R ~~O~~ D

DLI Physics **Mechanics Problem Set 1** **Applied Derivatives in Mechanics**

Name Dennis **Date** 18th.sep

1. A honeycrisp apple moves in a straight line with its position, x, given by the following equation:
$\boxed{x(t)} = t^4 - 4t^3 + 2t^2 + 3t + 6$
   a. Find its position after 1 second.
   b. Find its velocity after 2 seconds.
   c. Find its acceleration after 3 seconds.
   d. What is ~~the~~ rate of change of the acceleration at 1 second.
   e. Use Python to graph the position, velocity and acceleration as functions of time from t=0 to t=4 seconds.
   f. Use Python to graph the rate of change of acceleration vs. time.

(a) $\cancel{4}\,1 - 4 + 2 + 3 + 6 = \cancel{11}\,8m$

(b) $\frac{dx}{dt} = 4t^3 - 12t^2 + \cancel{8}4t + 3$

$\cancel{v(t)}\; v(2s) = 32 - 48 + 8 + 3$

$v(2s) = -5 m/s$

(c) $\frac{dv}{dt} = 12t^2 - 24t + 4$

$a(3s) = 108 - 72 + 4$

$= 40 m/s^2$

(d) $\frac{da}{dt} = 24t - 24$

$a(1) = 0$

intergral

2. A sky-diver of mass, m, opens her parachute and finds that the air resistance, Fa, is given by the formula Fa= bv, where b is a constant and v is the velocity.

a. Set up, but do not solve a differential equation for her velocity as a function of time.

b. Set up, but do not solve a differential equation for distance as a function of time.

c. Find the terminal velocity in terms of m, b, and g.

d. If in a different situation the formula for air resistance were Fa= bv +cv², where c is another constant find the terminal velocity in terms of the above plus c.

e. **If you are in Calc 2**, solve the differential equations from parts b and c.

(a) $\cancel{F \cdot \frac{dv}{dt} = bv}$

$F = ma,\ a = \frac{F}{m}$

$\cancel{\frac{dv}{dt} = \frac{bv}{F}}$

$F_A = -mg + bv$

$m\cancel{g}a = bv - mg$

$a\ \cancel{g} = \frac{bv}{m} - g$

$\frac{dv}{dt} = \frac{bv}{m} - g$

(b) $\frac{d^2x}{dt^2} = \frac{b}{m} \cdot \frac{dx}{dt} - g$

(c) when $a=0$, V_T is True.

$\frac{bV}{m} - g = 0$

(d) $F_{air} = bv + cv^2$

$F_g = mg$

$F_{TOT} = bv + cv^2 - mg$

$ma = bv + cv^2 - mg$

$a = \frac{bv}{m} + \frac{cv^2}{m} - g$

$\frac{dv}{dt} = 0$

$0 = \frac{bv}{m} + \frac{cv^2}{m} - g$

$0 = bv + cv^2 - mg$

$V = \frac{-b \pm \sqrt{b^2 + 4cmg}}{2c}$

(e) b: $\cancel{\int \frac{d^2x}{dt^2} = \int \frac{b}{m} \frac{dx}{dt} - g}$

$\int \frac{d}{dt}\left[\frac{dx}{dt}\right] = \frac{b}{m}\frac{dx}{dt} - g$

$\cancel{\frac{d}{dt}\left[\frac{dx}{dt}\right] = \frac{b}{m}\frac{dx}{dt} - g}$

$\int \frac{dx}{dt} = \frac{b}{m}x - gx$

$\cancel{\int dx =}$

x

$\cancel{\frac{d}{dt}\left[\frac{dx}{dt}\right] \frac{d}{dt} =}$

$\cancel{\frac{d}{dt}\int \frac{dx}{dt} x = \int \frac{b}{m} \frac{dx}{dt} - g\, dx}$

$x = \frac{b}{2m}x^2 - \frac{1}{2}gx^2$

$\cancel{\frac{d^2x}{dt^2} = \frac{b}{m}v - g}$

$\cancel{\int \frac{dv}{dt} = \frac{b}{m}v - g}$

$\cancel{\int dv = \int \frac{bv}{m} - g}$

3. Oompa-Loompas are pulling a 2 kg crate of golden eggs along a rough, but level, surface. In one case it is determined that the position of the block as a function of time is given by : $x(t) = .3t^3 - .1t^2 + .2t$.

a. Find the speed of the block at t = 2 sec.
b. Find an expression for acceleration as a function of time.
c. Find an expression for force as a function of time. ($\bar{a} = \frac{\bar{F}}{m}$)
d. Find the initial kinetic energy of the block ($KE = \frac{1}{2}mv^2$)
e. Find the change in kinetic energy of the block from t = 0 to t = 2 sec.
f. Another lab group determines that the Oompa-Loompa force as a function of distance is given by:

$F(x) = x^2 + 2x + 2$ ~~and the block is pulled at an angle of 15° to the horizontal~~.
Find the change in kinetic energy from x = 0 to x = 2 meters.

g. For the above group find a differential equation for power (Power = the time rate of change of kinetic energy).

(a) $\frac{dx}{dt} = 0.9t^2 - 0.2t + 0.2$

$v(2) = 3.6 \cancel{-0.8} 0.4 + 0.2$

$= 3.4\ m/s$

(b) $a = 1.8t - 0.2$

(c) $\frac{F}{m} = 1.8t - 0.2$

$F = (1.8t - 0.2)m$

(d) $K_E = \frac{1}{2} \cdot 2kg \cdot (0.2 m/s)^2$

$= 0.04J$

(e) $K_E(2) = \frac{1}{2} \cdot 2kg \cdot (3.4 m/s)^2$

$= 11.56J$

$\Delta KE = 11.52J$

~~(f) KE at x=0 = 0.04J~~

~~2 = 0.9t² - 0.2t + 0.2~~

~~t = 1 ± √63 / 9~~

(f) $F'(x) = 2x + 2$

$V(2) = 6,\ V(0) = 2$

$\Delta K_E = K_{Ef} - K_{Ei}$

$K_{Ef} = \frac{1}{2} \cdot 2kg \cdot (6)^2$

$= 36J$

$K_{Ei} = 4J$

$\Delta K_E = 32J$

(g) $P = \frac{dE}{dt} = \frac{d}{dt}\frac{1}{2}mv^2$

$P = mva$