Exercise Sheet Vectors

1. Let
$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ and } \vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$
Compute
   (a) $3\vec{a} + 2\vec{b}$
   (b) $2\vec{a} - \vec{b}$

2. Let
$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$
Compute the length of the vector $\vec{a}$.

3. Let
$$\vec{x} = \begin{pmatrix} 1/3 \\ a \\ 1/3 \end{pmatrix}$$
For which numbers a is $\vec{x}$ a unit vector (of length 1)?

4. Compute the distance between the points $A = (0, 0, 1)$ and $B = (1, 1, 2)$.

5. Let
$$\vec{a} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}, \vec{b} = \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} \text{ and } \vec{c} = \begin{pmatrix} 6 \\ -1 \\ 2 \end{pmatrix}$$
be vectors. Compute the coordinate representation and the lengths of the following vectors:
$$\vec{x} = -2\vec{a} + 3\vec{b} + 5\vec{c}$$
$$\vec{y} = 5(\vec{b} - 3\vec{a}) - 2\vec{c}$$
$$\vec{z} = 3(\vec{a} + \vec{b}) - 5(\vec{b} - \vec{c}) + \vec{a}$$

6. Let
$$\vec{a} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \text{ and } \vec{b} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$
be vectors. Compute the angles to the x-axis for both vectors. Compute the norms of the vectors.

7. Three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are applied to a mass point. $\vec{F}_1$ has the absolute value 4 Newton and the angle 45°. $\vec{F}_2$ has the angle 120° and absolute value 3 Newton, $\vec{F}_3$ has the angle 330° and absolute value 2 Newton.

   (a) Compute the coordinate representation of the three forces.

   (b) Compute the resulting force $\vec{F}$.

   (c) Draw the resulting force $\vec{F}$, including absolute value and angle.

   (d) Compute the resulting force $\vec{F}$, including absolute value and angle.

8. Let
$$\vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \vec{b} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \text{ and } \vec{c} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
be vectors. Compute the dot products
$$\vec{a} \cdot \vec{b}$$
$$\vec{a} \cdot \vec{c}$$
$$\vec{b} \cdot \vec{c}$$

9. Let
$$\vec{a} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \vec{b} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \text{ and } \vec{c} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$
be vectors. Use the dot product to determine which of these vectors are orthogonal to each other.

10. Given is a parallelogramm, spanned by the vectors

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ and } \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

The common origin of the vectors is $(1, 1, 1)$. Compute the vertices and the area of the parallelogramm.

11. Use vectors to find the relation between the edges of the triangle and the edges of "the middle triangle" (i.e. the triangle of the points dividing the edges in halves).

12. Let

$$\vec{a} = \begin{pmatrix} 2 \\ 1 \\ -c \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}$$

For which $c \in \mathbb{R}$ is

$$|\vec{a} - \vec{b}| = \sqrt{29}?$$

13. Use the computation rules for dot products to show

(a)
$$|\vec{a} + \vec{b}|^2 - |\vec{a} - \vec{b}|^2 = 4 \cdot \vec{a} \cdot \vec{b}$$

(b)
$$|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2|\vec{a}|^2 + 2|\vec{b}|^2$$

14. For which $a, b, c \in \mathbb{R}$ is

$$\begin{pmatrix} a \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ b \\ -4 \end{pmatrix} = \begin{pmatrix} a \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ b \\ -4 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ c \end{pmatrix} = 0?$$

15. Calculate the angle $\angle(A, B, C)$ and the area of the triangle $A = (1, 1, 1), B = (2, 2, 1), C = (2, 1, 2)$.