Marta Gonczar (*majority of write-up)* & Nicholas Garcia (*python code and portions of write-up)*
2022 / 10 / 4

I. Part 1- Ohms Law

This section will test the linear relationship between voltage and current in electrical circuits modelled by Ohm's law:

$$V = IR$$

II. Methodology

a. Method

A loop circuit was constructed using a power source, an ammeter connected in series, a resistor, and a voltmeter connected in parallel. The resistance was set manually to 100 ohms and was kept constant. The voltage is the independent variable; it was set manually to 0.452V and was increased by 0.010V 19 times until a final value of 0.652V was obtained. At each voltage, the dependent current was recorded. Once all values were measured and recorded, the resistance was measured using one of the multimeters; this value was recorded. Measurements of voltage and current were repeated for a potentiometer. Again, 20 measurements were conducted and resistance was measured at the end of all trials.

b. Materials

- Keysight Triple Output Programmable DC Power Supply
- 2 Keithley 130A Multimeters
- University of Toronto- Department of Physics LCR Breakout Box

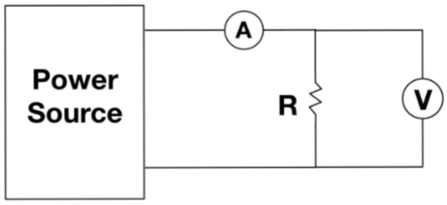

Figure1. Diagram of experimental setup. Resistor and ammeter are connected in series and a voltmeter is connected in parallel at both ends of a resistor.

Figure 2- Circuit experimental setup

III. Results

In the first iteration of the experiment the resistance was measured to be:

$$R = 98.95 \pm 0.5 \text{ ohms}$$

Figure3. resistance reading on the multimeter and multimeter setup.

Note that the precision of the resistance reading on the multimeter is one decimal place. A value of 98.95 ohms was chosen as the multimeter reading was "jumping" between 98.9 ohms and 99.0 ohm.

a. Data 1

The voltage, current, and their respective uncertainties were imported into python and data was graphed in the following plot:

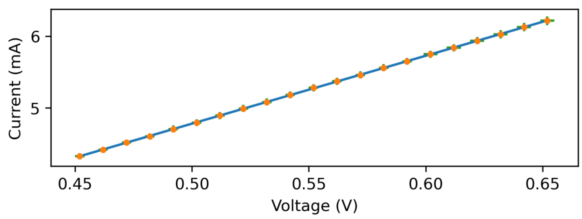

Figure4. The graph of the Current (mA) through the circuit was graphed as a function of Voltage for the 20 values of Voltage. Error bars represent multimeter uncertainties and the line of best fit is linear.

The residuals of the data were also graphed:

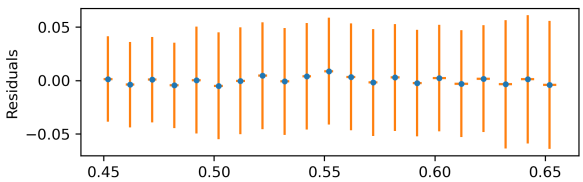

Figure5. The graph of the residual of Current (mA) through the circuit as a function of Voltage(V).

Figure4. shows the current in milliAmps as a function of Voltage (V). The line of best fit is denoted by the equation:

$$y = 0.00952727262\, x + 1.237403 \cdot 10^{-5}$$

This equation is linear, agreeing with the theoretical understanding of Ohm's law. The line of best fit is within error bars and upon further inspection seems to run through all data points. This supports the expectation that the relationship between current and voltage is linear. The graph of the residuals seen in Figure5. further supports this assumption.

According to Ohm's law, V=IR, the equation should be:

I =V/98.95

The resistance in the linear fit equation of the data is just 1/(the slope of the line of best fit):

$$R = 1/0.009527276$$

$$R = 104.96 \text{ ohms}$$

The resistance measured for this circuit was 98.95 ± 0.5 ohms. Resistance in the line of best fit is calculated using the curve_fit function in python which takes into consideration the data and all of its uncertainties. As seen in Figure.5 the residuals are not perfect therefore this is also accounted for when calculating resistance using the raw data. Because the resistance does not perfectly match the measured resistance, it can be expected that the line of best fit will not pass zero. This is evident in the linear equation of the data; when voltage is zero the line passes through I = $1.237403 \cdot 10^{-5}$A, which is I= $1.237403 \cdot 10^{-2}$A. Theoretically, when voltage is equal to zero, current should also be equal to zero thus the line should pass through I=V=0. This minor discrepancy is a result of experimental uncertainty, primarily the error in accuracy of the multimeters used; it should not be interpreted as an objection to Ohm's law.

Forcing the fit to got though I=V=0, a slightly lower resistance is obtained

$$R = 1/0.0095326 \pm 9 \cdot 10^{-7}\ ohms$$

$$R = 104.9 \text{ ohms}$$

b. Data 2

The resistor was switched to a potentiometer. The resistance of the potentiometer measured for this iteration was found to be:

$$R = 39.3 \pm 0.2\ ohms$$

The voltage, current, and respected uncertainties were imported into python. The current was again plotted against the voltage.

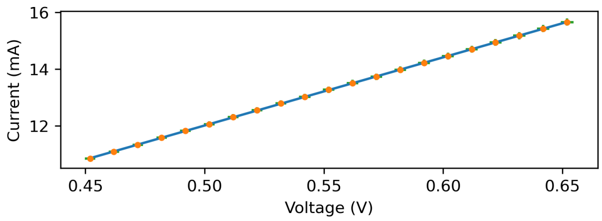

Figure6. The graph of the Current (mA) through the series circuit was graphed as a function of Voltage , this time for a lower resistance. Error bars represent multimeter uncertainties and the line of best fit is linear.

The residuals of current through the circuit as a function of time were plotted:

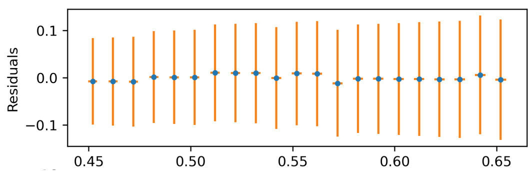

Figure7. The graph of the residual of Current (mA) through the circuit as a function of Voltage(V) at a lower resistance.

Figure6. shows the current, in milliamps, as a function of Voltage (V) . The line of best fit is denoted by the equation:

$$Y = 0.0240337662\,x - 5.6865795 \cdot 10^{-6}$$

Again, when voltage is 0, the line of best fit does not pass through I = V = 0. At V = 0 the Current is $5.6865795 \cdot 10^{-6}\,A$, or $5.6865795 \cdot 10^{-3}\,mA$. This is within the current uncertainty for 0.00 mA, +/- 0.01mA. Therefore, it can be interpreted that I = V = 0 at this point.

The resistance from the line of best fit is:

$$R = 1/0.024033 \pm 5 \cdot 10^{-6}\ ohms$$

$$R = 41.61\ \text{ohms}$$

The resistance from using curve fit is slightly larger than the resistance value measured with the multimeter ($R = 39.3 \pm 0.2\ ohms$). This again can be accredited to the uncertainties of all the readings; the resistance was calculated rather than measured thus it is reliant on the accuracy and precision of the data points.

Forcing the fit to got though I = V = 0, a slightly higher resistance is obtained:

$$R = 1/0.024023 \pm 2 \cdot 10^{-6}\ ohms$$

$$R = 41.63\ \text{ohms}$$

The value of χ^2_{red} for the linear reduction was computed to be 2.91×10^{-6}. This indicates that more data points should have been used, as the value is less than 1, indicating an overfit.

IV. Analysis and Discussion

The results support the linear relationship between voltage and current in electrical circuits modelled by Ohm's law.

a. Uncertainties

The uncertainties from this simulation came from the uncertainty of the Keithley 130A multimeters. This error of accuracy was used when graphing data in python. The accuracy reported in the keithley handbook was reported as(± percent uncertainty +count).The uncertainty for voltage, current, and resistance are ±0.25%+1, ± 2%+1, and ± 0.5%+4 respectively. These uncertainties were used to construct the error bars for the data, for all graphs, the line of best fit goes through either the data points directly or their error bars.

b. Accuracy

The relationship established between current and voltage is linear as expected from Ohm's Law. As demonstrated by the residual graphs, the results are precise but how do results compare with theoretical expected results?

For both data sets it suffices to examine one trial; it can be assumed that if one result is theoretically accurate the rest should be due to the high precision.

The theoretical expectation for current for the first data set is:

$$I = V/R$$

I = 0.452 ± 0.002 V / 98.95 ± 0.5 ohms

I = 4.57 ± $0.04 mA$

The uncertainty was calculated using the following steps:

$$U(I) = \frac{0.452}{98.95} \times (\frac{0.002}{0.452} + \frac{0.5}{98.95}) \times 100$$

$$U(I) = 0.04\, mA$$

The experimentally measured current was:

I = 4.32 ± 0.04 mA

The experimental current does not fit within the theoretical range. Thus it cannot be said that results are theoretically accurate. However, there is a correlation between the experimental results and the theoretically-predicted values. This implies that the uncertainty for measurements is larger than just the accuracy uncertainties. The uncertainty seems to be systematic as opposed to random as data follows a systematic pattern. The most probable explanation is a variability between resistance between trials. The resistance was measured once at the end of each experiment therefore there is no way to guarantee that it was kept constant. To strengthen the experimental design, it is recommended that resistance is measured for all trials.

For the second data set the same steps were repeated. The theoretically expected current is:

I = 0.452 ± 0.002 V / 39.3 ± 0.2 ohms

I = 11.5 ± $0.1\, mA$

The experimentally measured current was:

I = 10.85 ± $0.09\, mA$

Again the experimental current does not fit within the theoretical range. The results are less accurate for the second data set, implying that the potentiometer has higher experimental uncertainty than the voltmeter.

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I. Part 2- Power Law

In this section, the power law will be observed for blackbody radiation by plotting a voltage-current graph for a lightbulb.

II. Methodology

a. Method

A loop circuit was constructed using a power source, an ammeter connected in series to a lightbulb, and a voltmeter connected in parallel. The voltage is the independent variable; it was set manually to 0.00V. Once the voltage and current had stabilised, it was increased by 0.10V 14 times until a final value of 1.40V was obtained. At each voltage, the dependent current was recorded. Once all values were measured and recorded, data was interpreted using python and the resulting graphs were analysed.

b. Materials

- Keysight Triple Output Programmable DC Power Supply
- 2 Keithley 130A Multimeters
- Incandescent lightbulb

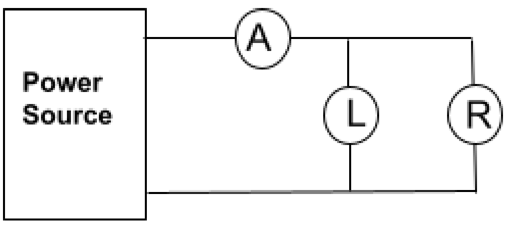

Figure 1-Diagram of circuit experimental setup; note that an unconventional symbol is used to denote the light bulb, "L".

Figure 2- Circuit experimental setup showing the ammeter connected in series and the lightbulb used.

III. Results

Originally, voltage was varied in too small of increments; the relationship between voltage and current seemed linear. This did not agree with the theoretic understanding that guided this experiment. To remedy this, the experiment was conducted again using larger increments of voltage. Instead of increasing voltage by 0.010 V, voltage was increased by 0.10 V. The data was plotted and a logarithmic relationship was observed:

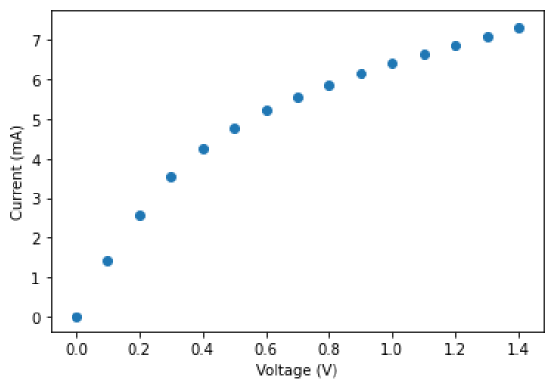

Figure 3- The effect of voltage on current through a series circuit with a fluorescent light bulb attached in series. Error bars are smaller than data points

thus are not visible in the graph. They represent the error in accuracy of the multimeters.

To better interpret data, a linear regression was performed on the natural logarithm of the voltage and the natural logarithm of the current. The following model was graphed:

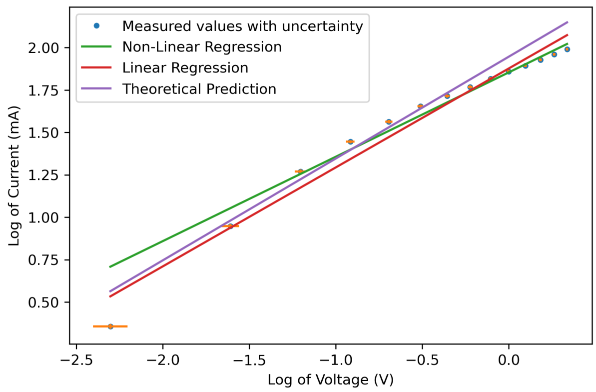

Figure 4- Log of the Current in the system as a function of the Log of the Voltage, comparing different curves with Data

A nonlinear regression (shown in Figure 5) was performed on the raw data collected during the experiment:

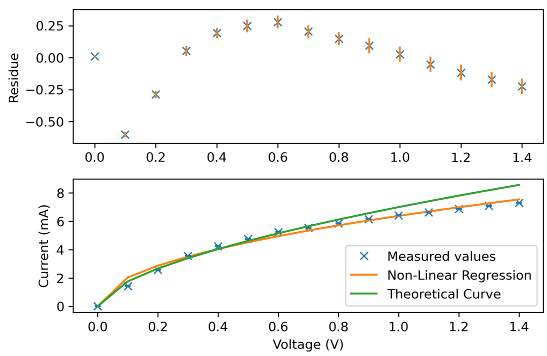

Figure 5- Comparisons of the Measured Values of Current from Controlled Voltage with a Non-Linear Regression model with residue on top. Error bars represent the uncertainty from the multimeters.

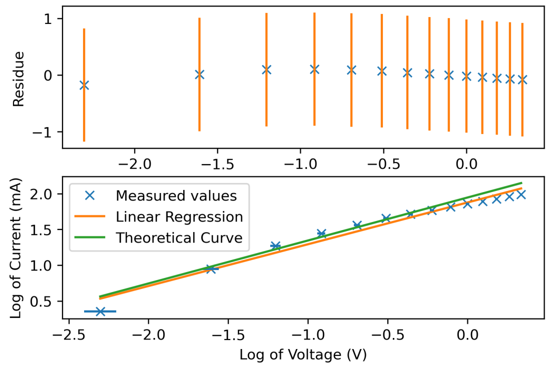

Figure 6- Comparisons of the Measured Values of Current from Controlled Voltage with a Linear Regression model with residue on top. Error bars represent the uncertainty from the multimeters.

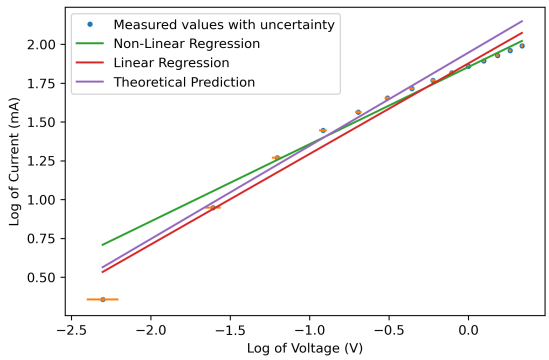

Figure 7- Comparisons of the Measured Values of Current from Controlled Voltage with a Linear and Non-Linear Regression model on a Log-Log scale. Error bars represent the uncertainty from the multimeters.

The linear regression method (shown in Figure 6) gave an exponent closer to the expected value, as is somewhat noticeable in the plots (particularly in Figure 7). The linear regression gave an exponent closer to the expected theoretical value of 0.6 at ~0.583 compared to the nonlinear regression ~0.497. Continuing with the linear regression, it was found that the standard deviation of the computed exponential value was ~0.0299. This level of uncertainty would allow the theoretical value of 0.6 to fall within the range of uncertainty of the computed value. The expected value for tungsten, 0.663, is still out of reach of the computed value with uncertainty, as the computed value could be maximum 0.612.

A chi-squared value of 0.0926 was computed for the linear regression model. This value is less than 1, indicating that not enough samples were taken, thus the model is overfit. To strengthen results, more samples should be taken.

IV. Analysis

Results support the expected power law between current and voltage for an ideal blackbody with a linear relation between resistance and temperature.

$$I \propto V^{3/5}$$

a. Uncertainties

The uncertainties from this simulation again came from the uncertainty of the Keithley 130A multimeters.This error of accuracy was used when graphing data in python. The same accuracy used in part one, Ohm's law, is applicable for this section; the uncertainty for voltage, current, and resistance are ±0.25%+1, ±2%+1, and ±0.5%+4 respectively. These uncertainties were used to construct the error bars for the data, for all graphs, the line of best fit goes through either the data points directly or their error bars.

b. Accuracy

The relationship established between current and voltage in the series circuit with a fluorescent light bulb is a power relation. It has already been established that the accuracy of the multimeters used is high. In figure 4 and 5, the theoretical curve is graphed and can be compared to the experimental results and regression models used to model data. The linear regression model gave an exponent that had the theoretical value 0.6 within its uncertainty range. However from the chi-squared assessment, the accuracy of the fit is not confirmed, thus it can be concluded with certainty that the results are theoretically accurate. In order to strengthen this conclusion, more samples should be taken.