

cs107e

§1 Baremetal systems programming

§2 Building a personal computer

- **Graphics**

- **Keyboard**

§3 Topics in systems programming

Systems Programming

Computer arithmetic (today)

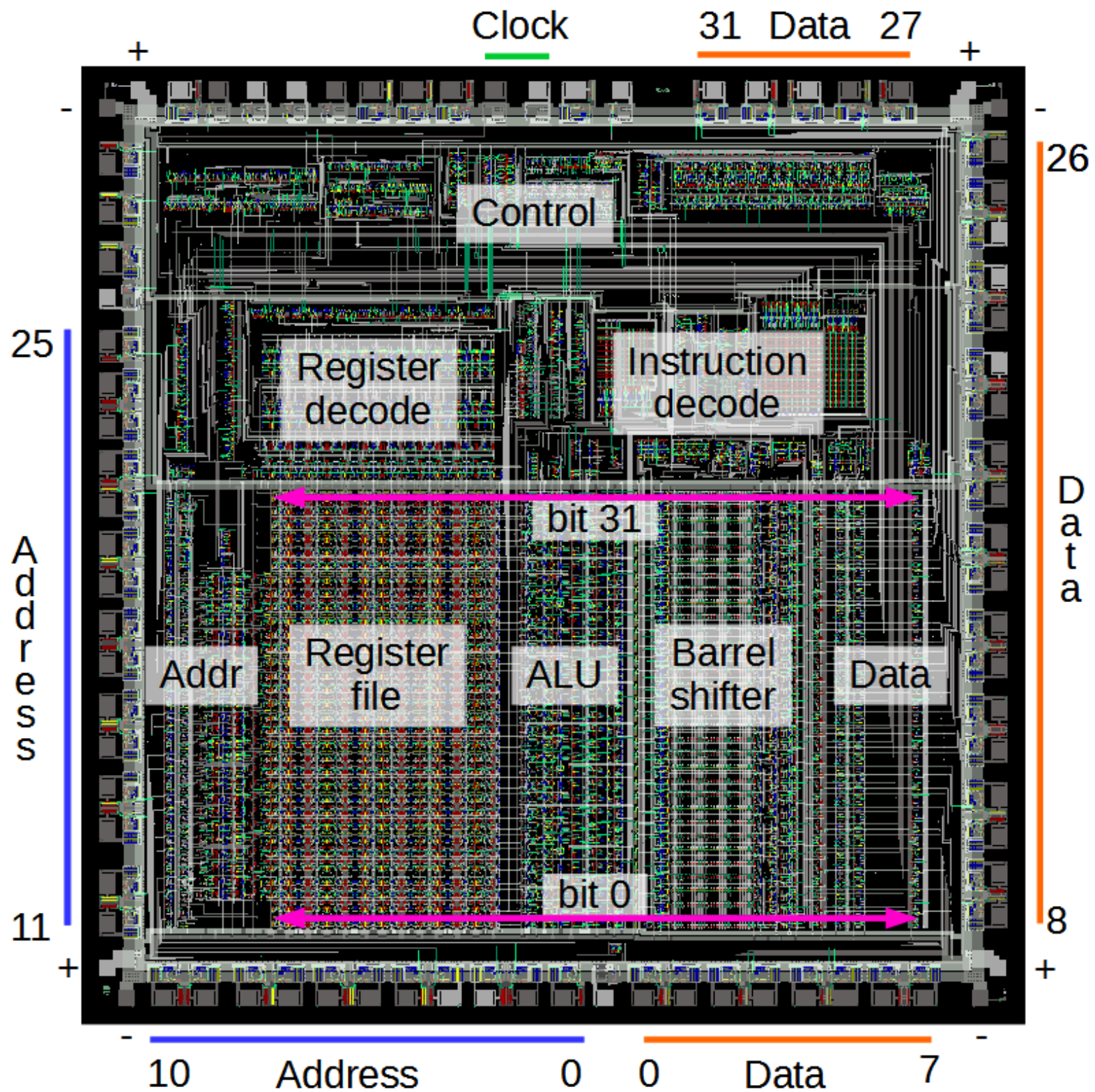
Caches; debugging discussion

New systems prog lang: Rust and Go

Veronica, Quinn Dunki

To Linux and beyond

Final project demonstrations



Learning Goals

Processor flags: Z, N, C, V

Addition and carry, subtraction and borrow

2's complement representation of negative numbers

Signed and unsigned conversions (Craziness)

Arithmetic-logic units (ALUs)

Addition

High School Addition

247

9

High School Addition

1
247
9

6

High School Addition

1
247
9

56

High School Addition

1

247

09

256

High School Addition (Hexadecimal)

F7

09

High School Addition (Hexadecimal)

1

F7

09

0

High School Addition (Hexadecimal)

11

F7

09

00

High School Addition (Hexadecimal)

11

F7

09

100

High School Addition (Binary)

11110111

00001001

High School Addition (Binary)

$$\begin{array}{r} 1 \\ 11110111 \\ 00001001 \\ \hline 0 \end{array}$$

High School Addition (Binary)

```
      11
  11110111
  00001001
  -----
           00
```


High School Addition (Binary)

```
      111
    11110111
    00001001
  -----
      000
```

High School Addition (Binary)

```
      1111
    11110111
    00001001
  -----
      0000
```

High School Addition (Binary)

```
    11111
  11110111
  00001001
  -----
    00000
```

High School Addition (Binary)

111111

11110111

00001001

000000

High School Addition (Binary)

1111111

11110111

00001001

0000000

High School Addition (Binary)

11111111

11110111

00001001

00000000

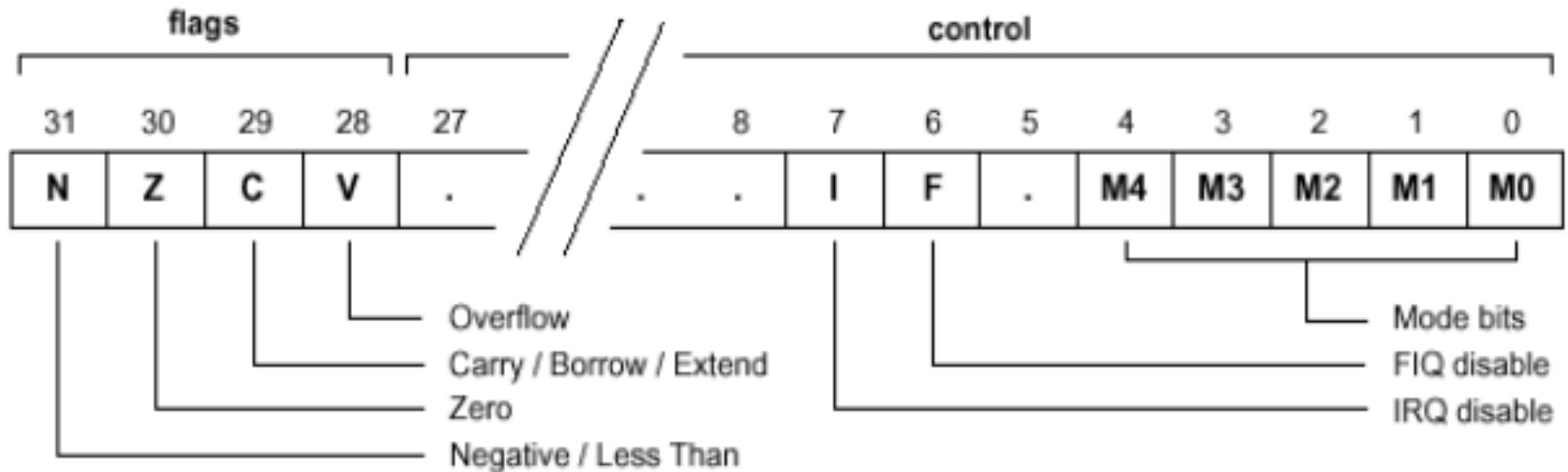
High School Addition (Binary)

```
11111111
 11110111
 00001001
-----
00000000
```

Result: 00000000 (only room for 8-bits)
Carry (C): 1 (extra bit)

To hold the result of adding two n-bit numbers requires n+1 bits

CPSR



Arithmetic instructions set Z, N, C, V
Logic instructions set Z, N

We will cover V later

cpsr.c : uadd32

```
// Multiple precision addition  
// https://gcc.gnu.org/
```

```
int64_t add64(int64_t a, int64_t b)  
{  
    return a + b;  
}
```

```
add64(int64_t, int64_t):  
    adds    r2, r2, r0 // adds, not add  
    adc     r3, r3, r1 // add w/ carry  
    mov     r0, r2  
    mov     r1, r3  
    bx      lr
```

Negative Numbers

Up to now, all binary numbers have been positive (unsigned)

How to define a negative number?

A clever way of defining -1 is to say that it is the number that when added to 1, results in 0 mod 256

$$0xFF + 0x01 = 0x100 = 0x00$$

0xFF can be *interpreted* as -1

In this system, adding unsigned numbers is exactly the same as adding signed numbers. Cool!

0x00 = 0

0xFF = -1

0xFE = -2

...

0x80 = -128 (could interpret as 128)

0x7F = 127

...

0x01 = 1

if we choose to *interpret* 0x80=-128,
then the most-significant bit of the number
indicates that it is negative (N) - sign bit

8-bit signed number range from -128 to 127

32-bit numbers from -2147483648 to 2147483647

How do we negate a number?

Subtract the number from 0!

```
11111111
10000000
 00000001
-----
11111111
```

11111111 (0xFF) is -1

Note the *borrow*s.

This is same as subtracting the number
from 100000000

11111111

100000000

00000001

11111111

But, 100000000 is more than 8-bits!

Rewrite as ...

$$100000000 = 11111111 + 1$$

11111111

-00000001

$$11111110 = \sim 00000001$$

11111110

+00000001

11111111

Negating

$$-x = \sim x + 1$$

Some nomenclature:

$\sim x$: One's complement

$\sim x + 1$: Two's complement ($2^{32} - x$)

Subtraction is negation and addition

$$a - b = a + (-b) = a + \sim b + 1$$

$$0x01 - 0x00 = 0x01 + 0xFF + 0x01 = 0x01 + C$$

$$0x01 - 0x01 = 0x01 + 0xFE + 0x01 = 0x00 + C$$

$$0x01 - 0x02 = 0x01 + 0xFD + 0x01 = 0xFF$$

Note that carry is set if $a \geq b$

$$\text{borrow} = \text{!carry}$$

cpsr.c : usub32

```
// Multiple precision subtraction  
// https://gcc.gnu.org/
```

```
int64_t sub64(int64_t a, int64_t b)  
{  
    return a - b;  
}
```

```
sub64(int64_t, int64_t):  
    subs    r2, r2, r0 // subs, not sub  
    sbc     r3, r3, r1 // sub w/ carry  
    mov     r0, r2  
    mov     r1, r3  
    bx      lr
```

Signed subtraction - overflow (sub32)

80000000+00000001=80000001 : z=0,=1,c=0,uge=0
80000000-00000001=7fffffff : z=0,=0,c=1,uge=1
7fffffff+00000001=80000000 : z=0,=1,c=0,uge=0
7fffffff-00000001=7ffffffe : z=0,=0,c=1,uge=1
-2147483648+1=-2147483647 : z=0,=1,c=0,v=0,ge=0
-2147483648-1= 2147483647 : z=0,=0,c=1,v=1,ge=0
2147483647+1=-2147483648 : z=0,=1,c=0,v=1,ge=1
2147483647-1= 2147483646 : z=0,=0,c=1,v=0,ge=1

"Overflow" happens when adding two numbers of the same sign, and getting a result with a different sign.

$V = (n1 == n2) \ \& \ (n \neq n1)$

Comparison

unsigned comparison (usub32)

00000000-00000000=00000000 : z=1,=0,c=1,uge=1
00000000-00000001=ffffffff : z=0,=1,c=0,uge=0
00000001-00000000=00000001 : z=0,=0,c=1,uge=1
00000001-00000001=00000000 : z=1,=0,c=1,uge=1
00000001-ffffffff=00000002 : z=0,=0,c=0,uge=0
ffffffff-00000001=ffffffffff : z=0,=1,c=1,uge=1
ffffffff-ffffffff=ffffffffff : z=0,=1,c=1,uge=1

bcs: branch carry set (greater than or equal)

bcc: branch carry clear (less than)

Signed comparison (sub32)

80000000+00000001=80000001 : z=0,=1,c=0,uge=0
80000000-00000001=7fffffff : z=0,=0,c=1,uge=1
7fffffff+00000001=80000000 : z=0,=1,c=0,uge=0
7fffffff-00000001=7ffffffe : z=0,=0,c=1,uge=1
-2147483648+1=-2147483647 : z=0,=1,c=0,v=0,ge=0
-2147483648-1= 2147483647 : z=0,=0,c=1,v=1,ge=0
2147483647+1=-2147483648 : z=0,=1,c=0,v=1,ge=1
2147483647-1= 2147483646 : z=0,=0,c=1,v=0,ge=1

bge: signed greater than or equal (n == v)

blt: signed less than (n != v)

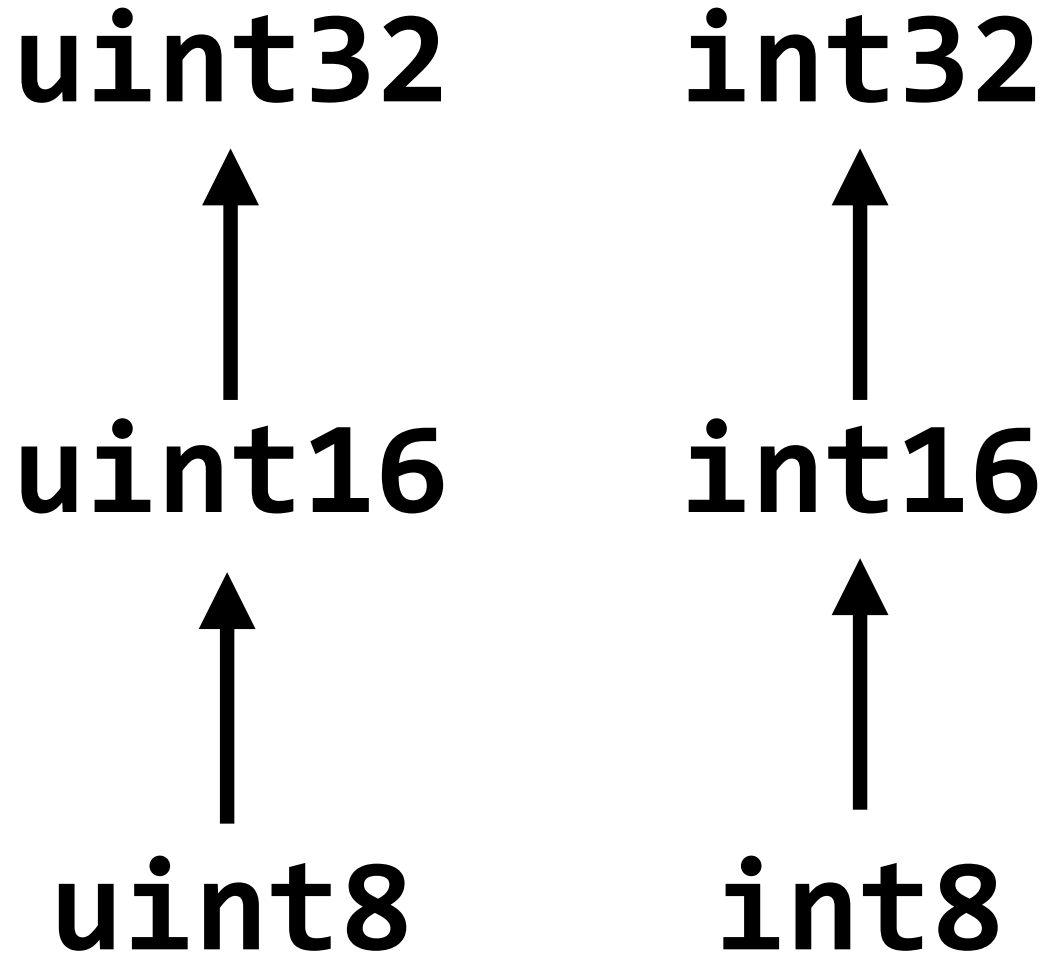
```
int ge() { return !v ? n : !n }
```

Code	Suffix	Flags	Meaning
0000	EQ	Z set	equal
0001	NE	Z clear	not equal
0010	CS	C set	unsigned higher or same
0011	CC	C clear	unsigned lower
0100	MI	N set	negative
0101	PL	N clear	positive or zero
0110	VS	V set	overflow
0111	VC	V clear	no overflow
1000	HI	C set and Z clear	unsigned higher
1001	LS	C clear or Z set	unsigned lower or same
1010	GE	N equals V	greater or equal
1011	LT	N not equal to V	less than
1100	GT	Z clear AND (N equals V)	greater than
1101	LE	Z set OR (N not equal to V)	less than or equal
1110	AL	(ignored)	always

Converting Signed and Unsigned Types in C

Ninja Job Interview Questions

Type Hierarchy



Types are sets of allowed values

Arrow indicate subsets: `uint8` \subset `uint16`

```
// Extend sign bit rightward  
// code/sign.c
```

```
int8 0xFE -> int32 0xFFFFFFFFFE  
int8 0x7E -> int32 0x0000007E
```

```
// Assembly language  
LSL r0,r0,#24  
ASR r0,r0,#24
```

uint32



uint16



uint8

int32



int16



int8

Truncate!

uint32 ← int32

uint16 ← int16

uint8 ← int8

Ok?

uint32 → int32

uint16 → int16

uint8 → int8

Ok?

uint32

int32

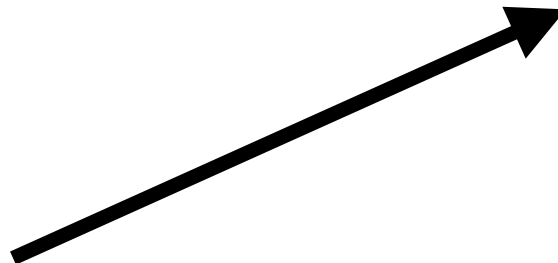
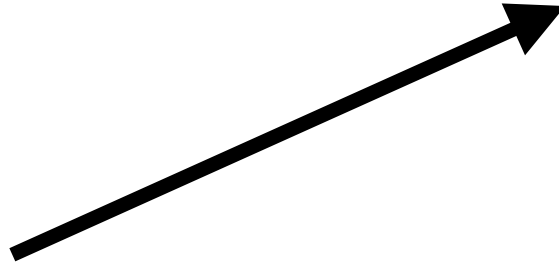
uint16

int16

uint8

int8

Ok?



uint32

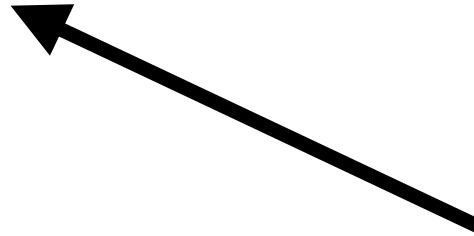
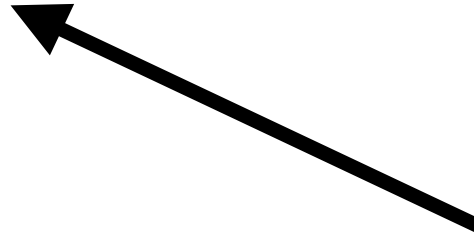
int32

uint16

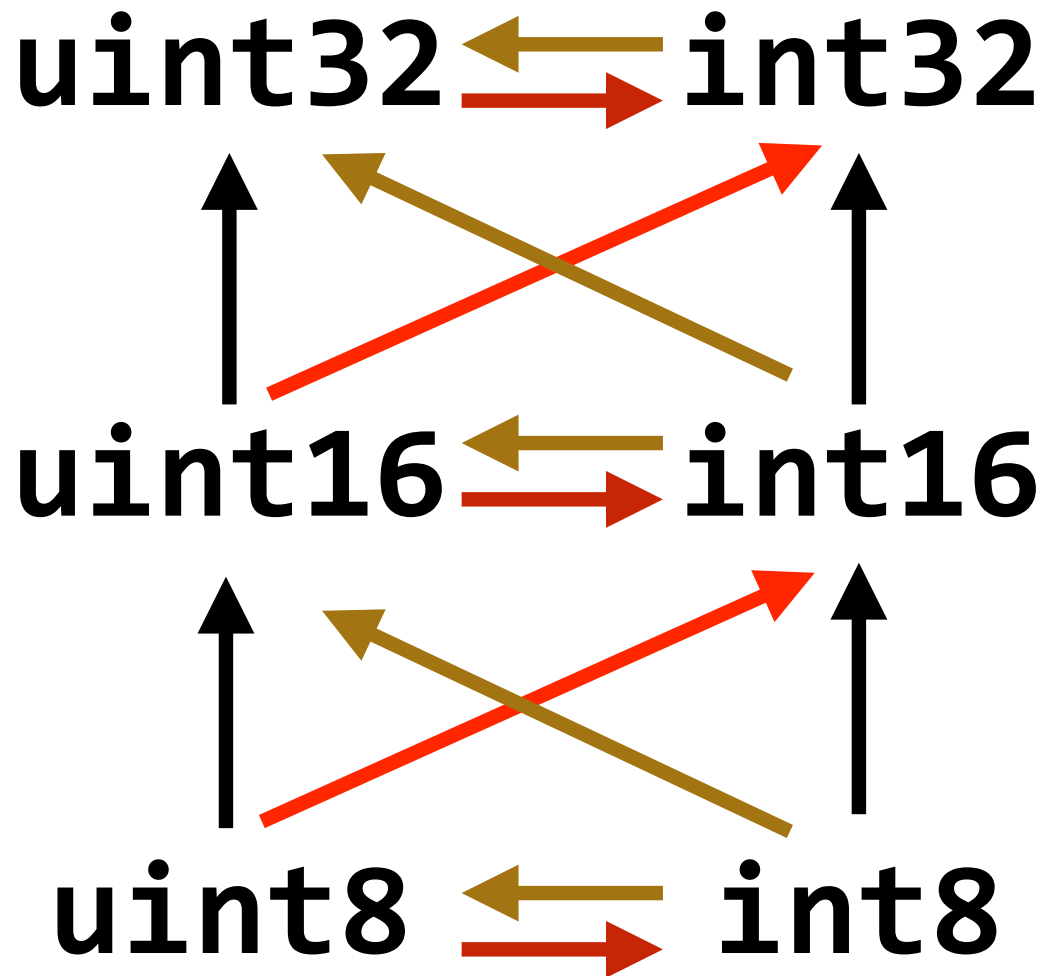
int16

uint8

int8



Ok?



→ technically, not defined
← defined

6.3.1.3 Signed and unsigned integers

1 When a value with integer type is converted to another integer type other than `_Bool`, if the value can be represented by the new type, it is unchanged.

2 Otherwise, if the new type is unsigned, the value is converted by repeatedly adding or subtracting one more than the maximum value that can be represented in the new type until the value is in the range of the new type.

3 Otherwise, the new type is signed and the value cannot be represented in it; either the result is implementation-defined or an implementation-defined signal is raised.

Operations involving signed and unsigned

`int32 + uint32?`

What is the type of the result?

Signed integers are changed to unsigned!

Why is this the right thing to do?

Unsigned conversion is well-defined

This leads to unexpected behavior!

```
#include <stdio.h>
```

```
int main(void)
```

```
{
```

```
    unsigned int a = 6;
```

```
    int b = -20;
```

```
    (b>a) ?
```

```
        puts("-20 > 6") :
```

```
        puts("-20 <= 6") ;
```

```
}
```

6.3.1.8 Usual arithmetic conversions

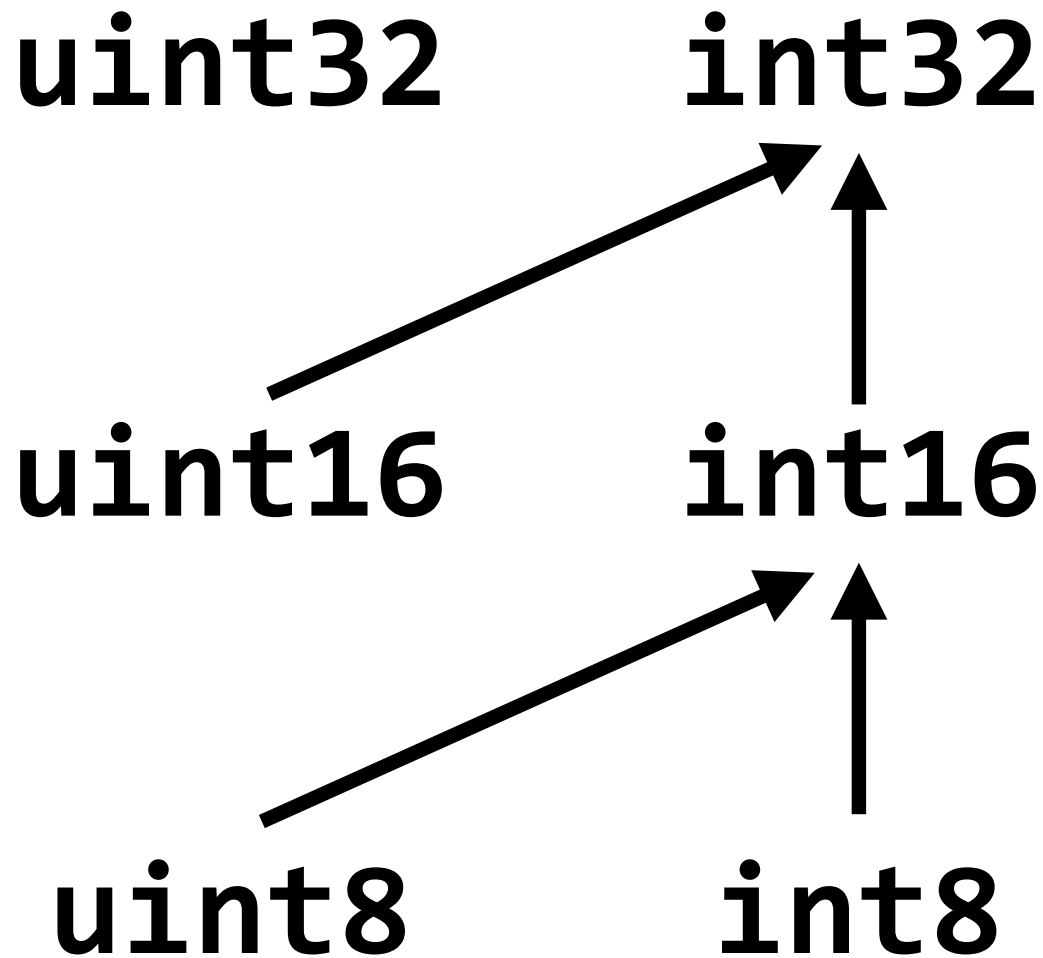
1 If both operands have the same type, then no further conversion is needed.

2 Otherwise, if both operands have signed integer types or both have unsigned integer types, the operand with the type of lesser integer conversion rank is converted to the type of the operand with greater rank.

3 Otherwise, if the operand that has unsigned integer type has rank greater or equal to the rank of the type of the other operand, then the operand with signed integer type is converted to the type of the operand with unsigned integer type.

4 Otherwise, if the type of the operand with signed integer type can represent all of the values of the type of the operand with unsigned integer type, then the operand with unsigned integer type is converted to the type of the operand with signed integer type.

5 Otherwise, both operands are converted to the unsigned integer type corresponding to the type of the operand with signed integer type.



Alternative: Promote to greatest common type

**"Whenever you mix signed and
unsigned numbers you get in trouble"**

Bjarne Stroustrup

1 -Bit Adder

Add 2 1-bit numbers

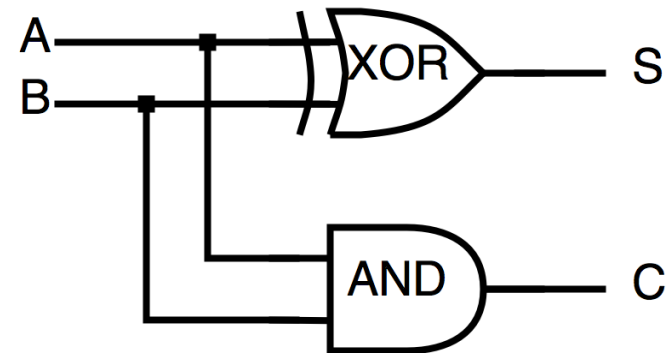
a	b	sum
0	0	00
0	1	01
1	0	01
1	1	10

Add 2 1-bit numbers (Half Adder)

a	b	sum
0	0	00
0	1	01
1	0	01
1	1	10

bit 0 of sum: $S = a \oplus b$

bit 1 of sum: $C = a \& b$



Have reduced addition to bitwise,
logical operations!

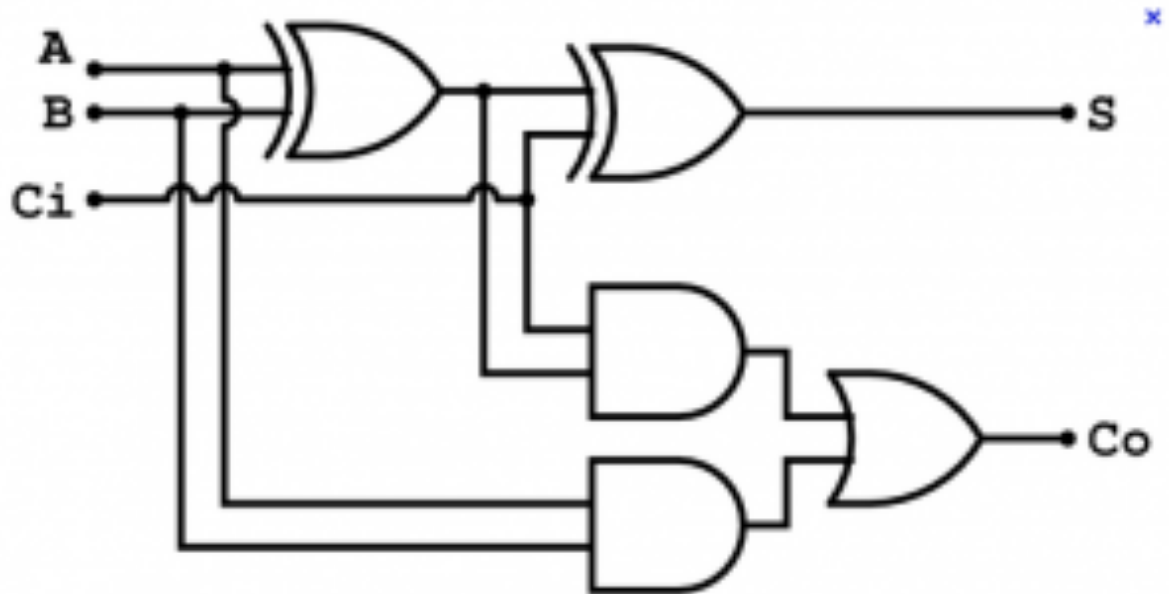
Add 3 1-bit numbers

a	b	c	=	s	c
0	0	0		0	0
0	1	0		0	1
1	0	0		0	1
1	1	0		1	0
0	0	1		0	1
0	1	1		1	0
1	0	1		1	0
1	1	1		1	1

Add 3 1-bit numbers

a	b	ci	=	s	co
0	0	0		0	0
0	1	0		0	1
1	0	0		0	1
1	1	0		1	0
0	0	1		0	1
0	1	1		1	0
1	0	1		1	0
1	1	1		1	1

Full Adder



8-bit adder

