VIETNAM NATRIONAL UNIVERSITY HO CHI MINH CITY

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CLASS 20CLC11

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Project 01: Color Compression

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I. General information

- Project 01: Color Compression
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- Environment: jupyter notebook
- Programing language: python

II. Requirements

1. Project's requirement:
   - Install a program that reduces the number of colors in an image using the K-Means algorithm.
   - Provides a main () function that allows users to enter their image file's name and to select the saving format for their image's output including "png" and "pdf".
2. Report's requirement:

No.	Task	Status
1	Present your idea generally, describe all your defined functions	Done
2	For each value of k (k = 3, 5, 7), return image after being compressed corresponding to k. You can try more value of k to gain insight into this algorithm	Done
3	Make your own comments on those results	Done
4	Your report must be paginated and included the reference section	Done

III. Conclusion

1. Present idea and describe functions

- General idea:
  - Color images commonly used are RGB images, where each pixel stores 3 color channel information (1 byte for each color channel): R (red), G (green), B (blue). Each component can take a value between 0 and 255, where the tuple - pixel (0, 0, 0) represents black and (255, 255, 255) represents white.
  - K-means is a clustering algorithm that is used to group data points into clusters (centroids) such that data points lying in the same group are very similar to each other in characteristics. K-means algorithm can be used to find subgroups in the image and assign the image pixel to that subgroup which results in image segmentation. We are

proactive in choosing k - the number of colors the image must represent.

- So, we're going to use K-means to present the initial image in the same size but only with K colors.

- Describe functions:
  - Steps of K-means:
    - Step 1: Generate K centroids.
    - Step 2: Assign the image pixel to these centroids.
    - Step 3: Generate K new centroids which values equal the mean of that cluster i.
    - Step 4: Repeat until the results are convergent.
  - Declare libraries:

```
# Ho Chi Minh city University of Science (HCMUS)
# Author: Phan Minh Phuc
# Student ID: 20127063
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image as img
from sklearn.cluster import KMeans
from sklearn.metrics import pairwise_distances_argmin
from sklearn.utils import shuffle
from time import time
```

  - generate_centroid(img_1d, k_clusters, init_centroids):

```
def generate_centroid(img_1d, k_clusters, init_centroids):
    if init_centroids == 'in_pixels':
        return A[np.random.randint(A.shape[0], size = k_clusters), :]
    if init_centroids == 'random':
        return np.random.randint(256, size = (k_clusters, 3))
```

    - Generate K centroids base on the value of init_centroids:
      - "random" → centroid has 3 channels, each channel is initial random in [0, 255].
      - "in_pixels" → centroid is a random pixels of original image.
  - classify_pixel(img_1d, centroid):

```python
def classify_pixel(img_1d, centroid):
    # distance between each pixel and the 1st controid
    distance = np.linalg.norm(img_1d - centroid[0], axis = 1)
    # convert the array to shape (img_1d.shape[0], 1)
    distance = distance.reshape((img_1d.shape[0], 1))

    # distance between each pixel and the rest controids
    for i in range (1, centroid.shape[0]):
        temp = np.linalg.norm(img_1d - centroid[i], axis = 1)
        temp = temp.reshape((img_1d.shape[0], 1))
        # adding these distances to the initial distance array
        distance = np.concatenate((distance,temp),axis=1)

    # return index of centroid nearest to each pixel
    #  return distance
    return np.argmin(distance,axis = 1)
```

- Calculate the distance between each pixel and the 1st centroid, save to distance.
- Then, calculate the distance between each pixel and the rest centroids similarly.
- Once again, save these results to distance by using numpy.concatenate() – this is the reason why you have to calculate the distance between each pixel and the 1st centroid before doing the same thing with the rest centroids.
- Return a list contains the index of centroid nearest to each pixel aka a list contains the index of centroid that each pixel belongs to.

➢ optimize_centroid(img_1d, k_cluster, centroid):

```python
def optimize_centroid(img_1d, k_cluster, centroid):
    label = classify_pixel(img_1d, centroid)
    is_changed = False
    for i in range (k_cluster):
        # choose all pixels belong to centroid[k]
        cluster = img_1d[label == i, :]
        # if no pixel belong to this centroid
        # => go on to the next centroid
        if len(cluster) == 0:
            continue
        # generate a new centroid from mean value of last chosen pixels
        new_centroid = np.mean(cluster, axis = 0)
        # if new centroid and centroid[k] is not convergent
        # => set centroid[k] with the new centroid's value
        if (not np.allclose(centroid[i], new_centroid, atol = 1, equal_nan = True)):
            centroid[i] = new_centroid
            is_changed = True

    return is_changed, centroid, label
```

- Generate label to save the index of centroid that each pixel belongs to.
- Generate is_changed to check whether the centroids are changed and assign the value False to it.
- For each centroid, generate cluster contains all pixels belong to it, then
- If there isn't any pixel belong to this centroid, the algorithm will go on to the next centroid.
- Else, generate new_centroid which value equal the mean of this cluster.
- If the difference in value of this centroid and the new_centroid is larger than the absolute tolerance parameter (atol) = 1, the function will update this centroid's value to new centroid's value and assign the value True to is_changed.
- Return whether the initial centroids are changed (is_changed), the current centroid values (centroid), and the current label (label).

➢ compress_pixel(img_1d, k_clusters, label, centroids):

```
def compress_pixel(img_1d, k_clusters, label, centroids):
    for i in range(k_clusters):
        img_1d[label == i, :] = centroids[i]
    return img_1d
```

- For each centroid in K centroids, change the value of all pixels belong to this centroid to its value.
- Return a matrix contain value of all pixels in the origianal image (img_1d) after being compressed.

➢ kmeans(img_1d, k_clusters, max_iter, init_centroids = 'random'):

```
def kmeans(img_1d, k_clusters, max_iter, init_centroids = 'random'):
    centroid = generate_centroid(img_1d, k_clusters, init_centroids)
    label = []
    while max_iter > 0:
        stop_condition, centroid, label = optimize_centroid(img_1d, k_clusters, centroid)
        if stop_condition == False:
            break;
        max_iter -= 1;
    # print(label.shape)

    return centroid, label
```

- Generate centroid contains K centroid by using generate_centroid(img_1d, k_clusters, init_centroids).
- Generate label – an empty list contains the index of centroid that each pixel belongs to.

- Start converging centroid by using optimize_centroid(img_1d, k_cluster, centroid) until the stop_condition is satisfied or max_iter is <= 0.
- Return the current centroid and label.

➢ main():

```python
def main():
    filename = input("Enter image's name: ")
    print("Filename: " + filename)
    output_format = input("Enter image's output format ('png' or 'pdf'): ")
    print("Image's output format: " + output_format)
    while output_format != "png" and output_format != "pdf":
        output_format = input("Enter image's output format ('png' or 'pdf'): ")
        print("Image's output format: " + output_format)

    img_1d = img.open(filename)
    # Convert the input to a matrix.
    img_1d = np.asarray(img_1d)

    # img_1d.shape[0]: number of rows
    # img_1d.shape[1]: number of columns
    # img_1d.shape[2]: number of channels
    row = img_1d.shape[0]
    column = img_1d.shape[1]
    # Reshape initial matrix to 1d array
    # img_1d.reshape(-1, 3) == img_1d.reshape(img_1d.shape[0] * img_1d.shape[1], img_1d.shape[])
    img_1d = img_1d.reshape(-1, 3)

    k_clusters = 5
    max_iter = 3000
    centroid, label = kmeans(img_1d, k_clusters, max_iter, init_centroids = 'random')

    compressed_img = compress_pixel(img_1d, k_clusters, label, centroid)
    compressed_img = compressed_img.reshape(row, column, 3)
    print(f"Number of clusters: {k_clusters}")
    print(f"Centroids: \n {centroid}")
    plt.clf()
    print(f"Compressed image ({k_clusters} colors, Project 1: Color Compression)")
    figure = plt.figure()
    figure.set_size_inches(column / 96, row / 96)
    plt.axis("off")
    plt.imshow(compressed_img)
    plt.savefig(f"compressed_img{k_clusters}." + output_format, dpi = 300)
```

- Generate filename, output_format and let users enter their image file's name and and select the saving format for their image's output including "png" and "pdf".
- Assign img_1d the result of opening image filename by using PIL.Image.open().
- Convert the img_1d into a matrix shape row × column × channel.
- Assign row the value of img_1d's row.
- Assign column the value of img_1d's column.
- Reshape initial matrix to 1d array by using numpy.reshape().
- Assign k_clusters respectively the value 3, 5, 7, 13, 27 and 63.
- Assign max_iter the value 3000.

- Assign centroid, label the result after executing kmeans(img_1d, k_clusters, max_iter, init_centroids = 'random').
- Assign compressed_img the result after executing compress_pixel(img_1d, k_clusters, label, centroids).
- Reshape compressed_img to the initial matrix's shape by using numpy.reshape().
- Use matplotlib.pyplot.figure() to create a new figure.
- Use matplotlib.pyplot.figure.set_size_inches() to set the figure size in inches. Row and Column are divide by 96 to convert from pixels to inches.
- Use matplotlib.pyplot.axis("off") to remove the axes and the plot borders completely.
- Show compressed_img by using matplotlib.pyplot.imshow() and save it as name "compressed_img{k_clusters}" (k_cluster: number of centroids) with output format output_format by using matplotlib.pyplot.savefig().

2. Testing result

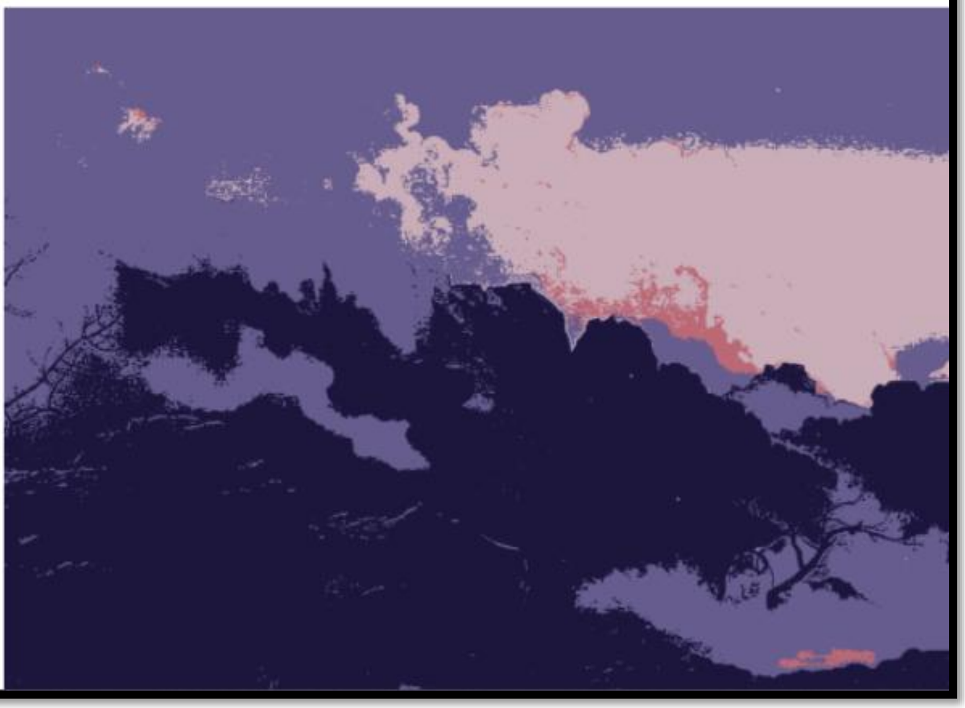
- The original iamge:

- K = 3:

```
In [59]: main()

Enter image's name: 1.jpg
Filename: 1.jpg
Enter image's output format ('png' or 'pdf'): png
Image's output format: png
Number of clusters: 3
Centroids:
 [[112  91 136]
 [199 156 167]
 [ 29  23  62]]
Compressed image (3 colors, Project 1: Color Compression)

<Figure size 432x288 with 0 Axes>
```

- K = 5:

```
In [57]: main()

Enter image's name: 1.jpg
Filename: 1.jpg
Enter image's output format ('png' or 'pdf'): png
Image's output format: png
Number of clusters: 5
Centroids:
 [[102  92 141]
 [ 28  22  61]
 [187 105 119]
 [ 66 231 249]
 [201 174 185]]
Compressed image (5 colors, Project 1: Color Compression)

<Figure size 432x288 with 0 Axes>
```

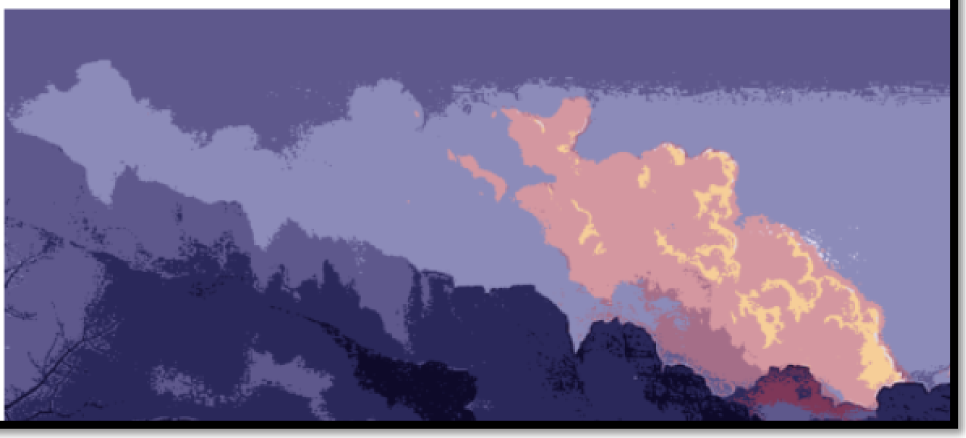


- K = 7:

```
In [61]: main()

Enter image's name: 1.jpg
Filename: 1.jpg
Enter image's output format ('png' or 'pdf'): png
Image's output format: png
Number of clusters: 7
Centroids:
 [[ 16  11  44]
 [ 42 217 107]
 [195  97 105]
 [229 192 184]
 [155 137 174]
 [101  89 138]
 [ 51  43  92]]
Compressed image (7 colors, Project 1: Color Compression)

<Figure size 432x288 with 0 Axes>
```

- K = 13:

```
In [63]: main()

Enter image's name: 1.jpg
Filename: 1.jpg
Enter image's output format ('png' or 'pdf'): png
Image's output format: png
Number of clusters: 13
Centroids:
 [[ 42  40  90]
 [ 14  10  41]
 [246 206 152]
 [140 139 185]
 [224 217 237]
 [165 110 136]
 [  6  96 217]
 [ 22   6 235]
 [ 83 231  79]
 [212 151 160]
 [219  94  85]
 [ 94  88 140]
 [126  55  88]]
Compressed image (13 colors, Project 1: Color Compression)

<Figure size 432x288 with 0 Axes>
```



- K = 27:

```
In [65]: main()

         Enter image's name: 1.jpg
         Filename: 1.jpg
         Enter image's output format ('png' or 'pdf'): png
         Image's output format: png
         Number of clusters: 27
         Centroids:
          [[ 25 215  18]
          [217  42  26]
          [178  99 117]
          [135  71 107]
          [  8   5  30]
          [ 98  60 103]
          [ 91  89 143]
          [ 63  28  71]
          [167  68 250]
          [225 129 127]
          [151  51  70]
          [ 71   7  21]
          [ 68 194  83]
          [133 100 138]
          [ 58 191 227]
          [174 255 107]
          [154 159 202]
          [ 23  25  72]
          [224  91  73]
          [ 50  57 110]
          [126 125 174]
          [180 132 155]
          [251 224 142]
          [225 169 169]
          [ 51 167 255]
          [229 223 241]
          [ 73 196  75]]
         Compressed image (27 colors, Project 1: Color Compression)
```

- K = 63:

```
In [67]: main()

Enter image's name: 1.jpg
Filename: 1.jpg
Enter image's output format ('png' or 'pdf'): png
Image's output format: png
Number of clusters: 63
Centroids:
 [[108  67 254]
 [ 30 249   4]
 [186 149 169]
 [ 42   6  21]
 [230  80  94]
 [179  97 116]
 [216 121 103]
 [140  75 108]
 [ 65 173 191]
 [ 39  43  96]
 [219 158 162]
 [ 36 220 191]
 [ 18  98 179]
```

```
In [67]: main()
 [ 18  98 179]
 [100  79 125]
 [235 181 170]
 [154  10 170]
 [217  38  31]
 [ 64  71 123]
 [ 79 162 171]
 [146 154 201]
 [151  53 217]
 [246 206 199]
 [168 127 156]
 [ 41 189 214]
 [ 68 228  54]
 [ 68 116 254]
 [161  60  78]
 [232 119 136]
 [138 211  27]
 [ 19  92 236]
 [252 238 138]
```

```
In [67]: main()
 [252 238 138]
 [ 88  90 146]
 [  2 234 142]
 [168 169 208]
 [248 240 247]
 [ 51 178   0]
 [  3  96 183]
 [216 219 248]
 [123  96 136]
 [ 44 141   9]
 [139  13  21]
 [ 99 194 167]
 [ 90  16  40]
 [108 109 164]
 [196 127 143]
 [ 17 195  61]
 [190 192 227]
 [ 55 160 179]
 [ 26 190 160]
```

```
In [67]: main()

 [ 26 190 160]
 [  5   4  28]
 [ 49 143 250]
 [ 67  49 187]
 [ 19  21  67]
 [133 131 178]
 [247  88  41]
 [103  51  92]
 [131 250  74]
 [148 108 141]
 [ 49  65 212]
 [210  94  64]
 [ 60  27  71]
 [ 25 119 234]
 [244 174 116]]
Compressed image (63 colors, Project 1: Color Compression)

<Figure size 432x288 with 0 Axes>
```

3. Comments

- When we increase the value of K, there’s a noticeable improvement in showing the image.
- The larger the value of K is, the clearer and sharper the image is shown and the longer it takes to finish the program.

4. Compare with KMeans algorithm in scikit-learn library

- Executing KMeans algorithm in scikit-learn library:

```
# compare results with KMeans in scikit-learn library
n_colors = 3

# Load the Summer Palace photo
pic_name = input("Enter image's name: ")
print("Filename: " + pic_name)

image = img.open(pic_name)
# Convert to floats instead of the default 8 bits integer coding. Dividing by
# 255 is important so that plt.imshow behaves works well on float data (need to
# be in the range [0-1])
image = np.array(image, dtype=np.float64) / 255

# Load Image and transform to a 2D numpy array.
w, h, d = original_shape = tuple(image.shape)
assert d == 3
image_array = np.reshape(image, (w * h, d))

print("Fitting model on a small sub-sample of the data")
t0 = time()
image_array_sample = shuffle(image_array, random_state=0, n_samples=1_000)
km = KMeans(n_clusters=n_colors, random_state=0).fit(image_array_sample)
print(f"done in {time() - t0:0.3f}s.")

# Get labels for all points
print("Predicting color indices on the full image (k-means)")
t0 = time()
labels = km.predict(image_array)
print(f"done in {time() - t0:0.3f}s.")

def recreate_image(codebook, labels, w, h):
    """Recreate the (compressed) image from the code book & labels"""
    return codebook[labels].reshape(w, h, -1)

print(f'Centroids: \n {km.cluster_centers_ * 255}')
plt.figure(2)
plt.clf()
plt.axis("off")
plt.title(f"Compressed image ({n_colors} colors, K-Means)")
plt.imshow(recreate_image(km.cluster_centers_, labels, w, h))
```

- K = 3:

```
Enter image's name: 1.jpg
Filename: 1.jpg
Fitting model on a small sub-sample of the data
done in 0.143s.
Predicting color indices on the full image (k-means)
done in 0.207s.
Centroids:
 [[114.87359551  90.10674157 133.38483146]
 [ 29.00877193  24.1754386   64.66666667]
 [196.92384106 154.59933775 168.01655629]]
```

Out[41]: <matplotlib.image.AxesImage at 0x25404d47df0>

Compressed image (3 colors, K-Means)

- K = 5:

```
Enter image's name: 1.jpg
Filename: 1.jpg
Fitting model on a small sub-sample of the data
done in 0.243s.
Predicting color indices on the full image (k-means)
done in 0.082s.
Centroids:
 [[ 91.80811808  79.29151292 128.6199262 ]
 [ 25.7980456   20.80130293  60.09120521]
 [154.68269231 134.79326923 172.19711538]
 [227.33018868 192.20754717 190.0754717 ]
 [196.44444444  99.53703704 102.52777778]]

Out[44]: <matplotlib.image.AxesImage at 0x25405be2d40>
```

Compressed image (5 colors, K-Means)

- K = 7:

```
Enter image's name: 1.jpg
Filename: 1.jpg
Fitting model on a small sub-sample of the data
done in 0.163s.
Predicting color indices on the full image (k-means)
done in 0.191s.
Centroids:
 [[ 13.81987578   9.80745342  41.30434783]
 [150.40935673 137.00584795 176.64912281]
 [135.42372881  56.52542373  84.61016949]
 [203.54621849 114.67226891 119.04201681]
 [227.22       194.82       192.61      ]
 [ 96.64903846  88.54807692 138.80288462]
 [ 40.13736264  39.26923077  88.98351648]]
```

```
Out[34]: <matplotlib.image.AxesImage at 0x1198b5dbb80>
```

Compressed image (7 colors, K-Means)

- K = 13:

```
Enter image's name: 1.jpg
Filename: 1.jpg
Fitting model on a small sub-sample of the data
done in 0.184s.
Predicting color indices on the full image (k-means)
done in 0.085s.
Centroids:
 [[191.08641975 134.82716049 155.43209877]
 [ 87.07142857  26.96428571  61.35714286]
 [ 90.64473684  87.07894737 139.34868421]
 [ 22.46527778  23.73611111  70.90972222]
 [157.25862069 159.86206897 201.93103448]
 [131.91954023 126.01149425 172.01149425]
 [139.22916667  62.60416667  91.33333333]
 [ 11.70408163   4.95918367  28.51020408]
 [237.46031746 186.3015873  166.77777778]
 [ 48.12765957  53.85106383 106.03191489]
 [218.51923077 105.59615385  93.42307692]
 [224.07407407 219.62962963 240.40740741]
 [147.54411765  99.39705882 131.82352941]]

Out[36]: <matplotlib.image.AxesImage at 0x1198c220b20>
```

Compressed image (13 colors, K-Means)

- K = 27:

```
Enter image's name: 1.jpg
Filename: 1.jpg
Fitting model on a small sub-sample of the data
done in 0.218s.
Predicting color indices on the full image (k-means)
done in 0.100s.
Centroids:
 [[ 67.58064516  36.5483871   79.80645161]
 [155.93023256 131.65116279 164.76744186]
 [192.38888889 195.72222222 231.88888889]
 [ 71.24615385  77.18461538 130.        ]
 [ 10.93939394  12.39393939  51.31818182]
 [179.4375     104.3125     120.09375   ]
 [229.09090909 166.         161.15151515]
 [ 98.0483871   99.19354839 155.80645161]
 [150.3902439  156.80487805 203.97560976]
 [124.48484848  61.78787879  99.54545455]
 [226.8125      91.9375      58.875     ]
 [193.63333333 153.56666667 173.16666667]
 [ 41.25        50.57352941 103.94117647]
 [109.72727273  18.90909091  43.27272727]
 [ 24.03846154  26.20192308  74.59615385]
 [ 43.17647059   5.41176471  18.76470588]
 [253.         232.33333333 135.55555556]
 [231.6        139.9        101.4       ]
 [137.10869565  97.89130435 135.34782609]
 [126.78846154 126.98076923 176.51923077]
 [240.30769231 234.53846154 247.84615385]
 [191.23809524 127.78571429 148.47619048]
 [218.72222222 101.27777778 112.22222222]
 [166.6875      63.75        75.        ]
 [  3.12244898   1.7755102   21.36734694]
 [ 99.62745098  81.88235294 129.21568627]
 [243.54166667 199.54166667 191.29166667]]
```

```
Out[39]: <matplotlib.image.AxesImage at 0x1198e74dba0>
```

Compressed image (27 colors, K-Means)

- K = 63:

```
Enter image's name: 1.jpg
Filename: 1.jpg
Fitting model on a small sub-sample of the data
done in 0.375s.
Predicting color indices on the full image (k-means)
done in 0.149s.
Centroids:
 [[141.6        146.9        193.35      ]
 [ 60.11764706  29.47058824  73.58823529]
 [205.46666667 139.26666667 155.4       ]
 [ 96.          93.875      153.29166667]
 [  3.46153846   1.26923077  12.38461538]
 [ 26.73333333  28.46666667  76.93333333]
 [167.2         84.          81.2       ]
 [249.44444444 240.         246.77777778]
 [ 49.          59.43478261 114.47826087]
 [241.41176471 198.88235294 198.88235294]
 [195.64705882 109.70588235 117.94117647]
 [138.66666667  99.55555556 136.33333333]
 [117.05555556  62.94444444 103.22222222]
 [229.5         64.25        71.        ]
 [ 15.81818182  13.13636364  44.72727273]
 [253.25       243.25       128.5       ]
 [247.28571429 193.         171.57142857]
 [ 73.75        79.25       133.20833333]
 [129.         129.48       181.68      ]
 [212.63636364 212.36363636 244.45454545]
 [166.52       141.64       170.36      ]
 [ 45.           3.27272727   7.54545455]
 [ 93.375       75.04166667 123.29166667]
 [172.7        173.5        199.1       ]
 [231.8        116.4        133.       ]
 [188.63636364 130.68181818 150.72727273]
 [ 94.11111111  22.44444444  52.77777778]
 [  2.93939394   3.42424242  35.       ]
 [126.52631579  89.36842105 128.42105263]
 [ 76.38461538  47.76923077  90.23076923]
```

```
[ 94.11111111  22.44444444  52.77777778]
[  2.93939394   3.42424242  35.        ]
[126.52631579  89.36842105 128.42105263]
[ 76.38461538  47.76923077  90.23076923]
[244.         159.33333333 107.66666667]
[143.63157895 130.68421053 168.21052632]
[248.         132.           9.        ]
[221.63636364 124.          92.81818182]
[157.5         42.5         51.5       ]
[219.375       89.5        109.625     ]
[ 41.75        49.53571429 102.53571429]
[131.25        12.5         32.5       ]
[150.68421053 159.10526316 210.57894737]
[198.57142857 155.92857143 179.92857143]
[181.         189.9        228.        ]
[  9.75757576  13.81818182  57.84848485]
[226.39130435 169.82608696 166.69565217]
[212.71428571 100.57142857  63.42857143]
[102.89285714 105.21428571 158.92857143]
[ 41.          10.125       43.375     ]
[234.44444444 154.55555556 146.        ]
[153.72222222 114.33333333 148.88888889]
[169.78571429  94.         119.92857143]
[ 17.82222222  21.77777778  70.02222222]
[252.         205.33333333 126.        ]
[151.09090909  69.18181818 101.54545455]
[247.          86.66666667  44.        ]
[106.31578947  87.47368421 131.94736842]
[ 32.09677419  39.4516129   90.41935484]
[253.66666667 246.66666667 169.        ]
[ 83.33333333  88.13333333 142.2       ]
[184.          58.66666667  65.        ]
[116.85       117.55       167.7       ]
[ 63.04347826  70.52173913 122.82608696]
[125.625       52.375       89.5       ]
[169.4        114.8        125.        ]
[177.30769231 115.53846154 144.53846154]]
```

```
Out[40]: <matplotlib.image.AxesImage at 0x1198c132fe0>
```

Compressed image (63 colors, K-Means)

- **Comment:**
  - Like our kmeans function, KMeans algorithm in scikit-learn library improve the quality of the image when we keep increasing the value of K aka the number of clusters.
  - Using only numpy library take us more time to handle the large matrices instead of using the others libraries (such as `Scipy`, `OpenCV`…).
  - The result (the value of centroids) of our kmeans is quite similar to scikit-learn ones.
  - The larger the value of K is, the more difference in the two results of those algorithms you get.

IV. References

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