Fig. 4.2.

4.4.2 Mutation magnitude

Whenever a company goes bankrupt, it chooses a new wage randomly among companies that made a profit to reflect startups imitating successful companies [28, 29]. Unfortunately, I need to add a small perturbation in the range $[-m, m]$ for the dynamics to work. I suspect this is because the system has an easier time growing, i.e. μ is large, when there is a lower diversity D. I see this in Fig. 4.1 c), where the average profit per worker is highest when D is lowest. If I try to set $m = 0$, the system goes into a pathological state where most companies never make a profit and effectively go bankrupt all the time. If no companies made a profit, entrants choose their wage randomly among the top 50% of companies that lost the least, which in most cases will be the companies that were not selected to transact (the better option would be to pick among the 50% of *transacting* companies, not all companies). Because a company starts with no workers, it skips the following transaction, but still raises the wage. The $m = 0$ pathology is seen in Fig. 4.7. In a), the average price μ/W is shown on the left y-axis and the fraction of companies that went bankrupt on the right y-axis, sharing the x-axis. While there are some downs in μ/W with an increased volatility in bankruptcies resembling crashes, the average bankruptcy fraction is around 60%, and μ/W continues to climb until it diverges. Below it in b), the average profit per worker $\mu/W - \bar{s}_{\text{comp}}$ is seen to grow exponentially more negative (the exponential likely comes from the percent increase in s), with a few extreme spikes. The bottom line is that the micro weirdness is intolerable, and any results are meaningless. The impact $m > 0$ has on the

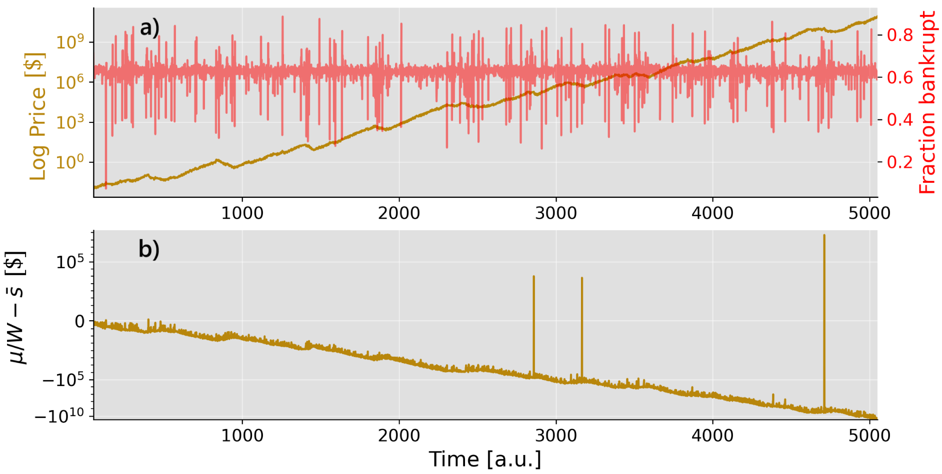

Figure 4.7: How the system breaks down for $m = 0$, in a) shown with μ/W (yellow-brown) and bankruptcy fraction (red), and in b) average profit per worker.

time series is qualitatively explored by plotting μ/W for different m on the same plot, and then quantitatively by comparing the extremum of the average price time series as m is varied. In Fig. 4.8 a), the average price $\mu(t)/W$, normalized by the time-average, can be seen for different m values. I normalize the values such that the dynamics can actually be seen, as the unnormalized values are over such a large spread that even in log-scale, it looks almost like a flat line. The takeaway point of a) is that the graphs are all quite similar in the shown timespan. In b), all time series have been run for $T = 10\,000$ time steps, and the minimum and maximum over time are found. This was repeated 15 times to get uncertainties, though the uncertainties are smaller than the data-markers. The extremums are divided by m to more easily compare them. We clearly see how the extremums relative to m are constant, meaning the maxima and minima of the average price scale linearly with m in the range of m values shown, because the dynamics becomes pathological for m approaching zero. I again compare the fragility using the lifespan, as

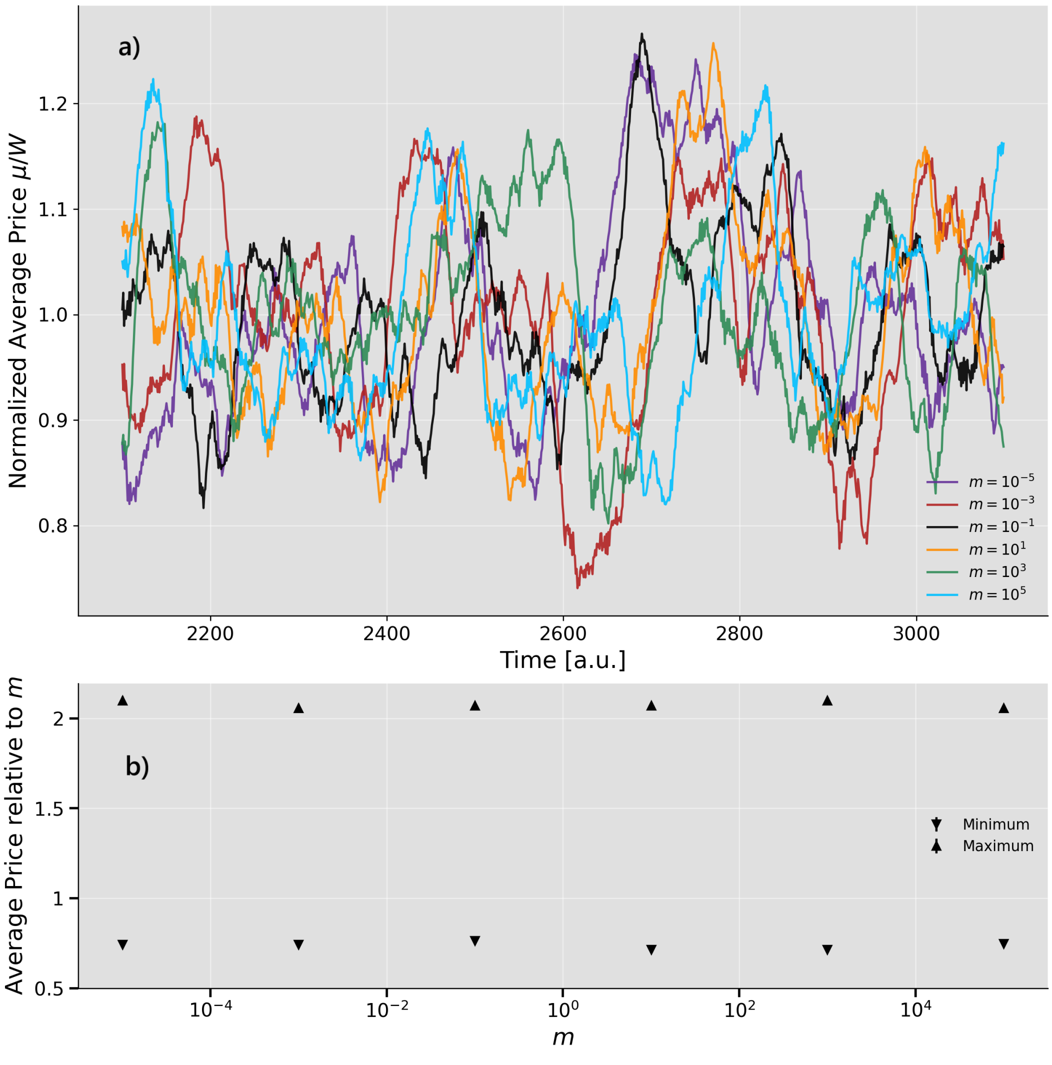

Figure 4.8: In a), the normalized μ/W time series for different m values is seen, and in b) the maximum and minimum of each of the (unnormalized) time series in a) are plotted against m, repeated 15 times and for $T = 10000$ time steps.

seen in Fig. 4.4 b). Here, all distributions but $m = 10^{-5}$ are so similar that any differences are more than likely from the stochastic nature of the model. That $m = 10^{-5}$ looks fine in Fig. 4.8 but not in Fig. 4.4 b) suggests that it is right at the border of being too low, and it is down to stochasticity if the system behaves or goes pathological. There does not seem to be an upper limit on m.

4.4.3 Minimum wage

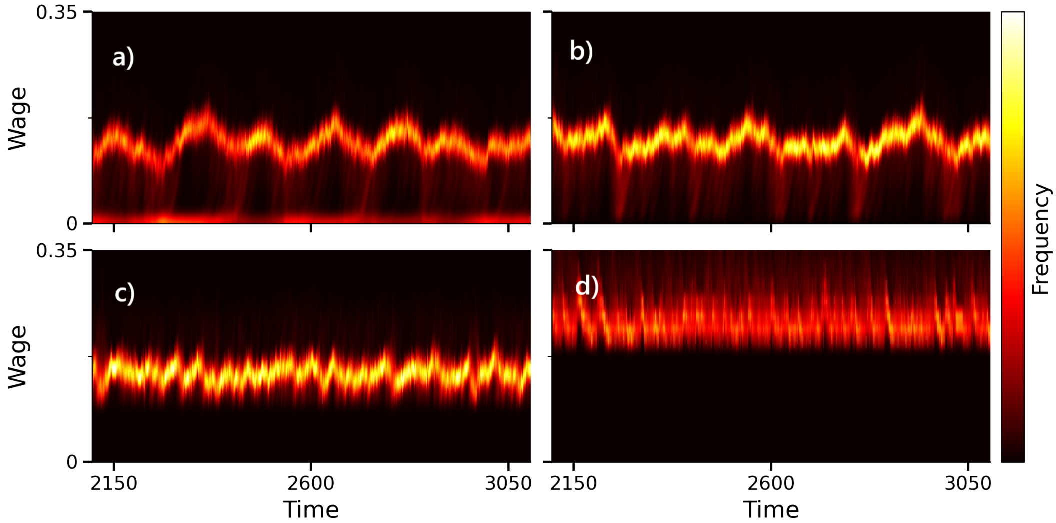

Figure 4.9: Wage density for four different $s_{\min}$, otherwise standard parameters are used. a) shows the lower extreme, which still has the normal cyclic behaviour but also has a "cloud" of low-wage companies. b) is the standard $s_{\min}$ value, and for c) and d) the dynamics break down as $s_{\min} \approx m$.

During mass bankruptcies, the smart choice for a startup is to pick a low wage, and I would expect most startups to choose a wage in the range $[s_{\min}, s_{\min} + m]$, aslow wage companies survive longer and are profitable, so startups are likely to pick one of the wages near $s_{\min}$. If the perturbation is negative, they get $s_{\min}$, and if the perturbation is positive, they get $\approx m$, which reduces to $[s_{\min}, m]$ when $s_{\min}$ is on a lower order of magnitude than m. However, when $s_{\min}$ approaches m, it becomes difficult for a startup to get a low wage, as the bounds in $[s_{\min}, s_{\min}+m]$ increase. As I will show in the following section, m sets the scale of the system, but a high $s_{\min}$ simply forces startups to pick high wages, creating constant bankruptcies. Looking at the wage densities over time in Fig. 4.9, we see how $s_{\min}$ influences the cycle dynamics. b) is the standard parameter version, and comparing the low $s_{\min}$ in a) to this, we see the only real difference is the "cloud" of low-wage companies, but the overall dynamics remain. In c), $s_{\min} = m$, and while the oscillation frequency is higher, there are still traces of the original oscillation dynamics as opposed to the mess in d). So, between the two extremes, the choice of $s_{\min}$ is almost insignificant, and the cycles only break down when $s_{\min}$ gets to the same order of magnitude as m, and especially when $s_{\min} > m$.

To further support this claim, I estimate the chance of going bankrupt when transacting, as opposed to the fraction of all companies going bankrupt. This is because as the number of bankruptcies increases, the number of transacting companies decreases, meaning fewer possible bankruptcies. It boils down to dividing the number of bankrupt companies by N or $N_{w>0}$, and I do the latter here. The average fraction of companies with non-zero workers $N_{w>0}$ going bankrupt in a) where $s_{\min} = 10^{-8}m$ and b) with $s_{\min} = 0.1m$ are both roughly 5%, in c) with $s_{\min} = m$ around 7.5%, and d) for $s_{\min} = 2m$, almost 25% go bankrupt every time step. While not shown, for even higher $s_{\min}$, the bankruptcy percentage becomes close to 100%. Plots showing the bankruptcy fraction over time can be seen in Appendix 7.1.

In Fig. 4.4 c), the cumulative lifespan distribution for the four $s_{\min}$ values shown in Fig. 4.9 can be seen. As expected from the wage density graphs, $s_{\min} = 10^{-8}m$ and $s_{\min} = 0.1m$ are almost identical, with $s_{\min} = 10^{-8}m$ having slightly longer lifespan. This is because companies with $s_k \approx 10^{-8}m$ realistically will have no workers until they have increased their wages a bit. The effect is not that large, but should be more noticeable for higher α, where small-wage companies have an even harder time getting workers. It seems strange at first that the $s_{\min} = 2m$ distribution has the companies that live the longest, given how many bankrupcties it has, but a possible reason could be that startups get too large wages, allowing non-bankrupt, low-wage companies to survive, but as said, the oscillation dynamics have largely broken down, and one should be careful about drawing too many conclusions from $s_{\min} = 2m$ other than the fact that values this high do not work. This further reinforces the conclusion that as long as $s_{\min}$ is on a lower order of magnitude than m, it has next to no influence on either micro or macro dynamics. However, it does make for prettier wage density plots when $s_{\min}$ is not small and does not have the low-wage clouds, so $s_{\min} = 0.1m$ in the standard parameters.

4.4.4 Wage update size

I would expect that the percent change in wage at each time step $\Delta s/s$ primarily serves as a change in the time scale. However, a larger wage update puts companies at a higher risk of reaching $s_k > \mu/W$, thus losing money and potentially facing bankruptcy. In other words, I expect company lifespan and similar metrics to be shorter for higher $\Delta s/s$, primarily because companies collectively reach too-high wages faster, and further on an individual level, a company is also more likely to increase its wage too much, leading to bankruptcy. The individual effect becomes more severe the higher $\Delta s/s$ becomes, reaching a point where wage updates are anticorrelated, i.e., if you increase your wage one step, you decrease it the next, preventing growth.

To find out how great this effect of individual companies overextending their wage is, a similar figure to that of Fig. 4.1 can be seen in Fig. 4.10. The median recession duration is around 70 time steps for $\Delta s/s = 0.1$ (detailed in Section 4.5), whereas the one shown in Fig. 4.10 has a duration of approximately 700 time steps (peak at 8500 and trough at 9200), thus a factor 10 larger. More surprisingly is how less volatile the $\Delta s/s = 0.025$ system is. There is an order of magnitude fewer bankruptcies (peaking at 0.3% vs. the 7% of $\Delta s = 0.1$), and companies are much more equal in size, i.e., high D, and therefore also in wages paid. The capital is more spread out, and unlike $\Delta s/s = 0.1$, the points of highest density are not near the point of bankruptcy $C = 0\$$, but instead around $C = 4\$$.

It is not surprising, then, that the cumulative lifespan distributions are similarly shaped but shifted, as seen in Fig. 4.4 d). It might very well then be, that a lower $\Delta s/s$ would have yielded more accurate results, yet because of the larger computational demand (as seen, a factor 10 more time steps are needed), this could not be done in time.

4.4.5 Companies and workers

The system size parameters N and W are closely related, and I will now show how the number of companies and the number of workers per company, W/N, influence the company diversity and median lifetime. Fig. 4.11. a) shows the time-averaged diversity $\bar{D}$ divided by system size, so $\bar{D}/N = 1$ means all companies have W/N workers, and $\bar{D}/N = 1/N$ means one company

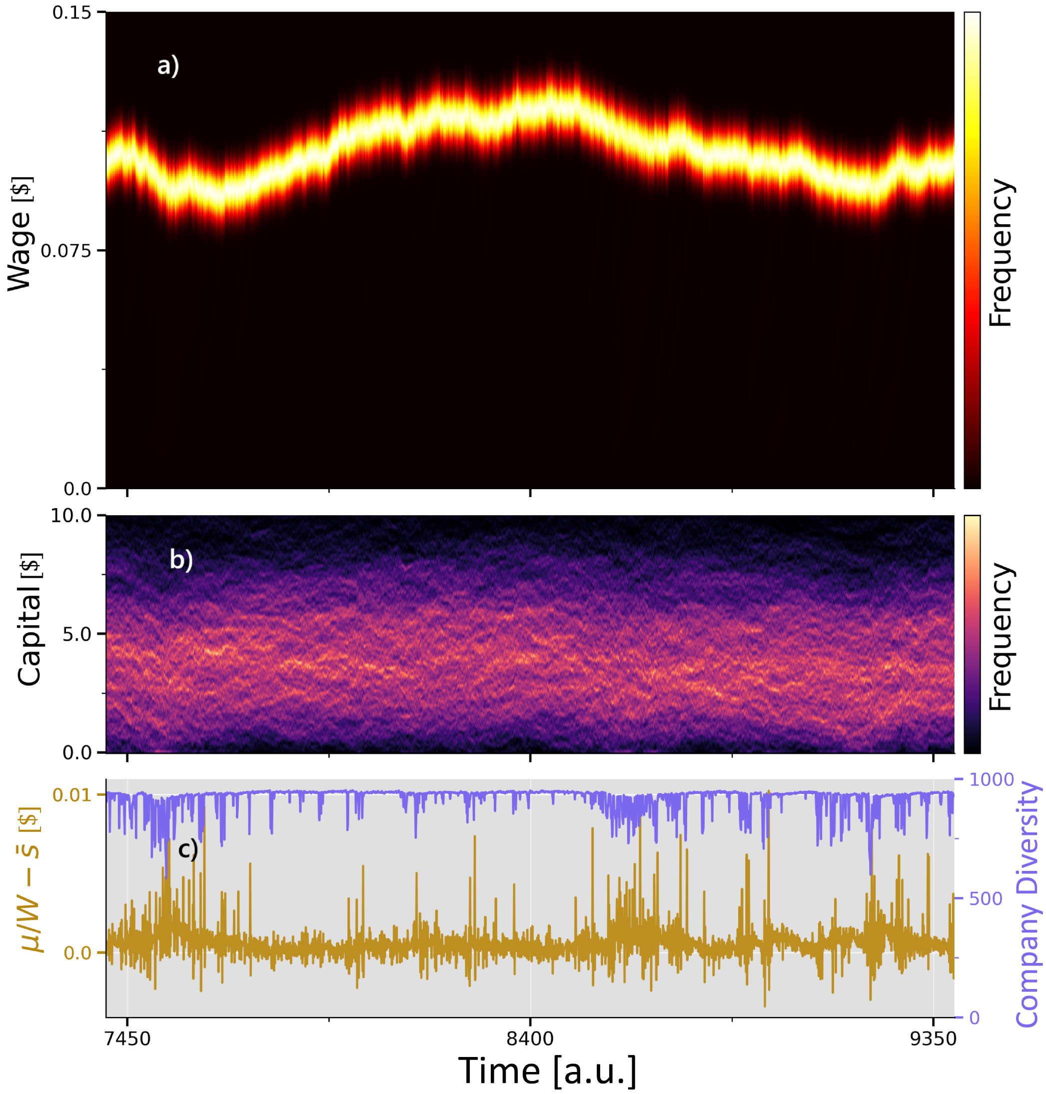

Figure 4.10: $\Delta s/s = 0.025$. a) illustrates the wage density in the system over time, b) the capital density over time, and c the average profit per worker and company size diversity. Compared to the standard $\Delta s/s = 0.1$, this system is slower and much less volatile.

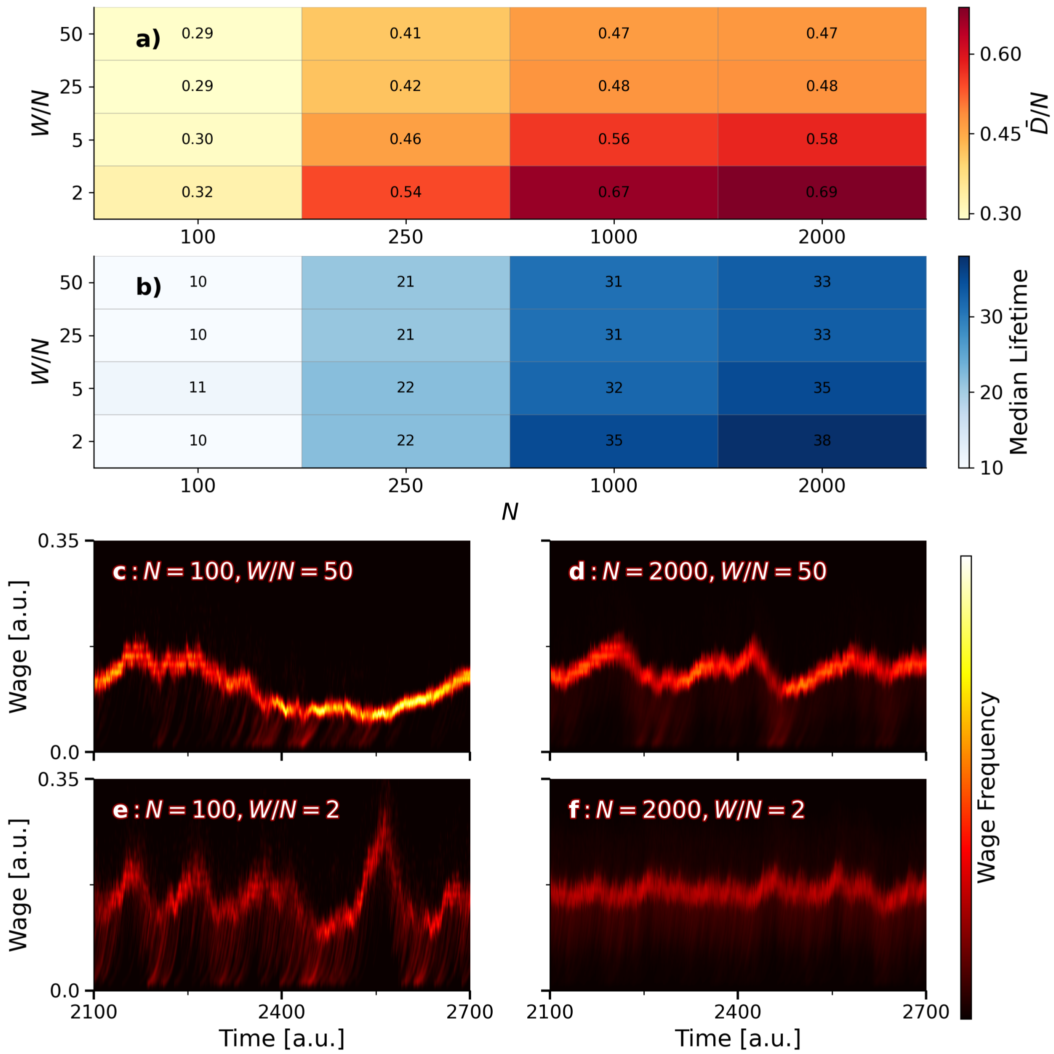

Figure 4.11: In a) the mean company diversity divided by N, and in b) the median company lifespan for different N and W/N values are illustrated. Below them in $c), d), e)$ and $f)$, the wage density over time for the parameters in the corners of a) and b) is shown. A large N increases stability, and a smaller W/N means fewer transaction companies and less pronounced cycles.

has all workers. I divide by the system size to better compare diversity values for different N, and while the minimum values change by doing the normalization, the $\bar{D}/N$ values are all much larger than their minimum values. In b), the median company lifespan is shown. To grasp the microbehaviour, the wage density over time for the parameters in the four corners of a) and b) are shown in c)-f). These wage densities show that it is not as straightforward as saying a high N and low W/N increases stability, which the high lifetime and diversity for $N = 2000$ and $W/N = 2$ in a) and b) might otherwise be interpreted as. Because, as seen in f), a low W/N means more companies with $w_k = 0$, resulting in fewer transactions, fewer opportunities for losing capital, and thus fewer bankruptcies and a longer lifetime. The high diversity in this area is due to a lack of clear cyclic dynamics, which the model using standard parameters shows.

What a) and b) can tell us, at least regarding diversity and lifespan, is that N has a higher influence than W/N, and that the results change little after $N > 1000$, giving no motivation to increase it further.

While not shown, there is a soft lower bound on how small the system can get before it turns pathological. A W/N less than 2 is almost guaranteed to fail, though larger N can increase the bound. This is not a problem, as markets of that size are unrealistic.

4.5 Time scale of the system

The three most straightforward variables to compare between the model and data to get the time scale of the model are: the lifespan of companies, the duration of recessions, and the time between recessions. The latter two should be similar as both are found from the peaks in μ. In Table 4.2, the medians of the two recession distributions and the half-life of the lifespan distribution are compared against the data. The uncertainties on the medians are found using bootstrapping, and since the dataset from NBER used for the recession medians was relatively small (35 values), the uncertainties should not be given too much quantitative weight. Table 4.3 shows the ratio between model times and data times, and we see that the model times are larger than the data times in all cases except the lifespan half-time for $\alpha = 8$. This by itself is not an issue, the issue is that the recession times are larger than the lifespan half-times. For $\alpha = 4$, the half-time is 30 time steps, which is roughly 3.5 times larger than the 8.5 years seen in the data from [22]. This would suggest that it takes 3.5 time steps in the model to make one year, but the two recession times would instead predict near 50 time steps per year. The discrepancy between the years that the half-time and the recession times is the lowest for $\alpha = 2$ at around a factor 10, which is still a concerning difference. The low half-time indicates that companies die too quickly, which is probably due to the simplistic nature of the model's companies. The reasons for the high company fragility are discussed in the discussion chapter.

In total, the comparison of times from the model to the datasets does not allow the model to give precise temporal predictions, yet it does provide a rough idea of the scale, so for $\alpha = 4$, a year is on the order of 10^1 time steps.

4.6 Robustness

The $m = 0$ case in Fig. 4.7 showed a pathological state that the system can enter when core parts of the model are changed. More such changes and their influences are now discussed.

Table 4.2: Model and data timescale values by α. The data row has been highlighted for easier reference.

	Time between recessions	Recession duration	Lifespan half-time
Data	4.08 ± 0.50	1.13 ± 0.15	8.53
$\alpha = 2$	555.00 ± 13.54	205.00 ± 6.50	134.00
$\alpha = 4$	185.00 ± 1.50	78.00 ± 1.50	30.00
$\alpha = 6$	171.00 ± 2.00	75.00 ± 1.50	14.00
$\alpha = 8$	217.00 ± 4.25	104.00 ± 4.00	7.00

Table 4.3: Ratio of model to data timescales by α used to set the time scale. The two recession columns have much higher ratios than the half-times.

	Time between recessions	Recession duration	Lifespan half-time
$\alpha = 2$	135.87 ± 16.95	181.74 ± 24.08	15.72
$\alpha = 4$	45.29 ± 5.55	69.15 ± 8.87	3.52
$\alpha = 6$	41.86 ± 5.14	66.49 ± 8.66	1.64
$\alpha = 8$	53.12 ± 6.58	92.20 ± 10.89	0.82

4.6.1 Startup wage picking

Since m only changes the scale of the system, but the system still needs some form of variation to avoid a too high D, it is tempting to think that m might be removed. I have tried a host of different methods for startup companies to pick their wage, but almost all give pathological outcomes. These methods include:

- Startups pick a random wage from a company in the last T time steps.
- Instead of having a fixed m, the mutation magnitude is a fraction of the mean wage.
- A wage between $s_{\min}$ and the mean wage is picked for startups.
- Startups choose another company's wage proportionally to how many workers that company has.
- The lowest wage of a non-bankrupt company with non-zero workers is chosen, plus a perturbation.

The only alternative option I found that gives satisfactory results, is to have the perturbation be drawn from a Normal distribution with $\mu = 0$ and σ equal to the standard deviation of the living companies' wages at that time, $q \sim \mathcal{N}(0, \sigma_{w(t)})$.

The wage density, capital density, and diversity together with average profit per worker, can be seen in Fig. 4.12 for the Gaussian perturbations. It is reminiscent of the usual results, yet has much higher wages and capital values, though the higher values can be achieved in the original model by increasing m, as seen in Fig. 4.8. The half-time of the company lifetime distribution is 36 time steps, which is slightly larger than the half-time of 30 time steps that the usual model has.

To sum up, it is much easier to break a model than to improve it or find equal alternatives. The normally distributed perturbations yield cyclic behaviour similar to that of the core model with its uniformly distributed perturbations. While this alternative approach gets rid of m, it does not reduce the number of parameters, as the normal distribution comes with 1 extra parameter, being the standard deviation.

4.6.2 The capital update

In the capital update in Eq. (3.1), companies balance their income and expenses, and greater profit is only achieved if companies collectively raise their wages, yet it seems reasonable that companies would want their income to be larger than their expenses independently of other companies. This could be done by adding a small factor $\beta > 1$ to the income term:

$$\Delta C_k = w_k(\beta \frac{\mu}{W} - s_k). \tag{4.4}$$

Yet even a small $\beta \gtrsim 1.01$ is enough to make the capital diverge to ∞. If one instead adds a factor $\gamma < 1$ to the expenses term, μ/W becomes smaller as well, and nothing is achieved.

Another approach, which I did not further analyze, would be to increase the wage not when $\Delta C_k > 0$, but when the profit was larger than some positive number, yet this might as well end up over-reducing μ/W through the lowered wages and also adds another parameter.

In conclusion, the capital update formula in Eq. 3.1 is central to the model, and any modifications radically change the system. This is both a strength and a weakness. Having this single, highly influential equation has allowed for easy interpretation of the inflation cycles and tracing them back to this equation. On the other hand, much of the system dynamics is heavily dependent on this equation, so any modifications may completely change the outcome of the model, reducing robustness.

4.6.3 Finding peaks

The peaks in the business cycle needed to find recessions in the model are found by:

1. Performing a rolling average with 10 steps on $\mu(t)$.
2. Finding the peaks of the averaged $\mu(t)$ using SciPy's find_peaks algorithm [30]. The hyperparameters (e.g., peak width, prominence, height) used here play a major role in the outcome.
3. Remove peaks and troughs such that peaks and troughs come in pairs by always taking the largest (or smallest for troughs) value if there are multiple candidates.
4. Recession duration is the time difference between neighbouring peaks and troughs, and the time between recessions is the time between neighbouring peaks.

The difference between a peak in the business cycle and an insignificant noise bump is not obvious from the results. To get a deeper understanding of the significance of the peak hyperparameters on the macro results, in Fig. 4.13, the recession duration distribution is shown for two sets of peak hyperparameters on the same dataset, one being "picky" on what is a peak and one being "relaxed". The two resulting distributions are rather different, but I went with the picky approach. There is no mathematical reason as to why the picky hyperparameters ended up being used, but they did give more qualitatively consistent recession distributions for different α values (not shown), and further, by eye the picky peaks/troughs in Fig. 4.13 found seem more correct than the relaxed ones.

4.7 Summary of main findings

Non-periodic inflation-deflation cycles emerge from the simple model. They are a result of wages going up until the market becomes vulnerable to shocks in the form of large companies not

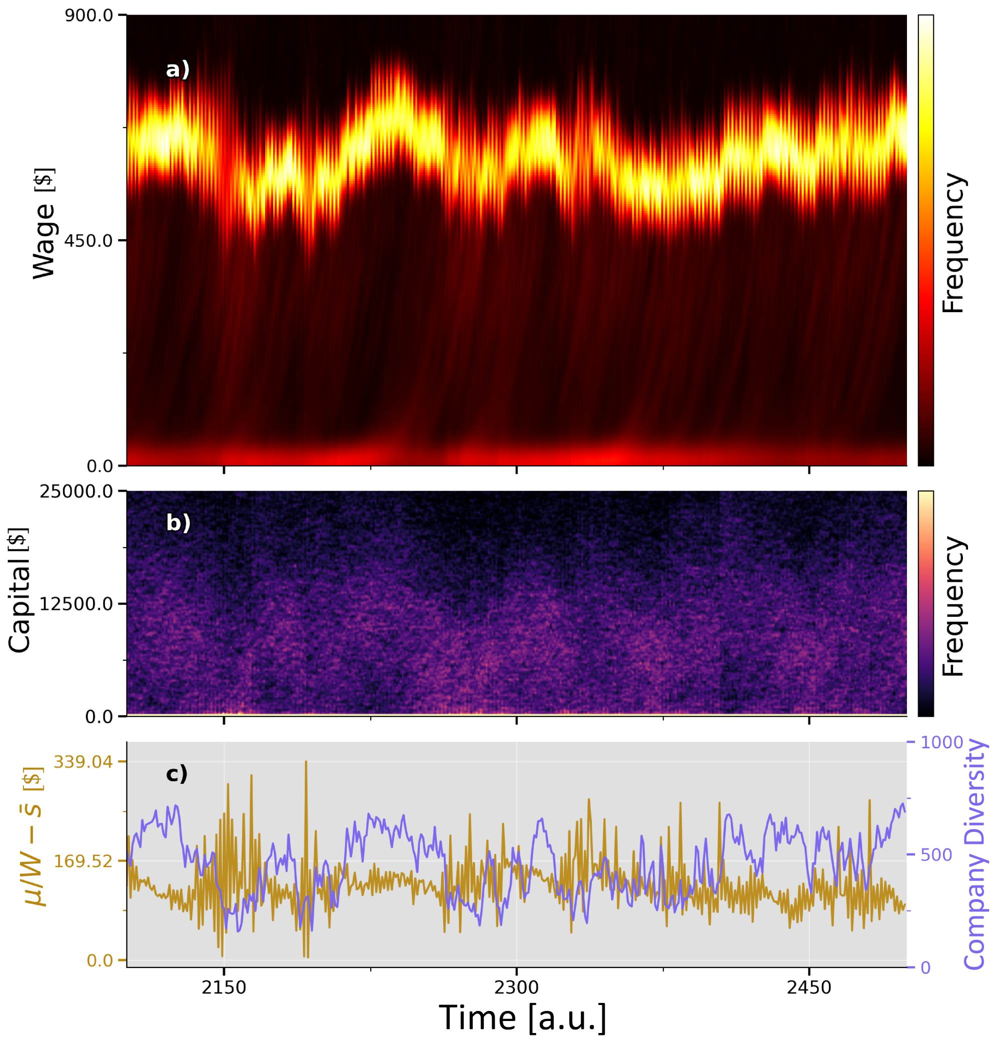

Figure 4.12: Normal distributed perturbations. a) illustrates the wage density in the system over time, b) the capital density over time, and c) the average profit per worker and company size diversity. This system has higher s and C than the standard model.

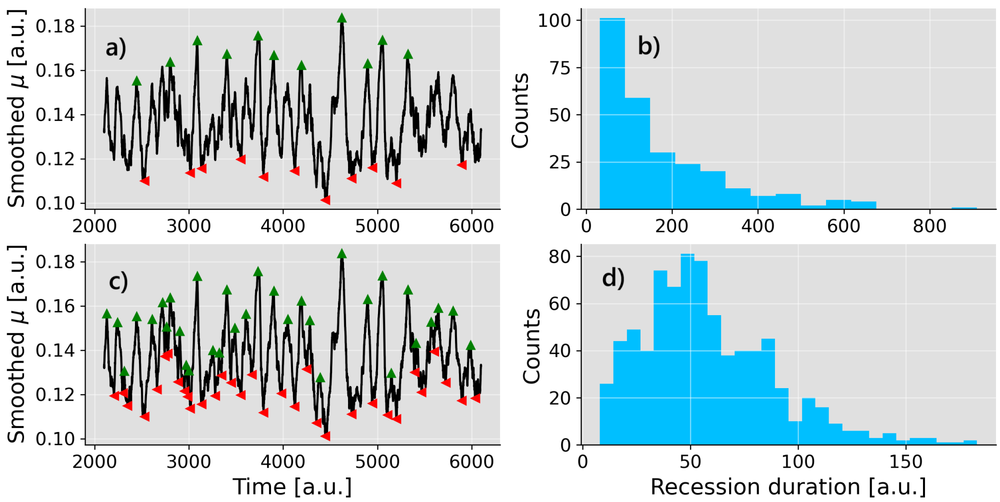

Figure 4.13: Significance of hyperparameters used in finding peaks using standard parameters. The first row is the picky selection of peaks, and the second row is the relaxed selection. Red and green triangles indicate troughs and peaks, respectively. The model was run for $T = 100000$ steps, but for clarity, only 4000 steps are shown on the peak plots.

transacting, the bubble bursts, and a recession starts, where low-wage startups push the wages further down and then slowly up again. Different indicators were validated against empirical data, and they generally agree on a qualitative level, except the company size distribution, which is not power-law distributed as expected.

The dynamics of the emergent cycle are heavily influenced by parameter choice, with the most interesting being workers' sensitivity to wage differences, α. A large α leads to larger companies, as high-wage companies will attract more workers, and since the economy is hurt more when large companies are not chosen to transact, it becomes more unstable, and bankruptcies increase. With too small an α, the diversity D is high almost all the time, and the system is too resistant against shocks. For high α, the company size distribution approaches a power-law, but the already short median company lifespan becomes lower than the median time between recessions.

The mutation magnitude m sets the scale of the wage and capital, and can be varied freely as long as it does not get close to zero, at which point the system goes into a pathological state of constant total bankruptcy.

As long as the minimum wage $s_{\min}$ does not get near or larger than m, it has little impact on the dynamics. When $s_{\min} \approx 10^{-8}$, there will always be a visible number of companies near $s_{\min}$, though the macro scale effect of this is minuscule.

The percent that companies change their wages each time step, $\Delta s/s$, clearly influences the cycles, but much the same as watching a movie at twice the speed changes the experience: the content does not change, it just happens faster. However, at higher $\Delta s/s$, more than just a speedup happens, as companies will have a harder time getting close to $s_k = \mu/W$ since they take larger steps in s-space.

The system size parameters may be the least interesting. A low number of companies N means the system becomes unstable, and little effect is seen when increasing N beyond 1000. When W/N is small, a large fraction of companies have no workers and thus cannot transact, slowing down the dynamics. The choice of $W/N = 20$ in the standard parameters is empirically motivated by [16].

While many of the results share qualitative features, such as the company lifespan distribution being exponential, when trying to estimate the time scale of the model, it becomes clear that it does not perform well at this scale of analysis. The lifespan is much shorter than the time between recessions, and the recession duration relative to the data. Depending on parameters, the ratio between the recession results to the recession data is around 10 to 20 times that of the lifespan ratio. It is thus not possible with good confidence to set a precise time scale for the system.

Chapter 5

Discussion

5.1 Interpretation of the Inflation-Deflation Mechanism

In the model, aggregate demand is determined by the total wages paid in the previous time step, distributed across the population, i.e., μ/W. This represents what workers are able and willing to spend. A company's "supply" is the number of workers it employs, since each worker produces one unit of output. When demand (μ/W) rises, companies attempt to match their supply by growing their workforce by offering higher wages. This further increases μ, which feeds back into demand, creating an endogenous mechanism for inflation.

However, as wages rise and companies expand in size, they also become more exposed to fluctuations. Since capital change ΔC_k scales with the number of workers w_k, large companies experience larger swings, both positive and negative. A shock occurs when a smaller-than-usual number of large, high-wage companies are selected to transact. This lowers μ, reducing demand, which then disproportionately hurts those same large companies because one: Their large wage means they likely have $s_k > \mu/W$ and thus a negative ΔC_k, and two, because of their large wage, ΔC_k is going to be greater in size. The resulting capital losses prompt wage reductions, further lowering μ, and thus amplifying the downturn. This feedback loop leads to disinflation and, in severe cases, deflation.

A similar cyclic pattern exists in real-world data and is often described through the Phillips curve [4, 31], which posits an inverse relationship between inflation and unemployment. Inflation tends to rise during economic expansions and fall during recessions, typically with a lag [27], and unemployment is strongly linked with economic downturns. This was exactly what was observed in the inflation graphs in the final row of Fig. 4.2, where the shaded areas that mark recessions are generally in periods with disinflation/deflation and vice versa for the good periods. The model thus reproduces this dynamic qualitatively, but without explicit unemployment. This highlights two simplifications: (i) the absence of unemployment, and (ii) the lack of wage stickiness. In reality, wages tend to be rigid downwards [32], while in the model, they adjust symmetrically in response to profit and loss. As a result, while the inflation-deflation cycle in my model mirrors empirical patterns, it arises from a slightly different and more stylised set of mechanisms, and better matches the following theories.

Because, even though my model does not include debt, it exhibits a similar deflationary feedback loop to that of Fisher's debt-deflation cycle [5]. In my model, the collapse is not driven by debt and the deleverage thereof, but bankruptcies still feed into declining demand, as measured by μ/W. The resulting contraction in consumer spending acts as an endogenous shock, leading to further wage cuts and company failures. This mirrors the destabilizing mechanisms Fisher described, albeit through a simplified wage-capital interaction rather than credit markets. In this way, this model can be viewed as a wage-based analogue to Fisher's debt-deflation dynamics.

Although the model does not explicitly include financial assets or debt, it produces a pattern remarkably similar to the endogenously generated instability in Minsky's financial instability hypothesis [6]. In the model, during boom phases, widespread profitability leads companies to raise wages, which attracts more workers to the largest companies, and amplifies growth, analogous to increasing financial leverage. However, this also raises systemic fragility, as higher wages make companies more sensitive to negative capital shocks. Eventually, such a stochastic fluctuation shock triggers losses and wage cuts, leading to a cascade of bankruptcies and a lowered μ/W. In this sense, the model mimics the qualitative structure of Minsky's cycle: stability breeds instability, and crises emerge from the system's internal dynamics.

5.2 Strengths and limitations of the Model

The model's greatest strength, namely its simplicity, is also its greatest weakness. The simplified, lacking micro behaviour, such as the lack of unemployment, high company fragility and bankruptcy, and downward wage rigidity, could likely be fixed, but only at the cost of added complexity through more rules, equations, and parameters, contrary to the purpose of this thesis.

The fragility of companies is perhaps the most outstanding problem for the model. This is seen in Table 4.3, where the recession duration and periodicity are much larger (around 15 to 20 times for $\alpha = 4$) compared to the data than the half-time of companies, and the fact that around 5% of companies go bankrupt each time step. There are several reasons why companies in the model are more fragile than in real life, such as the model companies not having any savings and thus have no protection against bad times, their wage decisions are made purely based on how they did in the current time step, the worker relocation means some companies may suddenly get a huge number of workers and lose a critical amount of capital, and new startup may choose a too high wage and instantly go bankrupt the next time they transact. Companies are effectively run by over-optimistic people who only look at the present and believe they will be bailed out of any failures [1]. Again, this could be fixed, but that would require rules for, e.g., how companies save capital. A better choice of wage for startup companies might exist, limiting that part's effect on the fragility. A more detailed way of making the wage update decision is examined in the following section.

At first, it may seem contradictory that the model allows both a problematically short company lifespan and longer, healthier recession dynamics. This is because of the fluid nature of workers and the low importance of capital, allowing a startup company to choose a high wage and get a large number of workers immediately and thus effectively be a huge player in the market in a short amount of time, though, as we have seen from following the trajectory of a single company in Fig. 4.3, such a move likely results in bankruptcy, but if it does not, the company can theoretically become the largest in a single time step. This may also suggest that C_k should play a larger role or even be removed. Currently, companies only care about the absolute value of their capital regarding bankruptcy, and the change in capital ΔC_k is much more important as it determines if s_k is increased or decreased.

It is likely that the long time scale observed in the lifespan distribution in Fig. 4.2 c) arises from startups choosing a wage near $s_{\min}$, and the short time scale from startups picking a too

[1] Where this type of behaviour can be seen in real life is left as an exercise for the reader.

high wage, and often, companies will pick a high wage and go bankrupt multiple times until a wage near $s_{\min}$ is chosen. This suggests that the current method of startup companies picking their wage among living companies that made a profit is flawed, though it is also entirely possible that the problem is the fallback condition when no companies made a profit, of choosing among companies that lost the least. Had time permitted, it would be insightful to see when and how often no companies made a profit and the impact it has.

As outlined in Section 3.4 and confirmed in the results, the model works best at a macro scale, and one should be cautious about interpreting at the level of individual agents. Nevertheless, the model successfully captures several important aggregate features, including endogenously driven business and inflation cycles, and recession dynamics, all without the need for complex institutional rules or fine-tuning parameters.

This supports the idea that simple, rule-based interactions can give rise to meaningful economic phenomena, even if the micro-level realism is limited. Much like how the ideal gas law neglects molecular structure yet reliably predicts bulk properties of gases, this model demonstrates that stylised agent interactions can qualitatively reproduce macroeconomic behaviour. The core takeaway is therefore not a claim about realistic microeconomics, but about the power of making simplifications at the micro level to get emergent, easily interpretable macro results. In other words, do not use the results of this thesis to aid your next stock purchase; instead, use it as a springboard to create similar models.

5.3 Possible Extensions

Below are a few extensions and alternative versions of the model, at varying levels of detail.

Salary-based

It does not seem controversial to claim that most working people are paid every month independently of their employing company's success that month, unlike our model, where employees are paid per work done. I tried implementing a salary-based market by changing the transaction phase. Currently, companies pay their workers and receive money simultaneously, but with salaries, all companies would pay their workers, and then when a company is selected for transacting, it makes money but does not pay its workers for the job done. However, this leads to a more swingy system with more bankruptcies, as about a third of the companies are not picked every time step but still pay their workers, while other companies are selected multiple times and make a fortune.

Unemployment

Our model can be extended to include unemployment. This could, for instance, be done by updating workers incrementally instead of the total relocation. Workers are picked to compare their wage with that of a random other company, and if their current wage is worse, they quit their job and become unemployed. Companies that make a profit then hire workers from the pool of unemployed workers, with the top wage companies having priority in the case where more companies want to hire than there are unemployed workers. This means in times of large wage differentials, unemployment will rise (a larger spread will make it more likely for workers to find a company with higher wage paid) in agreement with [33].

Debt

An earlier version of the model included debt, but it turned out that in almost every case where a company went negative and took a loan, it would inevitably go bankrupt. For simplicity, we had combined capital (then called "money") and debt into one variable, so if negative, you paid interest on the value. The bankruptcy criteria thus also had to be different. I settled on a company going bankrupt if it paid more in interest than it had income. Two variations were made, one had a "debt reservoir", and one where money was lent from a central bank that also controlled the interest rate.

Ultimately, debt was not included as the overall dynamics did not change much by removing it, as going into debt was a dead sentence anyway. This is obviously not the case for actual companies, where having debt is normal, and thus it should be possible to make a model where debt is not fatal, but a possible asset.

Productivity growth

While the system shows cyclic behaviour, unlike the real world, the wages and capital do not collectively grow over time. This is also reflected in the inflation graphs Fig. 4.2 k) and l), where the model's inflation changes around 0%, while the data go around a small positive value. As seen in section 4.4.2, the perturbation m sets the scale of the system. As such, increasing m (and keeping $s_{\min} = 0.1m$) over time should increase the wages and capital. This is a rather artificial way of introducing growth, as m has little in common with real economic variables.

Alternative Wage update decision criteria

All agents have one choice to make at each time step: Do I want to offer my potential workers a higher or lower wage? While simple, it has a large impact on whether or not the company will make a profit in the next time step and how large the change in capital will be. I have employed the simplest (besides randomly guessing) method: Did you make a profit or not? If yes, raise the wage. If no, lower it. This simple criterion does not reflect how businesses are run, though, as I have demonstrated, the model produces reasonable results anyway. There are several ways to make this decision more sophisticated, one could be for companies to estimate the expected capital change from lowering the wage versus increasing it, and do whichever is the most profitable. Another would be to have the decision made by a reinforcement learning agent. Both approaches will now be discussed.

Expected capital change

The expected change in capital $\Delta\hat{C}_k(t+1)$ in the following time step is found by replacing $w_k(t)$ in $\Delta C_k(t+1)$ with an estimate for the expected number of workers. This $\hat{w}_k(t)$ estimate is the probability of getting a worker given the updated wage $s_k(t)$ times the total number of workers W

$$\hat{w}_k(t) = \frac{s_k^\alpha(t)}{\sum_i^N s_i^\alpha(t-1)} W. \tag{5.1}$$

The reason for using $s_i^\alpha(t-1)$ in the denominator is that the wages at time t are unavailable, explained further below.

A reminder on notation and the order of model phases: The transactions happen before the wage and worker updates. So $\Delta C_k(t)$ is found, then the decision on how to update $s_k(t)$ is made, and finally, workers are relocated such that $w_k(t)$ is found. Companies need to estimate how much money they will make in the following time step before they update their wages in the current time step. Thus, $\Delta\hat{C}_k(t+1)$ is calculated in time step t after the transaction phase and before the wage update phase. For this reason, we must use the current sum of wages to estimate the expected number of workers in the next time step, and that is the reason we use $s_i^\alpha(t-1)$ in the denominator of (5.1) and not $s_i^\alpha(t)$, which would be the correct choice if the knowledge was available. However, outside of the periods of mass bankruptcies, the sum of wages does not change dramatically, and even so, this knowledge gap is realistic and could be interpreted as a benefit of this approach. At the risk of being verbose, here is the algorithm for making the wage update decision for each company spelled out in detail:

1. Calculate the wage when increasing and decreasing it:

$$s_k(t;+) = s_k(t-1)\,(1+\Delta s/s) \tag{5.2}$$
$$s_k(t;-) = s_k(t-1)\,(1-\frac{\Delta s/s}{1+\Delta s/s}). \tag{5.3}$$

2. Then, find the estimate of the expected number of workers for both cases:

$$\hat{w}_k(t;+) = \frac{s_k^\alpha(t;+)}{\sum_i^N s_k^\alpha(t-1)} \tag{5.4}$$
$$\hat{w}_k(t;-) = \frac{s_k^\alpha(t;-)}{\sum_i^N s_k^\alpha(t-1)}, \tag{5.5}$$

3. and then finally calculate the capital changes

$$\Delta\hat{C}_k(t+1;+) = \hat{w}_k(t;+)\frac{\mu(t)}{W} - s_k(t;+)\hat{w}_k(t;+) \tag{5.6}$$
$$\Delta\hat{C}_k(t+1;-) = \hat{w}_k(t;-)\frac{\mu(t)}{W} - s_k(t;-)\hat{w}_k(t;-) \tag{5.7}$$

4. and pick whichever yields the most profit:

$$s_k(t) \leftarrow \begin{cases} s_k(t;+), & \text{if } \Delta\hat{C}_k(t+1;+) > \Delta\hat{C}_k(t+1;-) \\ s_k(t;-), & \text{else} \end{cases} \tag{5.8}$$

Reinforcement learning

The second approach, using reinforcement learning (RL), adds vastly more complexity, though a lot of it is "hidden". RL is a branch of machine learning well-suited for game-like scenarios, where an agent learns an optimal strategy through interactions between it and its environment. The basic loop of RL is that an agent (in our case, a company) takes an action (increase/decrease wage), which influences the environment (All companies' C_k, w_k, and s_k) and gives a reward based on the output of the action. The reward and the changed environment are then used to determine the next action. Over time, the agent will have learned which actions give a high reward in the given environment. An agent typically does not see the entirety of the environment, but only a subset called the state. The learning and action-taking are handled by the agent's policy, and this is where the complexity is hidden. An illustration of the setup can be seen in Fig. 5.1

Striving for simplicity, an example RL setup for the model could be that the state $\mathcal{S}$ would

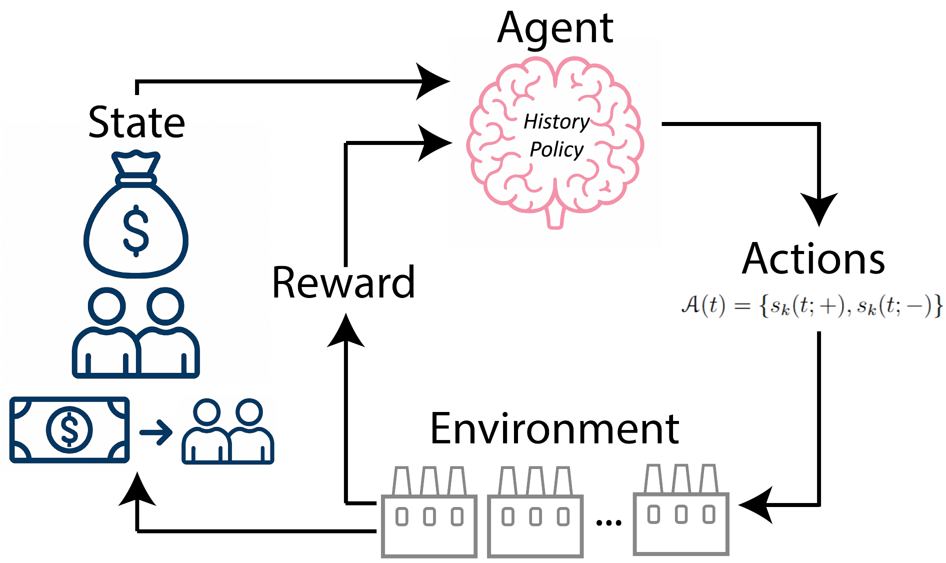

Figure 5.1: Illustration of the model contextualized in an RL framework. A company uses its history of past states (capital, workers, and wages paid) and policy to take that action on whether to increase or decrease its wage. The impact the action had on the agent's state then guides the following actions. Inspired by Fig 3.1 in [34].

only be the company's own values:

$$\mathcal{S}(t) = \{C_k(t), w_k(t-1), s_k(t-1)\}. \tag{5.9}$$

The two possible actions would be to either increase or decrease the wage (or, alternatively, choose its own $\Delta s/s$ including a sign.)

$$\mathcal{A}(t) = \{s_k(t;+), s_k(t;-)\}. \tag{5.10}$$

The reward $\mathcal{R}$ is a bit trickier. Whatever behaviour $\mathcal{R}$ promotes is what will be optimized towards. So, if one naively picks the simple option of saying that the agent should be rewarded for merely staying alive, it is highly likely that the agent will learn to stay at zero workers to prevent ΔC_k from ever being negative. Also, one should not reward short-term behaviour such as ΔC_k being positive, as generally you want the RL agent to discover the short-term strategies itself. There is no perfect solution when the system does not have an explicit end state, but I propose staying above the average capital $\bar{C}(t)$ gives a reward, and going bankrupt gives a large punishment:

$$\mathcal{R}(t) = \begin{cases} 1/T, & C_k(t) > \bar{C}(t) \text{ (above mean)} \\ -1, & C_k(t) < 0 \text{ (bankrupt)} \end{cases}. \tag{5.11}$$

Where T is the number of time steps the simulation is run for.

The RL approach is perhaps mostly interesting if you want to know the effect of having a single, intelligent company. You can make RL be responsible for all company wage update decisions, but the major disadvantage is the loss of information that the increased complexity adds.

Chapter 6

Conclusion

This thesis explored a simple agent-based model in which companies compete through wage-setting and workers relocate probabilistically based on offered wages. Despite its minimal micro-level rules and lack of complex institutional features such as debt, unemployment, or productivity growth, rich macroeconomic dynamics emerged, such as non-periodic inflation-deflation cycles, overconfidence akin to Minsky's financial instability hypothesis [6], capital accumulation.

Key findings include the emergence of endogenous boom-bust cycles driven by inflationary wage feedback loops. As wages rise, so does aggregate demand, which increases profitability and triggers further wage increases, until the system becomes vulnerable to shocks, meaning fewer large, high-wage companies are chosen to transact. Fewer workers earning wages reduce aggregate demand, triggering a deflationary spiral and cascading bankruptcies, after which low-wage startups re-enter the system and the cycle restarts.

Comparison with empirical data showed qualitative agreement on several fronts: company lifespans followed exponential decay except at a short time scale; asset returns were fat-tailed and asymmetric; and inflation/deflation was closely linked to the business cycle. However, the model failed to reproduce the power-law distribution of company sizes and did not succeed in finding a consistent timescale when comparing different model results to the data.

The central strength of the model is its simplicity and the resulting ease of interpretability. It demonstrates that complex, empirically sound macroeconomic patterns can emerge from simple microeconomic rules without relying on equilibrium assumptions, agent rationality, or dozens of parameters. The model also lends itself well to extensions, some of which, such as unemployment dynamics and alternative wage decision rules, were explored conceptually in the discussion. These additions would bring the model closer to economic realism, but must be balanced against the cost of added complexity.

The system dynamics were influenced more by some parameters than others. Most notably, the wage sensitivity parameter α strongly shapes macroeconomic volatility and company fragility. High values of α produce larger companies and a more unequal distribution of company sizes, which increases the system's fragility and frequency of collapse. In contrast, low α values lead to persistently high diversity and resilience, but suppress cyclic behaviour. Other parameters had less of an influence, such as the wage mutation magnitude m and the update fraction $\Delta s/s$, which only affect the system's scale and tempo.

In sum, this thesis has demonstrated that a simple, agent-based model can reproduce key features of macroeconomic dynamics, such as inflation-deflation cycles and recession durations, without requiring detailed micro-level realism. While the model does not capture the complexity of individual agent behaviour and fails to match all chosen empirical distributions, it succeeds in generating business cycle-like patterns and qualitatively agrees with several economic benchmarks. This reinforces the value of minimal models in exploring macroeconomic phenomena, especially when the goal is not to simulate individual behaviour, but to understand the systemic, aggregate consequences of basic interaction rules.

Chapter 7

Appendix

7.1 Minimum wage graphs

In this section, I present graphs showing the bankruptcy percentages for different $s_{\min}/m$ values.

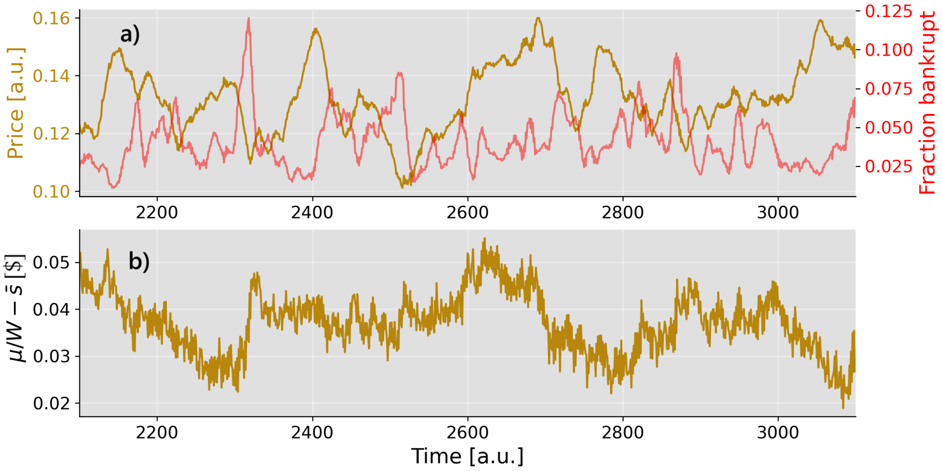

Figure 7.1: $s_{\min} = 10^{-8} \cdot m$. Top panel shows μ/W together with the fraction of companies gone bankrupt. The bottom panel shows the average profit per worker.

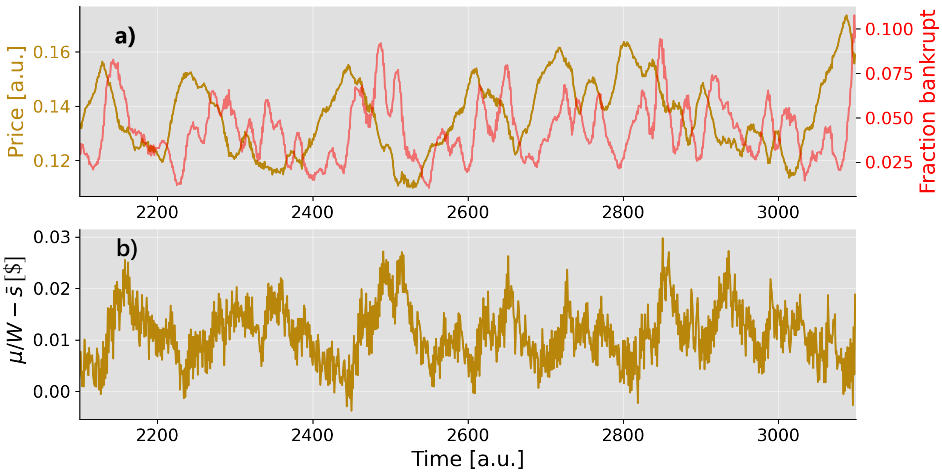

Figure 7.2: $s_{\min} = 0.1m$. Top panel shows μ/W together with the fraction of companies gone bankrupt. The bottom panel shows the average profit per worker.

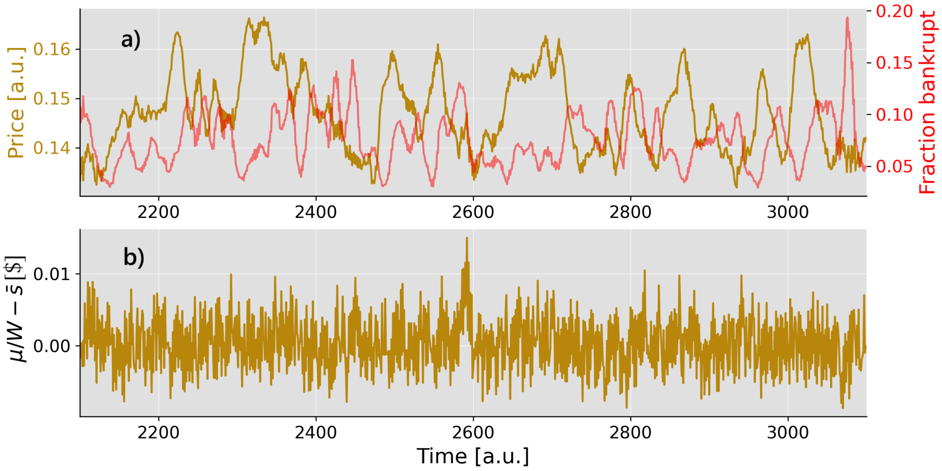

Figure 7.3: $s_{\min} = m$.Top panel shows μ/W together with the fraction of companies gone bankrupt. The bottom panel shows the average profit per worker.

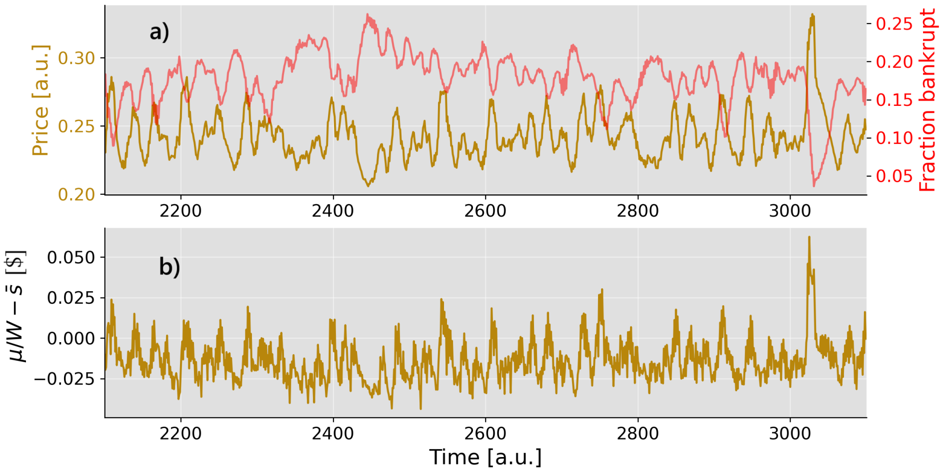

Figure 7.4: $s_{\min} = 10m$. Top panel shows μ/W together with the fraction of companies gone bankrupt. The bottom panel shows the average profit per worker.

7.2 Retrieval of company lifespan data

Data points were extracted from Figure 2c in [22] using a custom computer vision pipeline implemented in Python (using OpenCV and SciPy). The original figure was converted to a grayscale image, and a binary threshold was applied to isolate the plotted data points. Contours corresponding to individual points were detected and filtered by size to exclude axis labels and noise.

The pixel coordinates of the point centroids were then mapped to data coordinates using linear interpolation. This interpolation was calibrated against known axis tick values and gridlines manually identified in the figure. The resulting dataset closely approximates the original plot, with accuracy verified by overlaying detected points on the image for visual confirmation.

After this, three points had not been correctly identified, and we manually identified them by reading the graph. These were: $(30, 2), (52.5, 0, 57.5, 0)$.

This method was only used due to a lack of access to the original dataset and in accordance with fair use for research and reproduction of published scientific results. The original author was contacted to request the data directly.

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