Least square problem for polynomial regression

import library

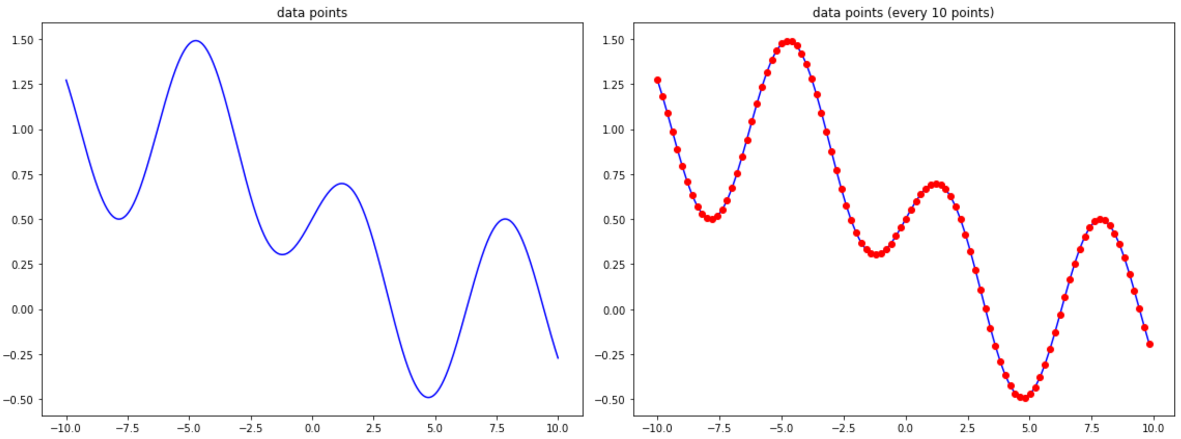
In [11]:

```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
import matplotlib.colors as colors
```

load point data

In [12]:

```
filename    = 'assignment_05_data.csv'
data        = np.loadtxt(filename, delimiter = ',')

x   = data[0, :]    # independent variable
y   = data[1, :]    # dependent variable

x2  = x[::10]
y2  = y[::10]

plt.figure(figsize=(16,6))

plt.subplot(121)
plt.plot(x, y, '-', color = 'blue')
plt.title('data points')

plt.subplot(122)
plt.plot(x2, y2, '-', color = 'blue')
plt.plot(x2, y2, 'o', color = 'red')
plt.title('data points (every 10 points)')

plt.tight_layout()
plt.show()
```



In [13]:

```
#
# A : [ x_1^0, x_1^1, ... , x_1^{p-1} ; x_2^0, x_2^1, ... , x_2^{p-1} ; ... ; x_n^0, x_n^1, ... , x_
#
def construct_matrix_A(x, p):

    # use functions including : np.power
    A=np.zeros((len(x),p))
    for i in range(len(x)):
        A[i] = np.power(x[i],[range(0,p)])

    return A
#
# x : independent variable
# y : dependent variable
# p : power of the polinomial (theta_0 * x^0, theta_1 * x^1, ..., theta_{p-1} * x^{p-1})
# alpha : coefficient for the regularization term : ₩| theta ₩|_2^2
#
def solve_polynomial_regression(x, y, p, alpha = 0):

    # use functions including : np.identity, np.matmul, np.linalg.inv
    A = construct_matrix_A(x, p)
    theta = np.matmul(np.linalg.inv(np.matmul(np.transpose(A), A) + alpha), np.matmul(np.transpose(A
    h = np.matmul(A, theta)

    return h

h_01        = solve_polynomial_regression(x, y, 1)
h_02        = solve_polynomial_regression(x, y, 2)
h_04        = solve_polynomial_regression(x, y, 4)
h_08        = solve_polynomial_regression(x, y, 8)
h_16        = solve_polynomial_regression(x, y, 16)
h_32        = solve_polynomial_regression(x, y, 32)

h_24_0      = solve_polynomial_regression(x, y, 24, 0)
h_24_00001  = solve_polynomial_regression(x, y, 24, 0.0001)
h_24_0001   = solve_polynomial_regression(x, y, 24, 0.001)
h_24_001    = solve_polynomial_regression(x, y, 24, 0.01)
h_24_01     = solve_polynomial_regression(x, y, 24, 0.1)
h_24_1      = solve_polynomial_regression(x, y, 24, 1)
```

In [14]:

```
def plot_polynomial_regression_3x2(x, y, h_01, title_01, h_02, title_02, h_04, title_04, h_08, title

    plt.figure(figsize=(16,18))

    plt.subplot(321)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_01, '-', color='red')
    plt.title(title_01)

    plt.subplot(322)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_02, '-', color='red')
    plt.title(title_02)

    plt.subplot(323)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_04, '-', color='red')
    plt.title(title_04)

    plt.subplot(324)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_08, '-', color='red')
    plt.title(title_08)

    plt.subplot(325)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_16, '-', color='red')
    plt.title(title_16)

    plt.subplot(326)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_32, '-', color='red')
    plt.title(title_32)

    plt.tight_layout()
    plt.show()
```

* *

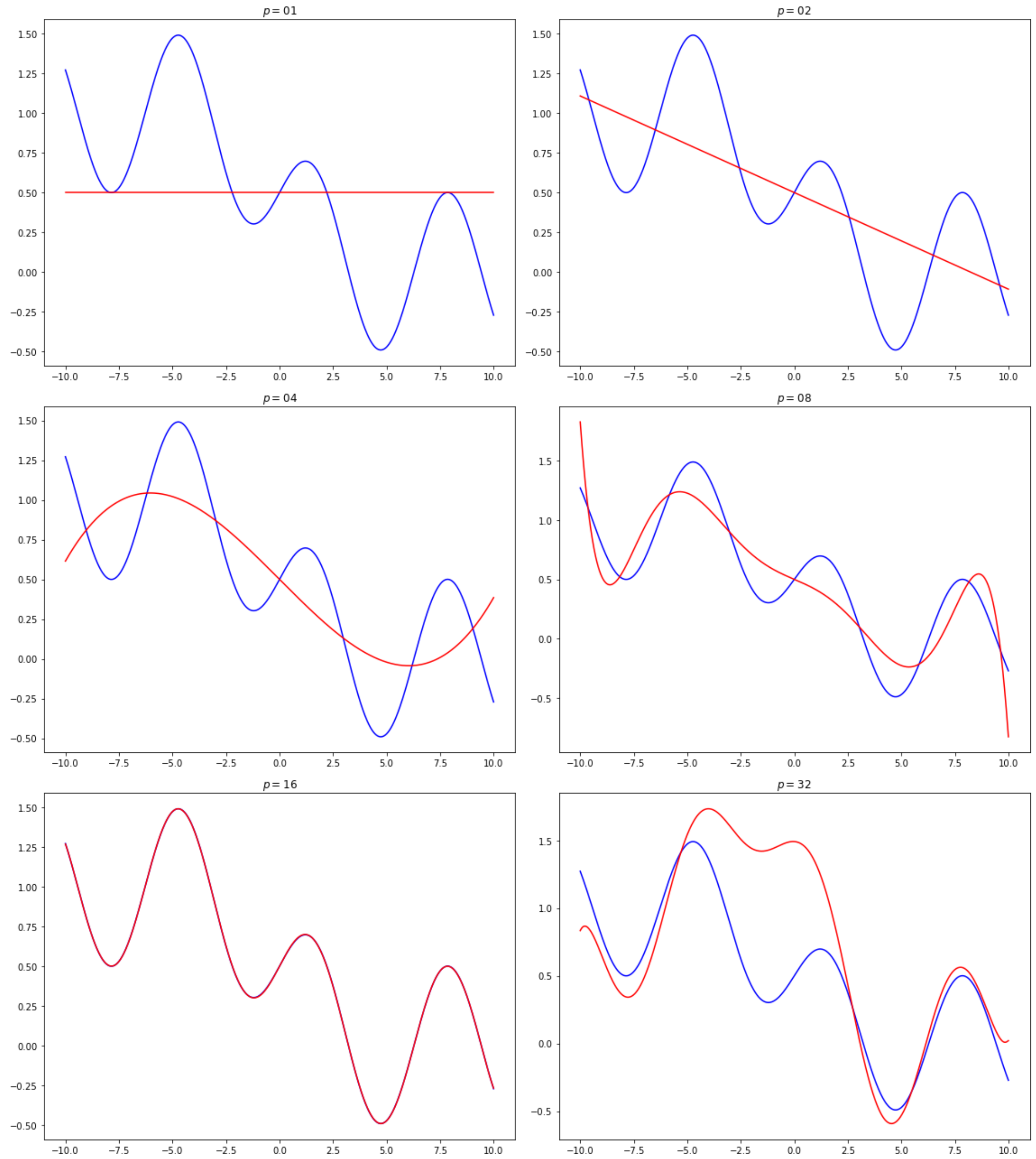
* results

* *

01. plot the input data in blue and the polynomial approximations with varying degrees in red ($p = 1, 2, 4, 8, 16, 32$)

In [15]:

```
plot_polynomial_regression_3x2(x, y, h_01, '$p = 01$', h_02, '$p = 02$', h_04, '$p = 04$', h_08, '$p
```



02. plot the input data in blue and the polynomial approximations with varying regularization parameters at $p = 24$ ($\alpha = 0, 0.0001, 0.001, 0.01, 0.1, 1$)

In [16]:

```
plot_polynomial_regression_3x2(x, y, h_24_0, '$p = 24, alpha = 0$', h_24_00001, '$p = 24, alpha = 0.
```