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Marta Gonczar (Introduction, analysis & discussion) and Nicholas Garcia (code, methodology, results, conclusion)

Spring-Mass Oscillation Experiment with Manipulated Dampening

Abstract

This report analyzes a mass-spring system to solve the equation of motion and calculate the energy of a spring mass system. Simulation data will be used to plot and visualise the position of the bob vs. time and the phase plot.

Introduction

The physics of a spring can be explained by hooke's law; the force of the spring is based on the spring constant k and the vertical displacement of the spring y.

$$(1)\ F_{Spring} =- ky$$

Using Newton's second law of motion, the equation of motion can be derived:

$$(2)\ F = ma$$

This equation can be approximated by equating the Force of the spring equation to Newton's second law. To make this approximation, the spring-mass system will be assumed to be completely linear:

$$- ky = ma$$

$$\frac{-ky}{m} = a$$

$$a = \frac{v}{t} = \frac{d^2y}{dt^2}$$

$$(3)\ 0 = \frac{d^2y}{dt^2} + \frac{ky}{m}$$

To find numerical solutions to this equation, ordinary differential equations can be used:

$$(4a)\ \frac{dp}{dt} = F$$

$$(4b)\ \frac{dq}{dt} = \frac{q}{m}$$

Where the coordinate is q, momentum is p, and force is F. These equation can be approximated using the forward Euler method as follows:

$$(5a)\ p(t + \Delta t) = p(t) + \Delta t F(q(t))$$

$$(5b)\ q(t + \Delta t) = q(t) + \frac{\Delta t}{m}(p(t))$$

Which can be written in recursive for as follows:

$$(6a)\ p_{i+1} = p_i + \Delta t F(q_i)$$

$$(6b)\ q_{i+1} = q_i + \frac{\Delta t}{m}(p_i)$$

where

$$p_i = p(t_i)$$

$$q_i = q(t_i)$$

$$t_i = t_0 + i\Delta t.$$

The numerical approximation for the spring mass system using position y and velocity v can be derived using equations 2 and 5:

$$(7a)\ \frac{dy}{dt} = v(t) \approx \frac{1}{\Delta t}(y(t + \Delta t) - y(t)$$

$$y(t + \Delta t) = \Delta t \times v(t) + y(t)$$

This can be written in recursive form as follows:

$$(7b)\ y_{i+1} = y_i + \Delta t v_i$$

The same can be done for velocity:

$$(8a)\ \frac{dv}{dt} = a(t) = \Omega_0^2 y(t) \approx \frac{1}{\Delta t}(v(t + \Delta t) - v(t))$$

$$v(t + \Delta t) = v(t) - \Delta t \Omega_0^2 y(t)$$

This can be written in recursive form as follows:

$$(8b) v_{i+1} = v_i - \Delta t \Omega_0^2 (y_i)$$

Note that at t=0, the initial position y is the starting amplitude of the oscillating system and the velocity is zero . Equation 7b approximates the vertical position of the spring mass system given a start time, t, at a change in time,Δt. Equation 8b approximates the velocity of the spring mass system given a start time, t, angular frequency, Ω_0, position at the start time, y(t), velocity at start time v(t), at a change in time, Δt.

Using experimental data of a mass spring system, these approximations can be calculated and manipulated to solve the equation of motion. These approximations can also be used to calculate the energy of the spring mass system. For a spring mass system, mechanical energy is conserved; the total energy of the system and be equated to the kinetic energy and potential energy:

$$E_{total} = K(y) + U(y)$$

$$(9)\ E_{total} = \frac{1}{2}mv^2 + \frac{1}{2}ky^2$$

The potential energy is elastic, note that the spring mass system has gravitational potential energy but it can be omitted from this equation if the reference for potential energy is at equilibrium. The equation for gravitational energy is the following:

$$(10) Ug = mg\Delta h$$

Where m and g are constant and h is the distance from the reference point. In a spring mass system, the system oscillates above and below its equilibrium; the maximum and minimum values of h are the amplitudes of an oscillation. Δh for an oscillation is A-A=0 thus at an equilibrium reference point, the gravitational potential energy of the system is 0J.

The physics in a spring mass system changes when a damping force such as drag is added; drag is caused by the velocity of the body relative to the surrounding medium. This can be achieved experimentally by adding a damping disk the the weight. The equation of motion changes weh the damping force is accounted for. The equation for damping force is the following:

$$(11a)\ F_d =- \frac{1}{2}C\rho A|v|v$$

$$(11b)\ Re = \frac{\rho vl}{\eta}$$

C, the drag coefficient, depends on the velocity through Reynold's number. Reynold's number is used to characterise the dependence of the drag coefficient on velocity.

For the sake of computing the spring constant and angular frequency, it is also worth remembering the following identities:

$$(12a)\ \omega_0^2 = \frac{k}{m}$$

$$(12b)\ \omega_0 = 2\pi f$$

Methodology

Materials:

1. 200g mass with hook ("bob")
2. Metal damper disk
3. Clamp on mast
4. Spring
5. Ultrasonic motion sensor
6. Digital scale
7. MotionSensor.vi Labview Application
8. Ruler

Figure 1. *Experimental setup* with *damper disk attached below the mass and the ultrasonic motion sensor aligned parallel with the spring under the mass*

The reported 200 g mass was weighed with the hook using the digital scale. The mass was then hooked to the spring, as shown in Figure 1, but without the damper disk. The ultrasonic motion sensor was placed underneath the mass, aligned parallel to the spring. The data collected by the sensor was analysed using the LabView application *MotionSensor.vi*.

The data was collected at 100 samples per second (set in the *Setup/Calibrate* menu of the LabView application). With care taken to reduce as much horizontal motion as possible, the bob was raised a couple of centimetres and released to induce the oscillatory motion. The oscillation data was collected for about 10 seconds. After the data was collected, 7 arbitrarily-chosen sequential oscillations were analysed using the application's *Analyse* panel, and the time between the 7 oscillations was used to calculate the period. The absolute value of the difference between the maximum and minimum position values was used to calculate the amplitude of the oscillation. The frequency was found by reciprocating the period. The spring constant was found using Equation 12. The position and velocity data was then saved as a text file.

Next, to observe the effects of damping , the damping disk was attached to the bob to increase air resistance, as shown in Figure 1. The mass was reweighed, with the hook and the damping disk, and the diameter of the damping disk measured. With the same methodology outlined above, analysis of the spring oscillation was repeated, except this time the data was collected for 2 minutes. The position and velocity data was then saved as a text file.

To calculate Reynold's number for the damped oscillation, the air density was determined by searching the internet for the local air pressure (*e.g.: air pressure of Toronto*) and calculating the pressure using the ideal gas law assumed at room temperature, as well as looking up the dynamic viscosity of air at room temperature. The characteristic length in the direction of flow was taken to be the diameter of the damping disk, and the root-mean-square of the record velocity data was used as the velocity.

To calculate the γ constant for the damped oscillation, a non-linear fit was applied to the amplitude of the oscillation as a function of time (which itself was found using a python function to find the amplitude at time t).

Plots for both trials were generated using Python with the data saved in each.

Results

Undamped Oscillating Spring Mass System:

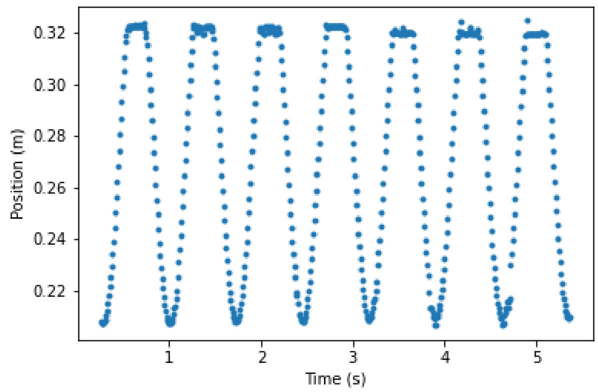

Figure 2. *Measured vertical position of the undamped oscillating spring mass through time.*

For the undamped trial, the mass weighed 200.7 ± 0.05 g, with the uncertainty coming from instrument error. The damper disk was measured to be 10.2 ± 0.05 cm in diameter, with the uncertainty coming from reading error. The period was calculated by dividing the observed time interval for seven oscillations by the number of oscillations; the uncertainty was also divided by seven:

$$(13)\ T_{undamped} = \frac{(5.27 \pm 0.01)\ s}{7} = 0.753 \pm 0.001\ s$$

The uncertainty of 0.01 s in the time interval measurement was used due to the precision of the time position graph: there was a 0.01 s time difference between points. This meant that it could not be determined whether an extrema was being observed, or if the true extrema was located in between data points. The frequency was determined by reciprocating $T_{undamped}$:

$$(14)\ f_{undamped} = \left(T_{undamped}\right)^{-1} = 1.33 \pm 0.002\ Hz,$$

The uncertainty was calculated using the exponent propagation rule:

$$(14b)\ u\left(f_{undamped}\right) = \left(f_{undamped}\right)\frac{\left(u\left(T_{undamped}\right)\right)}{\left(T_{undamped}\right)}(-\ 1) = (1.33\ Hz)\frac{((0.001\ s))}{(0.753\ s)}(-\ 1) = 0.002\ Hz$$

The angular frequency was calculated using Equation 12b, it was found to be:

$$(15)\ \omega_{undamped} = 2\pi f_{undamped} = 2\pi(1.33 \pm 0.002\ Hz) = 8.35 \pm 0.01\ s^{-1},$$

where the uncertainty was propagated as follows:

$$(15b)\ u\left(\omega_{undamped}\right) = 2\pi \times u\left(f_{undamped}\right) = 2\pi \times (0.002\ Hz) = 0.01\ s^{-1}.$$

The spring constant was calculated using Equation 12a. It was found to be:

$$(16)\ k_{undamped} = \omega_{undamped}^2 m_{undamped} = \left(8.35 \pm 0.006\ s^{-1}\right)^2 (200.7 \pm 0.05\ g)$$

$$k_{undamped} = \left(8.35 \pm 0.006\ s^{-1}\right)^2 (0.2007 \pm 0.00005\ kg) = 14.0 \pm 0.002\ N/m$$

Again, the uncertainty was propagated using the following equation:

$$(16b)\ u\left(k_{undamped}\right) = \sqrt{\left(\frac{u\left(\omega_{undamped}^2\right)}{\left(\omega_{undamped}^2\right)}\right)^2 + \left(\frac{u\left(m_{undamped}\right)}{\left(m_{undamped}\right)}\right)^2} = \sqrt{\left(\frac{\left(\left(\omega_{undamped}^2\right)\frac{\left(u\left(\omega_{undamped}\right)\right)}{\left(\omega_{undamped}\right)}(2)\right)}{\left(\omega_{undamped}^2\right)}\right)^2 + \left(\frac{u\left(m_{undamped}\right)}{\left(m_{undamped}\right)}\right)^2}$$

$$u\left(k_{undamped}\right) = \sqrt{\left(\frac{\left(0.01\ s^{-1}\right)}{\left(8.35\ s^{-1}\right)}(2)\right)^2 + \left(\frac{(0.00005\ kg)}{(0.2007\ kg)}\right)^2} = 0.002\ N/m.$$

The amplitude was determined by taking the absolute difference from the maximum and minimum position values in the recorded data set for the undamped oscillation and dividing by two:

$$(17)\ A_{undamped} = \frac{|(0.0032482 \pm 0.00000005\ m) - (0.0020653 \pm 0.00000005\ m)|}{2}$$

$$A_{undamped} = 0.0005914 \pm 0.00000004\ m,$$

The original uncertainties were determined from instrument error, and propagated as such:

$$(17b)\ u\left(A_{undamped}\right) = \frac{\sqrt{2}}{2}(0.00000005\ m) = 0.00000004\ m.$$

Using the Forward Euler method, the oscillatory motion of the undamped spring was predicted based on $k_{undamped}$, $m_{undamped}$, and the initial recorded values of $A_{undamped}$, as shown in Figure 3:

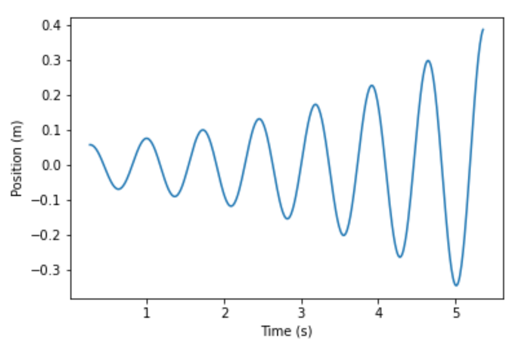

Figure 3. *Vertical position of the undamped Oscillating Spring Mass as a function of time, calculated using the Forward Euler method from initial experimental conditions.*

The predicted plot using the Forward Euler method shows an exponential increase in amplitude. This is characteristic of the Forward Euler method, it tends to predict that the total energy increases over time. Using the Symplectic Euler method instead predicted the following oscillatory motion in Figure 4:

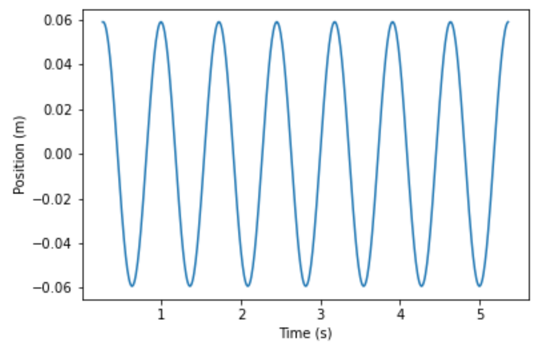

Figure 4. *Vertical position of the undamped Oscillating Spring Mass as a function of time, calculated using the Symplectic Euler method from initial experimental conditions*

The plot predicted by the Symplectic Euler method shows that, as time progresses, the oscillation's amplitude remains unaffected. This prediction is theoretically correct if the oscillatory system is assumed to be closed with conserved energy. Figure 4 more resembles the experimental data seen in Figure 2 than Figure 3.

Furthermore, the Forward Euler method was used to predict the oscillatory velocity of the spring-mass system based on the same initial conditions as with the prior position plots, as shown in Figure 5:

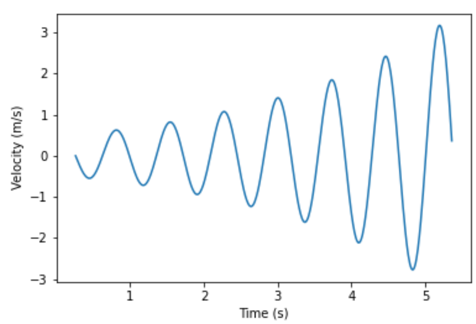

Figure 5. *Vertical velocity of the undamped Oscillating Spring Mass as a function of time, calculated using the Forward Euler method from initial experimental conditions*

Again, the amplitude of the velocity increases exponentially as time passes. Using the Symplectic Euler method, this relationship is no longer exhibited, as can be seen in Figure 6:

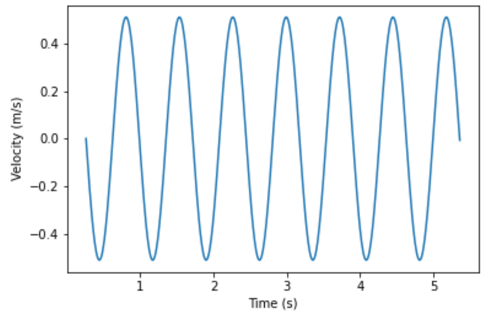

Figure 6. *Vertical velocity of the undamped Oscillating Spring Mass as a function of time, calculated using the Symplectic Euler method from initial experimental conditions*

Similarly to Figure 4, the plot predicted by the symplectic method shows that, as time progresses, the amplitude of the velocity is consistent throughout the oscillations. This prediction is theoretically correct if the oscillatory system is assumed to be closed with conserved energy.

The behaviour observed in Figures 2 to 6 imply that the behaviour of the oscillation is directly related to the energy of the system. Using the Forward Euler method to to predict the energy time relation of the spring-mass, the Figure 7 is produced:

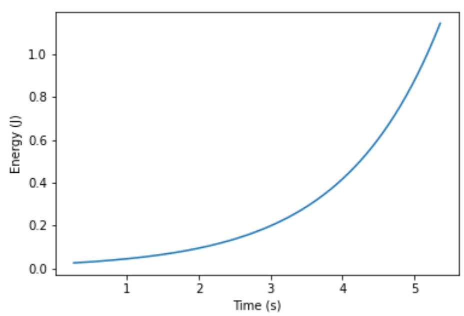

Figure 7. *Energy of the undamped Oscillating Spring Mass as a function of time, calculated using the Forward Euler method from initial experimental conditions*

The Symplectic Euler method would thus, by contrast, likely produce a constant energy graph. To further interpret the results of the two Euler methods, phase plots (Figures 8 and 9 for the Forward and Symplectic Euler Methods, respectively) were created for each of the methods' predicted values:

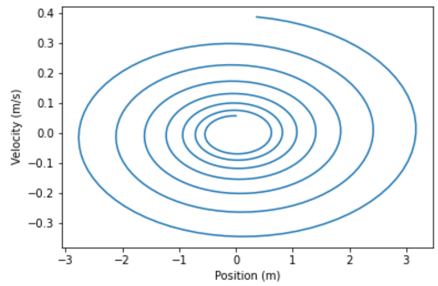

Figure 8. *Vertical velocity of the undamped Oscillating Spring Mass as a function of it's vertical position, calculated using the Forward Euler method from initial experimental conditions*

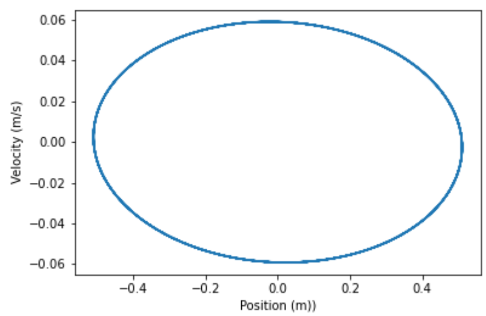

Figure 9. *Vertical velocity of the undamped Oscillating Spring Mass as a function of it's vertical position, calculated using the Symplectic Euler method from initial experimental conditions*

Damped Oscillating Spring Mass System:

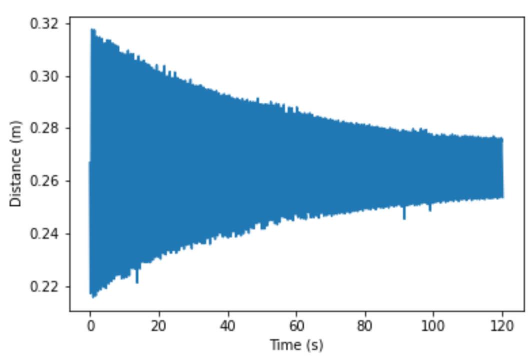

Figure 10. *Measured vertical distance from ultrasonic detector through time for the damped oscillating bob. Exponential amplitude decay is observed with the increased data collection time.*

For the damped oscillator, the combined mass of the bob and damper disk was $217.6 \pm 0.05\, g$ with instrument uncertainty. With identical uncertainty and propagation as in the undamped trial, the period was calculated to be:

$$(18)\ T_{damped} = \frac{(5.08\pm0.01)\ s}{7} = 0.726 \pm 0.001\ s$$

Similarly as with the undamped mass, the frequency, angular frequency, and spring constant were calculated. The same methods of uncertainty propagation were used:

$$(19)\ f_{damped} = 1.38 \pm 0.002\ Hz$$

$$(20)\ \omega_{damped} = 8.66 \pm 0.01\ s^{-1}$$

$$(21)\ k_{damped} = 16.311 \pm 0.003\ N/m$$

To define the amplitude of the damped oscillation, a function was defined in Python to determine the amplitude of the function at each recorded point in time. Its results are shown in Figures 11 and 12. The function works by finding the absolute value of the difference in the extrema of the data from time t to $t + \Delta t$, for some Δt:

$$(22)\ A_{damped} = \frac{|(maximum\pm0.00239\ m)-(minimum\pm0.00239\ m)|}{2}$$

The calculation was repeated for every point $t_i \in [t, t + \Delta t]$. The 0.00239 m uncertainty in each extremum comes from the uncertainty in measurement reported by the ultrasonic position sensor, the Amplitude uncertainty was calculated as follows:

$$(22b)\ u\left(A_{damped}\right) = \frac{0.00239+0.0239}{2} = 0.00239\ m$$

To determine the γ decay constant of the oscillation amplitude, a non-linear regression was fit using a model function of the form:

$$(24)\ f(t) = A_0 e^{-\gamma t}$$

Where A_0 is the initial amplitude of the oscillation. The uncertainty of γ was given as the covariance of the fit for that parameter.
To determine Reynold's number for the damped oscillating system, the velocity was taken to be the root-mean-square of the recorded velocity data. The characteristic length in the direction of flow was taken to be the approximate length of the damper disk. The other constants were sourced from the internet, assuming the air was dry and at 20℃.[1] Equation 11b was used, with the uncertainty propagation combining exponent and product propagation rules:

[1] Viscosity of Air, Dynamic and Kinematic
Engineers Edge:
https://www.engineersedge.com/physics/viscosity_of_air_dynamic_and_kinematic_14483.htm

$$(25)\ u(v^2) = u(v) \times 2 \wedge u(\bar{v}^2) = \frac{\sigma_{v^2}}{12019}$$

$$u(v) = u(\sqrt{\bar{v}^2}) = \sqrt{\bar{v}^2}\frac{u\left(\bar{v}^2\right)}{\bar{v}^2}\left(\frac{1}{2}\right) = \frac{\sigma_{v^2}}{24038\sqrt{\bar{v}^2}} = \frac{\left(0.03266\, m^2/s^2\right)}{24038(0.18073\, m/s)} = 0.000007\, m/s$$

$$(26)\ R_E = \frac{\rho v l}{\eta} = \frac{\left(1.204\, kg/m^3\right)(0.18073 \pm 0.000007\, m/s)(0.102 \pm 0.0005\, m)}{\left(1.825 \times 10^{-5}\, kg/m/s\right)}$$

$$R_E = \left(6.597 \times 10^4\, s/m^2\right)(0.18073 \pm 0.000007\, m/s)(0.102 \pm 0.0005\, m)$$

$$R_E = \left(6.597 \times 10^4\, s/m^2\right)\left(0.0184 \pm 0.005\, m^2/s\right)$$

$$R_E = (1.22 \times 10 \pm 3) \times 10^2$$

To observe the exponential decrease of oscillation amplitude over time, the amplitude was plotted as a function of time in Figure 11 and 12:

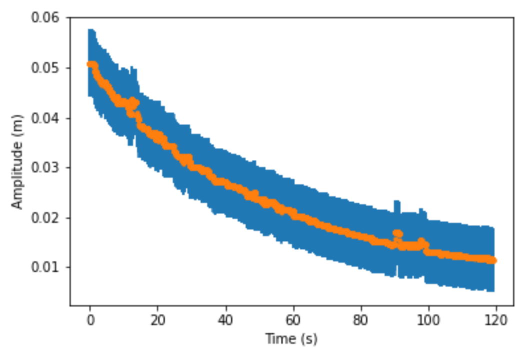

Figure 11. *Vertical amplitude of damped oscillating bob on spring, with uncertainty, determined using function of time (t) which finds the absolute difference in extrema from time* t *to* $t + \Delta t$ *for some* Δt*. The orange markers represent the amplitude of oscillations at different times and the blue error bar represents the uncertainty in the amplitude which was the difference in time between data points. Amplitude seems to decrease exponentially in relation to time.*

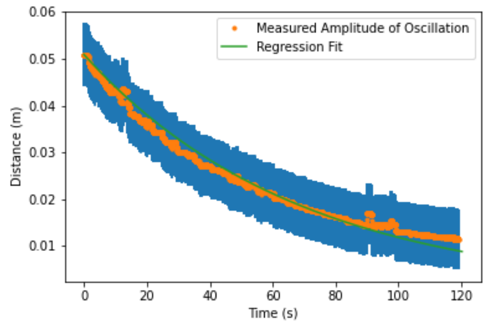

Figure 12. *Vertical amplitude of damped oscillating bob on spring, with uncertainty, determined using function of time* (t) *which finds the absolute difference in extrema from time* t *to* $t + \Delta t$ *for some* Δt*, with uncertainty error bars and a non-linear regression fit overlayed.* $\left(\chi^2 \approx 62.87862\right)$

For the gamma coefficient, the model function used was

$$(27)\ f(t) = (0.05092)e^{-\gamma t}$$

Where 0.05092 is the magnitude of the initial amplitude of the oscillation, found using the previously-discussed amplitude function. Figure 12 shows the fit of f to the amplitudes of the oscillation. The result of the python code was:

$$(28)\ \gamma = \left(1.463 \times 10^{8} \pm 2\right) \times 10^{-10}$$

where the uncertainty of γ was the given covariance of the fit.
In the same fashion as the undamped oscillation, the Forward and Symplectic Euler methods were used to predict the behaviour of the damped oscillator, shown in Figures 13 through 19:

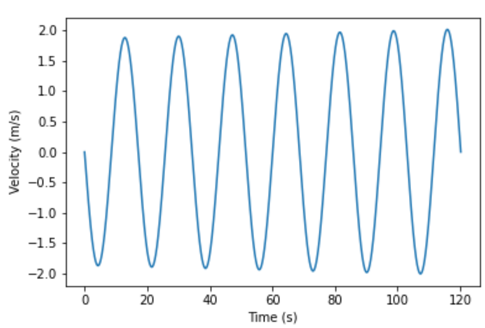

Figure 13. *Vertical velocity of the damped Oscillating Spring Mass as a function of time, calculated using the Forward Euler method from initial experimental conditions*

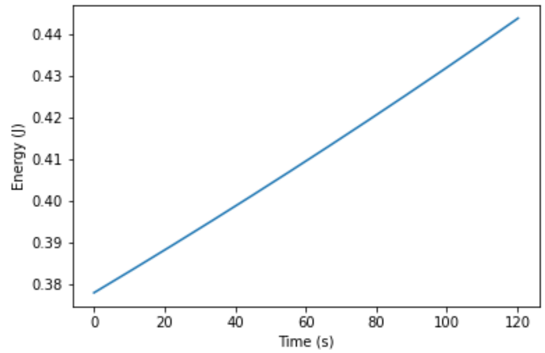

Figure 14. *Energy of the damped Oscillating Spring Mass system as a function of time, calculated using the Forward Euler method from initial experimental conditions*

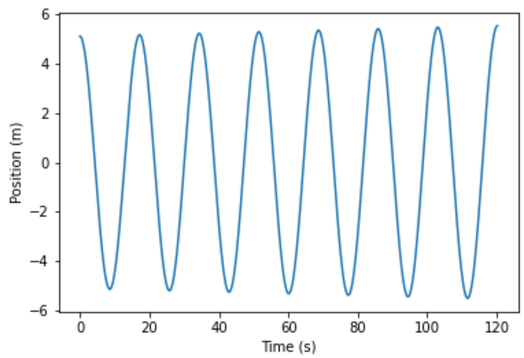

Figure 15. *Vertical position of the damped Oscillating Spring Mass as a function of time, calculated using the Forward Euler method from initial experimental conditions*

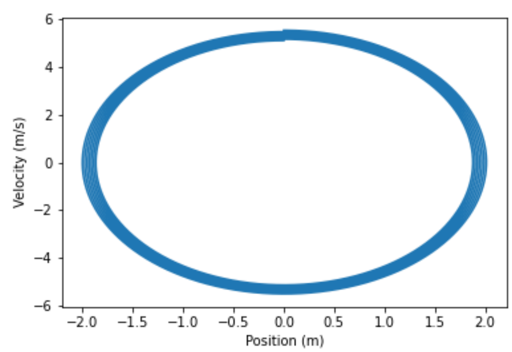

Figure 16. *Velocity-Position Phase plot of the damped Oscillating Spring Mass, calculated using the Forward Euler method from initial experimental conditions.*

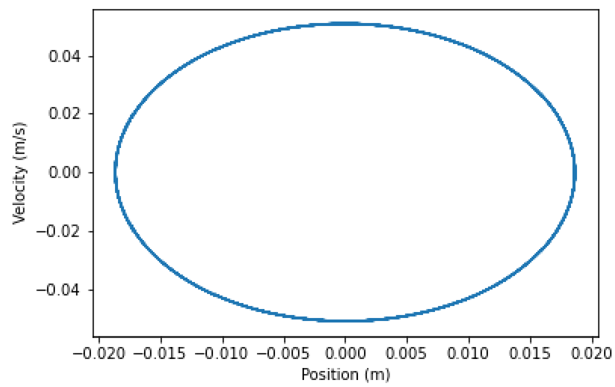

Figure 17. *Velocity-Position Phase plot of the damped Oscillating Spring Mass, calculated using the Symplectic Euler method from initial experimental conditions.*

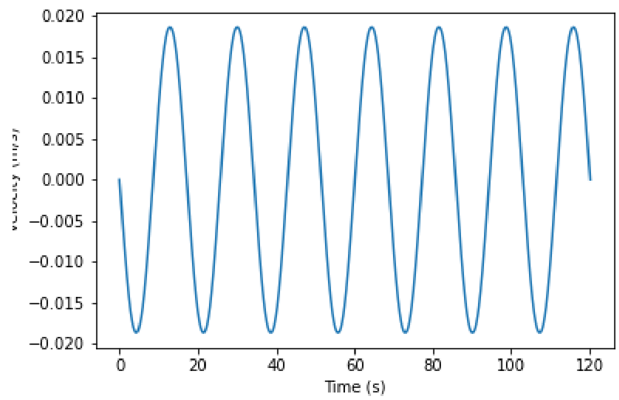

Figure 18. *Vertical velocity of the damped Oscillating Spring Mass over time, calculated using the Symplectic Euler method from initial experimental conditions.*

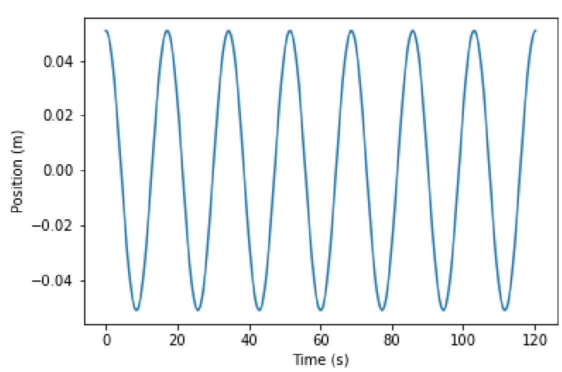

Figure 19. *Vertical Position Phase plot of the damped Oscillating Spring Mass over time, calculated using the Symplectic Euler method from initial experimental conditions.*

Analysis & Discussion

Undamped Oscillation:
As seen in Figure 2, the position's amplitude seems to be consistent for undamped oscillations through time. This is to be expected, as the spring mass system is safely assumed to be closed, and the trial was conducted over a short time span. Over a longer time interval, it would be expected that the position amplitude would diminish as energy exchange inevitably occurs in our imperfectly-closed system (e.g. heat dissipation, compression of supporting pole holding spring). Thus, as the energy of the system is dissipated, the vertical position amplitude should decrease proportionally; this can be seen in Equation 9.

The energy plot using the Forward Euler method, Figure 7, suggests that as time passes, the energy of the spring-mass system increases exponentially. This finding is congruent with the results of the position-time and the position-velocity graphs plotted using the same method (Figures 3 and 5, respectively); this is due to the nature of the Forward Euler method, as it does not conserve energy: it assumes energy increases over time. This energy-time relationship also explains the Velocity-Position plot using the Forward Euler method (Figure 8). The velocity amplitude increases at an increasing rate as time progresses, as does the position amplitude. Both position amplitude and velocity amplitude are dependent on the energy of the system. Since the energy of the system exhibits an exponential increase with the Forward Euler method, it makes sense that this same pattern is observed in position and velocity.

The predicted energy behaviour changes when using the Symplectic Euler method, as is evident in Figures 4 and 6. As time increases, the velocity and position amplitudes stay consistent throughout oscillations. This implies conservation of energy within the oscillatory system. This is theoretically expected under perfect, closed conditions, and exhibited by the experimental plots under a small time interval, as can be seen in Figure 2.

Both velocity and position are sinusoidal functions of time, with a 90º phase shift between the two. In theory, the oscillatory system's total energy is composed of potential energy, which depends on position, and kinetic energy, which depends on velocity. In theory, it is expected that energy of a closed system is conserved, which would produce an ellipse on a velocity-position plot. This can be seen in Figure 9: an elliptical plot of the velocity as a function of the vertical position, calculated using the Symplectic Euler Method, which accounts for conservation of energy, forms a clear ellipse. By contrast, the phase plot Figure 8 was calculated using the Forward Euler Method, which does not account for energy conservation, and the plot is characteristic of a system whose energy increases over time. Figure 9 is more representative of the experimental data.

Damped Oscillation:
As is visible in the position-time plot for the damped oscillator, Figure10, the damping force on the oscillating mass, the air drag from the additional damped disk, causes the position's amplitude to decrease exponentially with time. Thus, damping seems to decrease the mechanical energy of the system with time. Plotting the predicted energy time plot using the Forward Euler method shows energy increasing linearly with time, as can be seen in Figure 14. Experimentally, it is evident that this is not the case; damping decreased the mechanical energy of the system.

The Forward Euler method tended to predict behaviour opposite in trend to the observed experimental data: instead of a decrease in mechanical energy, the Forward plots displayed an increase in mechanical energy; velocity-time and position-time plots using the Forward Euler method seem to remain constant in amplitude over time instead of decaying exponentially, as observed experimentally. Using the Symplectic Euler method yielded far greater correlation with the observed experimental data. The discrepancy in accuracy between the Forward and Symplectic Euler methods is most noticeable in their respective phase plots, Figures 16 and 17: only Figure 17 is shaped like a pure ellipse, whereas Figure 16 shallow spiral, albeit one that is elliptic in shape.

The difference in the quality of the two methods seems to rest on the Symplectic Euler method accounting for the conservation of total energy in the physical system it describes, whereas the Forward Euler method does not. Energy conservation, being a fundamental law of the universe, will greatly skew predictions if not taken into account.

Uncertainties & Fitting:

Uncertainties appeared in amplitude and period calculation: the extrema of the wave oscillation, as well as the start and end of the oscillations, were estimated using the best data point available. It is possible that the actual desired values lay in between data points. Thus, an uncertainty of 0.01 s (the time interval between data points) was introduced to account for the potential error.

The linear regression used to determine the γ decay constant of the oscillation turned out to have a χ^2 value of ~ 62.87862. Thus, the method can be ruled as a poor-fit and thus an inaccurate representation of the data. This can eliminate the validity of the determined γ constant. The amplitudes themselves are considered valid as the methodology was subjectively, relatively straightforward, using simple differences and quotients.

Conclusion

In this experiment, we analysed the oscillatory vertical motion of an undamped and damped mass on a spring. Some of our findings can be found in Table 1:

Table 1: *Calculated Findings from analysis of vertical oscillatory motion of bob mass on a spring in damped and undamped cases dependant on the attachment of the damper disk*

	Damped Motion	Undamped Motion
Period	0.726 ± 0.001 s	0.753 ± 0.001 s
Frequency	1.38 ± 0.002 Hz	1.33 ± 0.002 Hz
Angular Frequency	8.66 ± 0.01 s^{-1}	8.35 ± 0.01 s^{-1}
Spring Constant	16.311 ± 0.003 N/m	14.0 ± 0.002 N/m

For the undamped oscillation, the amplitude was determined to be 0.05914 ± 0.000004 cm. For the damped oscillation, the linear regression of $\chi^2 \approx 62.87862$ determined a γ constant of $\left(1.463 \times 10^8 \pm 2\right) \times 10^{-10}$ and a Reynold constant of $(1.22 \times 10 \pm 3) \times 10^2$.

Ultimately, apart from the γ constant since $\chi^2 \gg 1$. These findings are considered valid as the methodology was subjectively, relatively straightforward.