q 0: Assume $\Sigma = \{a, b\}$ A= "aa", B= "bb". Then
$A \in L$, $B \in L$. Z= "a". A+Z= "aaa", B+Z = "bba".
Then $(A+Z) \in L$. $(B+Z) \notin L$. Therefore, there is a distinction between A and B on L. Thus, $L = \{w \mid w \in \Sigma^*, rev(w) = w\}$ of all palindromes is not regular.

$G_0 = \{(S, A), (a, b), (S \to aAa \mid bAb, A \to \varepsilon \mid a \mid b \mid aAa \mid bAb)\}$

q 1: "aabaab" is not in $L(G_1)$

$\because S \to ABS \mid AB$ $\therefore$ S always ends with S or B.
$B \to bA$ $A \to aA \mid a$. Therefore S always ends with "a". However "aabaab" ends with "b"
Thus "aabaab" is not in $L(G_1)$

"aaaaba" is in $L(G_1)$.
$S \to ABS \mid AB$, $A \to aA \mid a$. therefore, A can be "aaaa".
$B \to bA$, therefore B can be "ba". Thus $AB \to$ "aaaaba"

"aabbaa" is not in $L(G_1)$.
$S \to ABS \mid AB$, $A \to aA \mid a$. $B \to bA$. Therefore $S \to aAbAS \mid aAbA \mid abA$.
However, b appears twice in a row in "aabbaa", which is impossible for $S \to aAbAS \mid aAbA \mid abA$. Therefore, "aabbaa" is not in $L(G_1)$

"abaaba" is in $L(G_1)$, $S \to ABS \mid AB$. $A \to aA \mid a$, $B \to bA$
Therefore. "abaaba" can be devided to "a|ba|a|ba|"
which will be $ABAB = S \to AB\underset{\substack{\downarrow \\ AB}}{S}$. Therefore, "abaaba" is in $L(G_1)$.

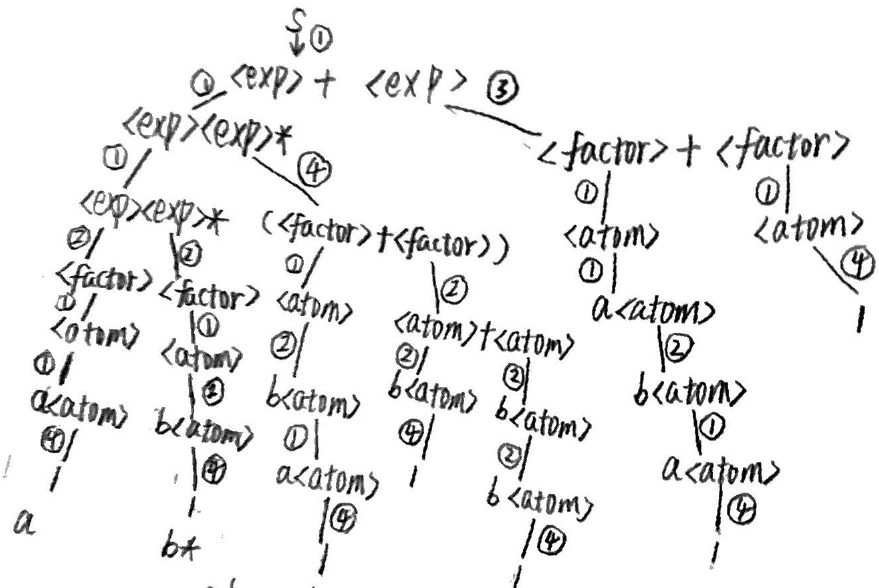
Q2: G_2:

S → <exp>+<exp> ① | <exp>*

exp → <exp><exp>* ① | <factor> ② | <factor>+<factor> ③ | (<factor>+<factor>) ④

factor → <atom> ① | <atom> + <atom> ②

atom → a<atom> ① | b<atom> ② | 0 ③ | 1 ④



ab*(ba + b+bb)* + aba +1

Q3: G_3 =

S: <exp>+<exp>

exp: <exp><exp>* | <factor> | (<factor>+<factor>)

factor: <atom> | <atom>+<atom>

atom: a<atom> | b<atom> | 0

Q4:
(1) By $S \to aSb \mid bSa \mid SS \mid \varepsilon$, we can define recursive functions for the number of as $a(w)$ and number of bs $b(w)$

$a(w)$:
$a(\varepsilon)=0$
$a(aSb) = 1 + a(S)$
$a(bSa) = 1 + a(S)$
$a(SS) = a(S) + a(S)$

$b(w)$ =
$b(\varepsilon) = 0$
$b(aSb) = 1 + b(S)$
$b(bSa) = 1 + b(S)$
$b(SS) = b(S) + b(S)$.

① Therefore, we can say the recursion only ends with $S = \varepsilon$.

$S = \varepsilon$ case: $a(\varepsilon)=0$, $b(\varepsilon)=0$, therefore $a(S)=b(S)$

$S = aSb$ case: $a(aSb) = 1 + a(S)$, $b(aSb) = 1 + b(S)$. Due to recursion only ends with $S=\varepsilon$ and $a(\varepsilon) = b(\varepsilon)$. Therefore, when $S = aSb$, $a(aSb) = n + 0$, $b(aSb) = n + 0$.
n = recursive times. Therefore, $a(aSb) = b(aSb)$

$S = bSa$ case: Due to $a(aSb) = a(bSa)$, $b(aSb) = b(bSa)$, $a(aSb) = b(aSb)$
then $a(bSa) = b(bSa)$.

From the proof above, we can say on $S \to aSb \mid bSa \mid \varepsilon$, $a(S) = b(S)$

$S = SS$ case. $a(SS) = a(S) + a(S)$, $b(SS) = b(S) + b(S)$, $S \to aSb \mid bSa \mid \varepsilon$.
therefore $a(SS) = b(SS)$.

In summary, for any S in $S \to aSb \mid bSa \mid SS \mid \varepsilon$. $a(S) = b(S)$.
Therefore, by the definition of $\mapsto^*$, if $S \mapsto^* w$, then $a(w) = b(w)$

② By strong induction on the length of w. let $w \in \Sigma^*$ be arbitrary, $n = |w|$, and assume that the lemma holds for all strings of length $< n$. we consider three cases:

case 0: $n = 0$. the $w = \varepsilon$. so by $S \mapsto \varepsilon$, and if $\alpha \mapsto \beta$, then $\alpha \mapsto^* \beta$. we get $S \mapsto^* w$

case 1: $n \geq 2$ w can not be divided in two parts u, v which $u(a) = u(b)$, $v(a) = v(b)$.

when $n \geq 2$, w at least contains one "a" and one "b"

Therefore, $w = a\beta b$, $w = b\beta a$ Due to length of $\beta < n$, $a(\beta) = b(\beta)$

since $a(w) = b(w)$, $a(\beta) = b(\beta)$ we get $\beta = a y b$ or $\beta = b y a$

There fore, we can conclude that $S \mapsto^* \beta$. so, we have

$S \mapsto aSb \mapsto bSa \mapsto a\beta b \mid b\beta a$. By if $\alpha \mapsto \beta$ then $\alpha \mapsto^* \beta$, $\alpha \mapsto^* \beta$, $\beta \mapsto^* y$, then $\alpha \mapsto^* y$ we have $S \mapsto^* w$.

case 2: $n \geq 2$, w can be devided in two parts. $w = u \cdot v$. which $a(u) = b(u)$ $a(v) = b(v)$ Thus $u = a\beta b$ or $b\beta a$, $v = a\beta b$, or $b\beta a$.

since $a(u) = b(u)$, $a(v) = b(v)$, $a(\beta) = b(\beta)$ Therefore,

$$S \mapsto aSb \mid bSa \mapsto a\beta b \mid b\beta a \mapsto u.$$
$$S \mapsto aSb \mid bSa \mapsto a\beta b \mid b\beta a \mapsto v$$

By if $\alpha \mapsto \beta$ the $\alpha \mapsto^* \beta$, $\alpha \mapsto^* \beta$, $\beta \to^* y$ then $\alpha \mapsto^* y$, we have $S \mapsto^* u$, $S \mapsto^* v$.

Therefore, we have:

$$S \mapsto SS \mapsto^* uS \mapsto^* uv.$$

In summary. $S \mapsto^* w$.

In conclusion, for all $w \in \Sigma^*$, if $a(w) = b(w)$, then $S \mapsto^* w$

95: $S \to aS \mid aSbS \mid \varepsilon$ generated a set that the number of as of strings in that set always more than or equals to the number of bs.

proof: $a(\varepsilon) = 0$ $\qquad$ $b(\varepsilon) = 0$

$a(aSbS) = 1 + a(S) + a(S)$ $\qquad$ $b(aSbS) = 1 + b(S) + b(S)$

$a(aS) = 1 + a(S)$ $\qquad$ $b(aS) = 0$.

Make length of string to be l, then.

when $l = 0$: $\omega = \varepsilon$. therefore $a(\varepsilon) = 0$, $b(\varepsilon) = 0$

thus $a(\varepsilon) = b(\varepsilon)$

when $l \geq 1$:

$S \to aS$ case: $a(aS) = 1 + a(S)$. $b(aS) = 0$. then,

since $a(S) \geq 0$. $1 + a(S) \geq 1$. since $b(aS) = 0$. We have

$a(aS) > b(aS)$.

$S \to aSbS$ case:

$a(aSbS) = 1 + a(S) + a(S)$ $\quad$ $b(aSbS) = 1 + b(S) + b(S)$.

$S \to \varepsilon$ case: $a(aSbS) = 1 + a(\varepsilon) + a(\varepsilon) = 1 + 0 + 0 = 1$

$b(aSbS) = 1 + b(\varepsilon) + b(\varepsilon) = 1 + 0 + 0 = 1$

therefore: $a(aSbS) = b(aSbS)$ when $S \to \varepsilon$.

$S \to aSbS$ case: $a(aSbS) = 1 + a(aS) + a(aS)$, $a(aS) \geq 1$, then.

$a(aSbS) = 1 + a(aS) + a(aS) \geq 1 + 1 + 1$

$a(aSbS) \geq 3$

$b(aSbS) = 1 + b(aS) + b(aS)$. $b(aS) = 0$.

therefore: $b(aSbS) = 1 + 0 + 0 = 1$. thus.

We get: $a(aSbS) > b(aSbS)$.

In conclusion: $a(S) \geq b(S)$, therefore, grammar $S \to aS \mid aSbS \mid \varepsilon$ generated the list that number of as of the strings in the list always more or equals to the number of bs

7b: For $\{a^i b^j a^i \mid i \geq 0, j \geq 1\}$, we have $S \to X \mid aSa$, $X \to b \mid Xb$

$S \to X \mid aSa$
$X \to b \mid Xb$

$\downarrow$

$S' \to S$
$S \to X \mid aSa$
$X \to b \mid Xb$

$\downarrow$

$S' \to b \mid Xb \mid aSa$
$S \to b \mid Xb \mid aSa$
$X \to b \mid Xb$

$\downarrow$

$S' \to b \mid Xb \mid aY$
$S \to b \mid Xb \mid aY$
$X \to b \mid Xb$
$Y \to Sa$

$\downarrow$

$S' \to b \mid XB \mid AY$
$S \to b \mid XB \mid AY$
$X \to b \mid XB$
$Y \to SA$
$B \to b$
$A \to a$