

Deriving

## Question 2 Test

### Rotation Along Z-Axis

$$x = r\cos(\theta)$$

$$y = r\sin(\theta)$$

$$x' = r\cos(\theta + \phi)$$

$$y' = r\sin(\theta + \phi)$$

*z, w remain the same*

$$x' = r[\cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)]$$

$$x' = r\cos(\theta)\cos(\phi) - r\sin(\theta)\sin(\phi)$$

$$\text{but } r\cos(\theta) = x \text{ \&\& } r\sin(\theta) = y$$

$$\therefore x' = x\cos(\phi) - y\sin(\phi)$$

$$y' = r[\sin(\theta)\cos(\phi) + \sin(\phi)\cos(\theta)]$$

$$y' = r\sin(\theta)\cos(\phi) + r\sin(\phi)\cos(\theta)$$

$$y' = y\cos(\phi) + x\sin(\phi)$$

We can represent the original co-ordinates of the system as a vector

$$\text{initial point} = [x \quad y \quad z \quad w]$$

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix}$$

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x\cos(\phi) - y\sin(\phi) \\ y\cos(\phi) + x\sin(\phi) \\ z \\ w \end{pmatrix}$$

$$\begin{cases} ax + by + cz + dw = x\cos(\phi) - y\sin(\phi) \\ ex + fy + gz + hw = y\cos(\phi) + x\sin(\phi) \\ ix + jy + kz + lw = z \\ mx + ny + oz + pw = w \end{cases}$$

$$\begin{cases} \therefore a = \cos(\phi), b = -\sin(\phi), c = 0, d = 0 \\ e = \sin(\phi), f = \cos(\phi), g = 0, h = 0 \\ i = 0, j = 0, k = 1, l = 0 \\ m = 0, n = 0, o = 0, w = 1 \end{cases}$$

$$\therefore \begin{pmatrix} \cos(\phi) & -\sin(\phi) & 0 & 0 \\ \sin(\phi) & \cos(\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} == \text{Rotation, whereby } z \text{ remains the same}$$