Marta Gonczar *(code, calculations, drafting written report)*
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2022 / 10 / 25

Introduction

This report outlines an experimental procedure to determine a power supply's output resistance: the internal resistance provided by the power supply without contribution from outside sources on an electrical network.

The quantitative basis of this experiment rests on equation 1, which applies to a power supply connected to a closed circuit:

$$1.\ R_{int} = \frac{V_{\infty} - V}{I}$$

In equation 1, R_{int} is the output resistance, V is the terminal voltage: the voltage measured between the two terminals of the power supply in the closed circuit, I is the current in the circuit, and V_{∞} is the open circuit voltage: the voltage between the power supply's terminals when no external electronic load is applied.

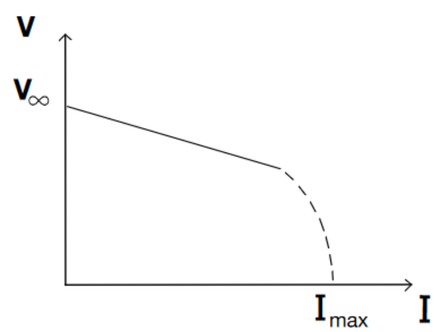

Figure 0: *Expected relationship between Voltage and Current of a closed circuit consisting of a power supply, external load resistor, voltmeter, and ammeter. The voltage will behave linearly for small currents and non-linearly for currents past a certain amperage. The limit as the current approaches zero is the open circuit voltage, and the voltage's minimum occurs at the limit of some maximum current amperage. The internal resistance of the power supply is the slope of the linear portion of V.*

According to Equation 1, if the internal resistance is constant, the relationship between the terminal voltage and current is expected to be linear. Furthermore, it anticipates that as external resistance is increased, the terminal voltage will also increase while the current will decrease. Finally, a linear relationship is anticipated for smaller current values and a non linear relationship is expected for larger current values, as depicted in Figure 0. The internal resistance of the power supply is the slope of the linear portion of V.

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Methodology

Materials:
- Keysight Triple Output Programmable DC Power Supply (*DC power supply)*
- 6.5V Battery (*battery)*
- 2 Keysight U1272A Multimeters
- University of Toronto - Department of Physics LCR Breakout Box (*breakout box)*

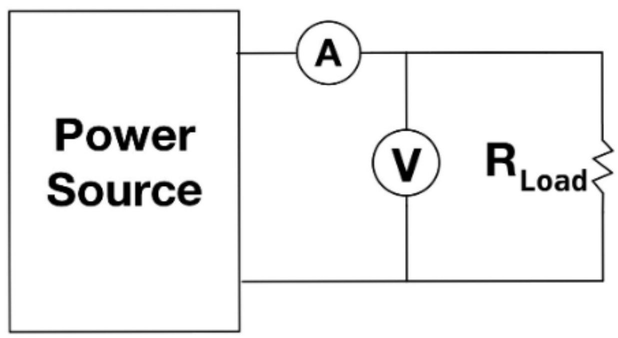

Figure 1. *Circuit schematic of the experimental setup. A series circuit with a voltme R_{Load} is physically implemented using the built-in external load resistors on the breakout box.*

A loop circuit was set up using the 6.5V battery, the ammeter-set multimeter connected in series, one of the external load resistors on the breakout box connected in series, and the voltmeter-set multimeter connected in parallel, as shown in Figure 1.

To begin, the breakout box's external resistor *displaying* 100 Ω was used, and then the terminal voltage was directly measured using the voltmeter-set multimeter, alongside the current using the ammeter-set multimeter. Then, the *actual* load resistance of the breakout box's external resistor was separately measured using one of the multimeters. This series of measurements was repeated three times while incrementally increasing the resistance of the external resistor on the Breakout Box based on the displayed resistance (220 Ω, 470 Ω, 2.7 KΩ).[1] These values were verified using a multimeter and more precise values were measured (101.52 Ω, 214.94 Ω, 464.5 Ω, 2694.8 Ω). Once all values were measured and recorded, the circuit was disassembled and a voltmeter-set multimeter was attached directly to the battery's terminals to measure and record the open circuit voltage.

The measurement process is repeated all over again with the DC power supply while varying the *display* voltages on the DC power supply (beginning with 6.5 V)[2], including measuring the current and terminal voltages with different load resistances, the terminal resistance for each external load resistor, and the open circuit voltage of the DC power supply for each *display* voltage. The different manipulated resistances used were 10 V, 15 V, and 20 V.

[1] The breakout box has six potential values for resistance. The first four were chosen for the load resistance as they could be measured with the same precision using the multimeters.
[2] 6.5 V was chosen first to allow for comparison with the battery

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Results

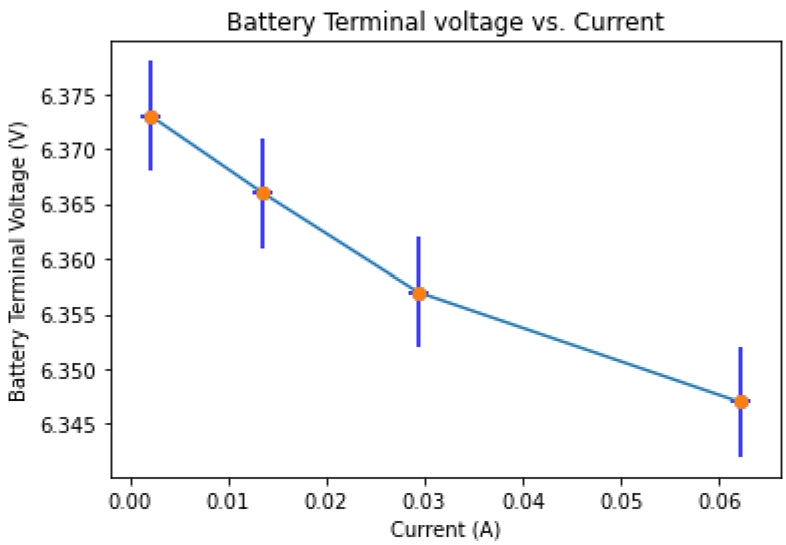

Figure 2. *The terminal voltage of a 6.389 V battery series circuit at varied external load resistance connected in series and respective circuit current. Error bars represent the uncertainty in the measurement apparatus.*

To determine the battery's output resistance, the battery's terminal voltage was plotted against current as shown in Figure 2. Due to the lack of data points, it is difficult to interpret the relationship between the two variables for certain. It appears to exhibit linear behaviour until a current of about 0.03 A. The relationship then begins decreasing at a decreasing rate. Due to a large uncertainty in the terminal voltage and minimal measurements, it is difficult to interpret any meaningful conclusion from the data.

The uncertainties in the readings were calculated using the propagation formulae in the Keysight U1272A multimeter user manual. They are provided as sums of a certain multiple of the reading's precision plus a certain percentage of the reading's magnitude, with the factors varying depending on what units are measured. Equations 2, 3, and 4 below show sample calculations from the first trial of measurements:

$$2.\ u(6.389\,V) = ((6.389\,V) \times 0.05\%) + (2 \times 0.001\,V) = 0.005\,V$$

$$3.\ u(101.52\,\Omega) = ((101.52\,\Omega) \times 0.2\%) + (5 \times 0.01\,V) = 0.3\,\Omega$$

$$4.\ u(0.0621\,A) = ((0.0621\,A) \times 0.3\%) + (1 \times 0.0001\,A) = 0.001\,A$$

Using Equation 1, the output resistance of the battery was calculated to be:

$$5.\ R_{int} = \frac{V_\infty - V}{I} = \frac{(6.389\ V) - (6.347\ V)}{(0.0621\ A)} = 0.676\ \Omega$$

The output resistance's uncertainty was calculated by adding the relative uncertainties of current and voltage difference together and multiplying by the output resistance of the battery:

$$6a.\ u(R_{int}) = R_{int}\sqrt{\left(\frac{u(V_\infty - V)}{V_\infty - V}\right)^2 + \left(\frac{u(I)}{I}\right)^2}$$

$$6b.\ u(R_{int}) = (0.676\ \Omega)\sqrt{\left(\frac{u(6.389\ V - 6.347\ V)}{(6.389\ V) - (6.347\ V)}\right)^2 + \left(\frac{u(0.0621\ A)}{(0.0621\ A)}\right)^2}$$

$$6c.\ u(R_{int}) = (0.676\ \Omega)\sqrt{\left(\frac{(0.005\sqrt{2}\ A)}{(0.042\ V)}\right)^2 + \left(\frac{(0.001\ A)}{(0.0621\ A)}\right)^2}$$

$$6d.\ u(R_{int}) = 0.2\ \Omega$$

Calculating the remaining output resistances at the respective external load resistance yielded Figure 3:

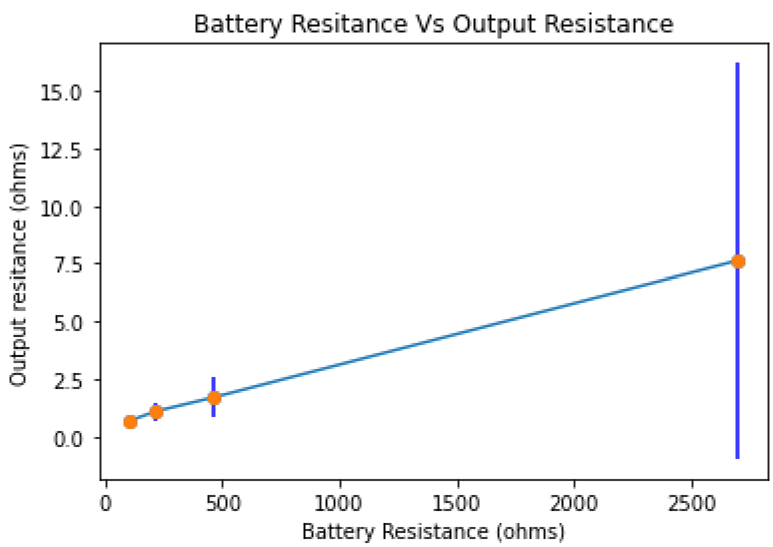

Figure 3. *The output resistance of a 6.389 V battery series circuit at varied external load resistances. Error bars represent the uncertainty of output resistance calculated using voltage and current uncertainty in the measurement. Trend line shows linear relationship.*

To determine the output resistance of the DC Power source, the terminal voltage was plotted against current for varying power source voltage, shown in Figure 4.

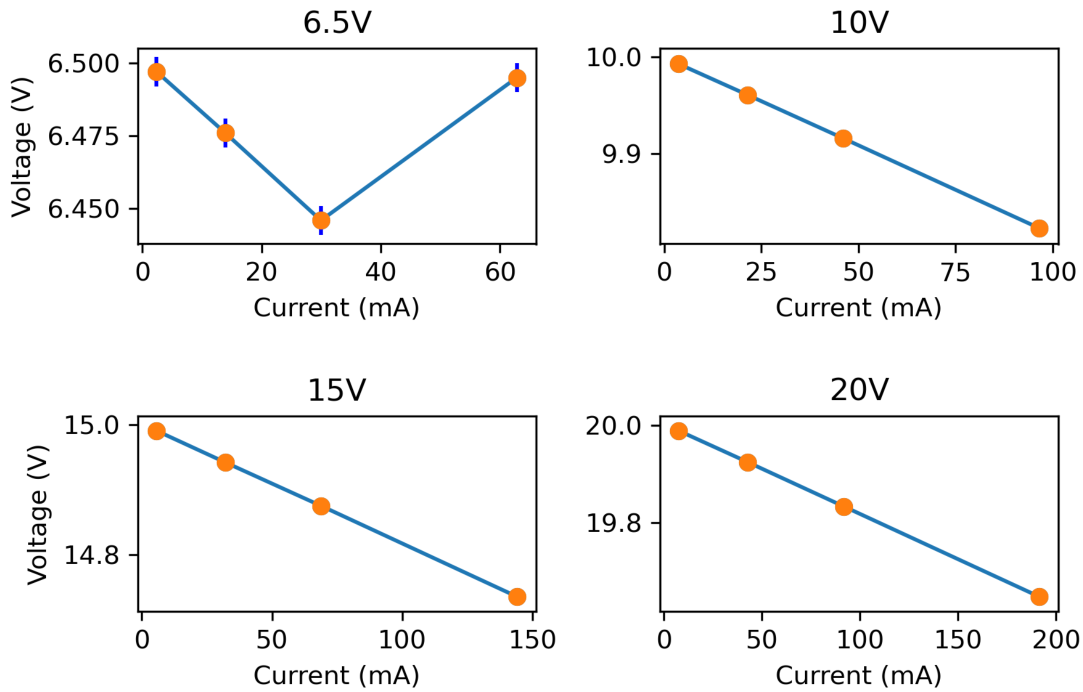

Figure 4. *The terminal voltage of the DC power supply series circuit, for each manipulated* display *voltage, at varied external load resistances connected in series, plotted over the measured current. Error bars represent the uncertainty in the measurement apparatus, current error is present but not visible as it is too small.*

As can be seen in Figure 4, the relationship between voltage and current is linearly decreasing, a result consistent with Equation 1. The expected non-linear behaviour at higher current is not observed in any of the graphs. The 60 mA data point on the 6.5 V subplot appears to be an outlier that may be caused by error factors to be discussed later. The maximum current can be estimated in a DC power supply circuit, by calculating the current as terminal voltage approaches zero. There appears to be a correlation between I_{max} and V_{∞}: $10 \times V_{\infty} = I_{max}$, where V_{∞} is measured in volts and I_{max} in milliamperes. The figure for 6.5 V display frequency is an outlier to this trend.

The output resistance of the power supply was calculated using the same process outlined in Equation 5:

$$7.\ R_{int}^{DC} = 0.010\ \Omega$$

The uncertainties for the output resistance were calculated in the same process outlined in Equation 5:

$$8.\ U(R_{int}^{DC}) = 0.2\ \Omega$$

Calculating the remaining output resistances at the respective external load resistance yielded Figure 8:

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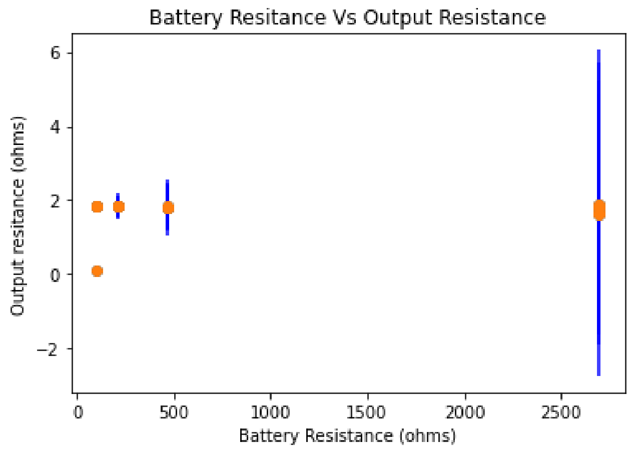

Figure 5. *The output resistance of a varied terminal voltage of a DC power supply series circuit at varied external load resistance connected in series. Error bars represent the uncertainty of output resistance calculated using voltage and current uncertainty in the measurement. Relationship is linear, output resistance appears constant with the exception of an outlier.*

As can be seen in Figure 5, the output Resistance is kept relatively constant in the DC power supply, as opposed to the battery in which the output resistance increases linearly in relation to external load resistance (as can be seen in Figure 3).

Discussion & Conclusion

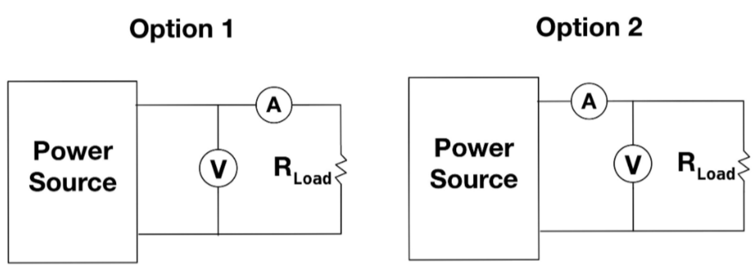

Figure 6. *Circuit schematics representing the two possible setups that could have been used for this experiment. Option 1 was the one ultimately chosen.*

It is worth addressing that the circuit setup for this experiment was not the only possible one, and could, in fact, be argued to be a sub-optimal choice. Figure 6 shows two possible circuit setups that could have been used, given the materials needed for this experiment. In theory, option 1 is the better choice. This is because the ammeter-set voltmeter, being in parallel, will be allowed to measure the voltage of the circuit *before* it is distrubed by the imperfect ammeter-set multimeter. By conducting the experiment again but changing the experimental setup to option 2, current could be compared and the better option could be experimentally confirmed.

To carry on with the imperfect assumptions of perfectness in the system, there were key sources of error present in this experiment. The largest source of error was almost certainly the time available to take measurements. When attempting to record voltage values, they would decrease at a steady pace the longer the multimeter was left alone. It would take multiple minutes for it to stop decreasing. Since this experiment was originally performed in a classroom setting where time was of the essence, eventually the readings were just read off at 30 seconds. That is to say, the degree of accuracy for the values recorded in this experiment is unknown.

Furthermore, the current through the voltmeter-set multimeter and the resistance in the ammeter-set multimeter respectively were neglected, although they certainly would have contributed, at least in small part, to measurement inaccuracy.[3] A potential solution to this problem would have been to attempt to measure the difference in current between the multimeter planned to be set as an ammeter attached to some circuit, and the current through the circuit without the ammeter-set multimeter attached, *using the other multimeter*. This would have given a reasonable reading for the difference in current of the multimeter, although it would not account for the resistance of the other multimeter that is doing the measuring. It is worth noting too that the DC power supplies, by design, disturb the results of this experiment by attempting to reduce as much output resistance as possible. This could explain why the dropping value reading on the multimeter occurred for longer and to a greater extent on the DC power supply as opposed to the battery.

In conclusion, this experiment was conducted to determine the output resistance of a 6.5 V battery and a variable-voltage DC power supply. The determined output resistance for the battery was $0.7 \pm 0.2\ \Omega$. The determined output resistances for the DC power supply was $0.0 \pm 0.2\ \Omega$. Ultimately, these values seem reasonable, especially given that the DC power supply is designed to continuously optimise its output resistance by minimising it whenever it is in operation.

[3] Particularly if the *Option 2* circuit setup was used