VGU WS18/19

Field of study: Computer Science

Instructor: Huong Tran

Practice Test: Algebra

Personal information:

Your full name:	
Student ID:	
Signature:	

Result (Please do not fill in these cells). 37 **points** is sufficient to pass the exam.

Problem	1	2	3	4	5	6	7	8	$\sum$
Maximum scores									
Obtained scores									

Remarks:

- You are allowed to bring a two-sided A4 sheet with any contents. No calculator please.
- Duration of examination: 70 minutes.
- Write on every page your name and your student ID.
- Hand in **all** results you want to be assessed.
- Copying and cheating in any form are strictly prohibited and result to a failing grade
- Write your answers in the blank space below each question. You could ask for another blank sheet in case you need more space.

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1. Dual Choice Questions.

   This exercise presents assertions, which you shall evaluate as true or false. You are not expected to reason about your answers. Please proceed as follows:

   - If you are convinced, that the assertion is true, please **underline** the letter *t* for *true* on the left margin of the assertion.
   - If you are convinced, that the assertion is false, please **underline** the letter *f* for *false* on the left margin of the assertion.

   Every correct answer yields you one point, every false answer results in a subtraction of one point. If you do not answer to an assertion, no point is given. If the sum of your points in this exercise is negative, this exercise is rated with 0 points.

   t / f: The inner product of two vectors $\vec{a} = (1, -3, 2, 9)$ and $\vec{b} = (2, -2, 1, 0)$ is 10.

   t / f: The following vectors are linearly dependent.

   $$u = (1, 0, 0), \quad v = (0, 1, 1), \quad w = (1, 1, 1).$$

   t / f: If a matrix M of size 5×5 with $\det(M) = 2$, the $\det(2M) = 4$.

   t / f: If the equation $Ax = 0$ has a unique solution, then $\det A = 0$.

   t / f: Suppose that A and B are subsets of a set U with 8 elements. Knowing that corresponding binary strings of A and B are 10100010 and 11001011 respectively. Then $A \cap B$ has 2 elements.

   t / f: Let $n \in \mathbb{N}_0$. If M is a set with $|M| = n$, then we have $|\mathcal{P}(M)| = 2^n$ for its power set $\mathcal{P}(M)$.

   t / f: The following logical equivalence holds: $(\mathcal{A} \vee \neg\mathcal{B}) \iff (\mathcal{A} \Rightarrow \mathcal{B})$.

   t / f: The relation $R = \{(1, 1), (0, 0), (1, 0), (2, 1), (2, 2)\}$ on $\{0, 1, 2\}$ is both reflexive and symmetric.

   t / f: In the group $(\mathbb{Z}_{16}^*, \cdot_{16})$, the inverse of the element 11 is 13.

   t / f: The group $(\mathbb{Z}_8, +_8)$ is cyclic and of order 4.

2. Linear algebra

(a) Let A, B, C be matrices as follows:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 0 & -3 \end{bmatrix}.$$

Find the matrix $AB + 2C^T$.

(b) Given a matrix D as below. Use at most two elementary row operations to transform D into a row echelon form whose leading entries are not necessary to be 1. Circle all leading entries of its row echelon form. Find the rank of D.

$$D = \begin{bmatrix} 1 & -2 & 2 & -1 \\ 0 & 0 & 2 & -1 \\ -2 & 4 & -4 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

3. Consider the following system of equations in x in real numbers:

$$\begin{cases} x_1 + 2x_2 + \lambda x_3 = -3, \\ -2x_1 + x_2 - 2\lambda x_3 = 6, \\ 2x_1 + 5x_2 - x_3 = 3, \end{cases} \tag{1}$$

where $\lambda \in \mathbb{R}$ is a parameter.

(a) Write down the coefficient matrix, and augmented matrix of system (1)

(b) Determine the determinant of the coefficient matrix of system (1). For which λ the system has a unique solution?

(c) For $\lambda = 1$, compute x_3 using Cramer's rule.

4. Find the inverse matrix of the following matrix using Gauss-Jordan elimination method.

$$\begin{bmatrix} 1 & 2 & 0 \\ -1 & -2 & 1 \\ -1 & 2 & -2 \end{bmatrix}$$

5. Propositional logic.

   (a) Please prove the following logical equivalence:

   $$(\mathcal{A} \Rightarrow \mathcal{B}) \quad \Leftrightarrow \quad (\neg\mathcal{B} \Rightarrow \neg\mathcal{A}) \; .$$

   (b) Please negate the following propositions or terms:

      i. $\mathcal{A} \wedge \neg\mathcal{B}$
      ii. $(\mathcal{A} \wedge \mathcal{B}) \Rightarrow \mathcal{C}$

   (c) Rewrite the following statements without using the conditional:

      i. If it is cold he wears a hat.
      ii. If productivity increases, then wages rise.

6. Relations

Given a relation $R = \{(1,5),(2,4),(3,5),(4,2),(5,1),(5,3)\}$ on $\{1,2,3,4,5\}$.

(a) Find the boolean matrix representation M_R for R.

(b) Find the relation R^2.

(c) Is R transitive or not. If it is not, please find the transitive closure for R.

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7. The group $(\mathbb{Z}_n^*, \cdot_n)$.

   (a) List all elements of the group $\mathbb{Z}_{15}^*$.

   (b) List all elements of the subgroup $H = \langle 8 \rangle$ of $\mathbb{Z}_{15}^*$

   (c) Find all cosets of H in $\mathbb{Z}_{15}^*$

   (d) What is the index $\mathbb{Z}_{15}^* : H$ of the subgroup H in $\mathbb{Z}_{15}^*$.

8. Symmetric groups.

   Given a permutation $\sigma = 3241765 \in S_7$.

   (a) Write down σ in the cycle notation form.

   (b) Compute σ^{-1} and σ^2 in one-line notation form.

   (c) Find the order $o(\sigma)$ of σ in the group S_7.