VIETNAM NATRIONAL UNIVERSITY HO CHI MINH CITY UNIVERSITY OF SCIENCE FACULTY OF INFORMATION TECHNOLOGY



SUBJECT: Applied Mathematics and Statistics

CLASS 20CLC02

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Project 01: Color Compression

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I. General information

➤ Project 01: Color Compression

➤ Author: Phan Minh Phúc – Student ID: 20127063

> Environment: jupyter notebook

Programing language: python

II. Requirements

- 1. Project's requirement:
 - ➤ Install a program that reduces the number of colors in an image using the K-Means algorithm.
 - ➤ Provides a main () function that allows users to enter their image file's name and to select the saving format for their image's output including "png" and "pdf".
- 2. Report's requirement:

No.	Task	Status
1	Present your idea generally, describe all your defined functions	Done
2	For each value of k ($k = 3, 5, 7$), return image after being compressed corresponding to k . You can try more value of k to gain insight into this algorithm	Done
3	Make your own comments on those results	Done
4	Your report must be paginated and included the reference section	Done

III. Conclusion

- 1. Present idea and describe functions
- General idea:
 - ➤ Color images commonly used are RGB images, where each pixel stores 3 color channel information (1 byte for each color channel): R (red), G (green), B (blue). Each component can take a value between 0 and 255, where the tuple pixel (0, 0, 0) represents black and (255, 255, 255) represents white.
 - ➤ K-means is a clustering algorithm that is used to group data points into clusters (centroids) such that data points lying in the same group are very similar to each other in characteristics. K-means algorithm can be used to find subgroups in the image and assign the image pixel to that subgroup which results in image segmentation. We are





proactive in choosing k - the number of colors the image must represent.

- ➤ So, we're going to use K-means to present the initial image in the same size but only with K colors.
- Describe functions:
 - > Steps of K-means:
 - Step 1: Generate K centroids.
 - Step 2: Assign the image pixel to these centroids.
 - Step 3: Generate K new centroids which values equal the mean of that cluster i.
 - Step 4: Repeat until the results are convergent.
 - ➤ Declare libraries:

```
# Ho Chi Minh city University of Science (HCMUS)
# Author: Phan Minh Phuc
# Student ID: 20127063
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image as img
from sklearn.cluster import KMeans
from sklearn.metrics import pairwise_distances_argmin
from sklearn.utils import shuffle
from time import time
```

generate_centroid(img_1d, k_clusters, init_centroids):

```
def generate_centroid(img_1d, k_clusters, init_centroids):
    if init_centroids == 'in_pixels':
        return A[np.random.randint(A.shape[0], size = k_clusters), :]
    if init_centroids == 'random':
        return np.random.randint(256, size = (k_clusters, 3))
```

- Generate K centroids base on the value of init_centroids:
 - o "random" \rightarrow centroid has 3 channels, each channel is initial random in [0, 255].
 - o "in_pixels" → centroid is a random pixels of original image.
- classify_pixel(img_1d, centroid):



```
def classify_pixel(img_1d, centroid):
    # distance between each pixel and the 1st controid
    distance = np.linalg.norm(img_1d - centroid[0], axis = 1)
    # convert the array to shape (img_1d.shape[0], 1)
    distance = distance.reshape((img_1d.shape[0], 1))

# distance between each pixel and the rest controids
for i in range (1, centroid.shape[0]):
    temp = np.linalg.norm(img_1d - centroid[i], axis = 1)
    temp = temp.reshape((img_1d.shape[0], 1))
    # adding these distances to the initial distance array
    distance = np.concatenate((distance,temp),axis=1)

# return index of centroid nearest to each pixel
# return distance
return np.argmin(distance,axis = 1)
```

- Calculate the distance between each pixel and the 1st centroid, save to distance.
- Then, calculate the distance between each pixel and the rest centroids similarly.
- Once again, save these results to distance by using numpy.concatenate() this is the reason why you have to calculate the distance between each pixel and the 1st centroid before doing the same thing with the rest centroids.
- Return a list contains the index of centroid nearest to each pixel aka a list contains the index of centroid that each pixel belongs to.
- > optimize_centroid(img_1d, k_cluster, centroid):

```
def optimize centroid(img 1d, k cluster, centroid):
    label = classify_pixel(img_1d, centroid)
    is changed = False
   for i in range (k cluster):
       # choose all pixels belong to centroid[k]
       cluster = img_1d[label == i, :]
        # if no pixel belong to this centroid
        # => go on to the next centroid
        if len(cluster) == 0:
            continue
        # generate a new centroid from mean value of last chosen pixels
       new_centroid = np.mean(cluster, axis = 0)
        # i\overline{f} new centroid and centroid[k] is not convergent
        # => set centroid[k] with the new centroid's value
        if (not np.allclose(centroid[i], new centroid, atol = 1, equal nan = True)):
            centroid[i] = new_centroid
            is_changed = True
    return is changed, centroid, label
```





- Generate label to save the index of centroid that each pixel belongs to.
- Generate is_changed to check whether the centroids are changed and assign the value False to it.
- For each centroid, generate cluster contains all pixels belong to it, then
- If there isn't any pixel belong to this centroid, the algorithm will go on to the next centroid.
- Else, generate new_centroid which value equal the mean of this cluster
- If the difference in value of this centroid and the new_centroid is larger than the absolute tolerance parameter (atol) = 1, the function will update this centroid's value to new centroid's value and assign the value True to is_changed.
- Return whether the initial centroids are changed (is_changed), the current centroid values (centroid), and the current label (label).
- compress_pixel(img_1d, k_clusters, label, centroids):

```
def compress_pixel(img_1d, k_clusters, label, centroids):
    for i in range(k_clusters):
        img_1d[label == i, :] = centroids[i]
    return img_1d
```

- For each centroid in K centroids, change the value of all pixels belong to this centroid to its value.
- Return a matrix contain value of all pixels in the original image (img_1d) after being compressed.
- > kmeans(img_1d, k_clusters, max_iter, init_centroids = 'random'):

```
def kmeans(img_1d, k_clusters, max_iter, init_centroids = 'random'):
    centroid = generate_centroid(img_1d, k_clusters, init_centroids)
    label = []
    while max_iter > 0:
        stop_condition, centroid, label = optimize_centroid(img_1d, k_clusters, centroid)
        if stop_condition == False:
            break;
        max_iter -= 1;
    # print(label.shape)
    return centroid, label
```

- Generate centroid contains K centroid by using generate_centroid(img_1d, k_clusters, init_centroids).
- Generate label an empty list contains the index of centroid that each pixel belongs to.





- Start converging centroid by using optimize_centroid(img_1d, k_cluster, centroid) until the stop_condition is satisfied or max_iter is <= 0.
- Return the current centroid and label.

\triangleright main():

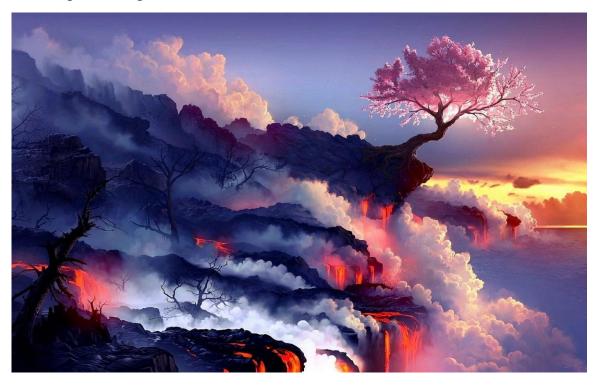
```
def main():
    filename = input("Enter image's name: ")
    print("Filename: " + filename)
    output_format = input("Enter image's output format ('png' or 'pdf'): ")
    print("Image's output format: " + output_format)
while output_format != "png" and output_format != "pdf":
        output_format = input("Enter image's output format ('png' or 'pdf'): ")
        print("Image's output format: " + output_format)
    img 1d = img.open(filename)
    # Convert the input to a matrix.
    img_1d = np.asarray(img_1d)
    # img 1d.shape[0]: number of rows
    # img 1d.shape[1]: number of columns
    # img_1d.shape[2]: number of channels
    row = img_1d.shape[0]
    column = img_1d.shape[1]
    # Reshape initial matrix to 1d array
    # img_1d.reshape(-1, 3) == img_1d.reshape(img_1d.shape[0] * img_1d.shape[1], img_1d.shape[])
    img 1d = img 1d.reshape(-1, 3)
    k clusters = 5
    max iter = 3000
    centroid, label = kmeans(img_1d, k_clusters, max_iter, init_centroids = 'random')
    compressed_img = compress_pixel(img_1d, k_clusters, label, centroid)
    compressed_img = compressed_img.reshape(row, column, 3)
    print(f"Number of clusters: {k_clusters}")
    print(f"Centroids: \n {centroid}")
    plt.clf()
    print(f"Compressed image ({k_clusters} colors, Project 1: Color Compression)")
    figure = plt.figure()
    figure.set size inches(column / 96, row / 96)
    plt.axis("off")
    plt.imshow(compressed_img)
    plt.savefig(f"compressed_img{k_clusters}." + output_format, dpi = 300)
```

- Generate filename, output_format and let users enter their image file's name and and select the saving format for their image's output including "png" and "pdf".
- Assign img_1d the result of opening image filename by using PIL.Image.open().
- Convert the img_1d into a matrix shape row × column × channel.
- Assign row the value of img_ld's row.
- Assign column the value of img_1d's column.
- Reshape initial matrix to 1d array by using numpy.reshape().
- Assign k_clusters respectively the value 3, 5, 7, 13, 27 and 63.
- Assign max_iter (max iterator) the value 3000.





- Assign centroid, label the result after executing kmeans(img_1d, k_clusters, max_iter, init_centroids = 'random').
- Assign compressed_img the result after executing compress_pixel(img_1d, k_clusters, label, centroids).
- Reshape compressed_img to the initial matrix's shape by using numpy.reshape().
- Use matplotlib.pyplot.figure() to create a new figure.
- Use matplotlib.pyplot.figure.set_size_inches() to set the figure size in inches. Row and Column are divide by 96 to convert from pixels to inches.
- Use matplotlib.pyplot.axis("off") to remove the axes and the plot borders completely.
- Show compressed_img by using matplotlib.pyplot.imshow() and save it as name "compressed_img{k_clusters}" (k_cluster: number of centroids) with output format output_format by using matplotlib.pyplot.savefig().
- 2. Testing result
- The original iamge:



- K = 3:











- K = 5:



```
In [57]: main()
         Enter image's name: 1.jpg
         Filename: 1.jpg
         Enter image's output format ('png' or 'pdf'): png
         Image's output format: png
         Number of clusters: 5
         Centroids:
           [[102 92 141]
           [ 28 22 61]
[187 105 119]
           [ 66 231 249]
           [201 174 185]]
         Compressed image (5 colors, Project 1: Color Compression)
          <Figure size 432x288 with 0 Axes>
```





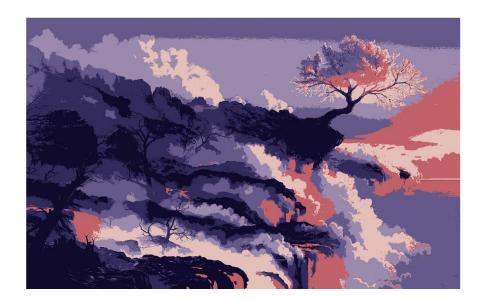
- K = 7:





```
In [61]: main()
         Enter image's name: 1.jpg
         Filename: 1.jpg
         Enter image's output format ('png' or 'pdf'): png
         Image's output format: png
         Number of clusters: 7
         Centroids:
          [[ 16 11 44]
          [ 42 217 107]
          [195 97 105]
          [229 192 184]
          [155 137 174]
          [101 89 138]
[51 43 92]]
         Compressed image (7 colors, Project 1: Color Compression)
         <Figure size 432x288 with 0 Axes>
```





- K = 13:





```
In [63]: main()
         Enter image's name: 1.jpg
         Filename: 1.jpg
         Enter image's output format ('png' or 'pdf'): png
         Image's output format: png
         Number of clusters: 13
         Centroids:
          [[ 42 40 90]
          [ 14 10 41]
          [246 206 152]
          [140 139 185]
          [224 217 237]
          [165 110 136]
          [ 6 96 217]
          [ 22 6 235]
          [ 83 231 79]
          [212 151 160]
          [219 94 85]
          94 88 140
          [126 55 88]]
         Compressed image (13 colors, Project 1: Color Compression)
         <Figure size 432x288 with 0 Axes>
```





- K = 27:



```
In [65]:
         main()
         Enter image's name: 1.jpg
         Filename: 1.jpg
         Enter image's output format ('png' or 'pdf'): png
         Image's output format: png
         Number of clusters: 27
         Centroids:
          [[ 25 215 18]
          [217 42 26]
          [178 99 117]
          [135 71 107]
             8
                5 30]
          [ 98 60 103]
          [ 91 89 143]
          [ 63 28 71]
          [167 68 250]
          [225 129 127]
          [151 51 70]
          71
                7 21]
          [ 68 194 83]
          [133 100 138]
          [ 58 191 227]
          [174 255 107]
          [154 159 202]
          [ 23 25 72]
          [224 91 73]
          [ 50 57 110]
          [126 125 174]
          [180 132 155]
          [251 224 142]
          [225 169 169]
          [ 51 167 255]
          [229 223 241]
          [ 73 196 75]]
         Compressed image (27 colors, Project 1: Color Compression)
```





- K = 63:

```
In [67]: main()
         Enter image's name: 1.jpg
         Filename: 1.jpg
         Enter image's output format ('png' or 'pdf'): png
         Image's output format: png
         Number of clusters: 63
         Centroids:
          [[108 67 254]
          [ 30 249 4]
          [186 149 169]
          [ 42 6 21]
          [230 80 94]
          [179 97 116]
          [216 121 103]
          [140 75 108]
          [ 65 173 191]
          [ 39 43 96]
          [219 158 162]
          [ 36 220 191]
            18 98 179]
```





```
In [67]: main()
          [ 18 98 179]
          [100 79 125]
          [235 181 170]
          [154 10 170]
          [217 38 31]
          [ 64 71 123]
          [ 79 162 171]
          [146 154 201]
          [151 53 217]
          [246 206 199]
          [168 127 156]
          [ 41 189 214]
          [ 68 228 54]
          [ 68 116 254]
          [161 60 78]
          [232 119 136]
          [138 211 27]
          [ 19 92 236]
          [252 238 138]
```

```
In [67]: main()
          [ 19 92 230]
          [252 238 138]
          [ 88 90 146]
          [ 2 234 142]
          [168 169 208]
          [248 240 247]
          [51 178 0]
          [ 3 96 183]
          [216 219 248]
          [123 96 136]
          [ 44 141
                   9]
          [139 13 21]
          [ 99 194 167]
          [ 90 16 40]
          [108 109 164]
          [196 127 143]
          [ 17 195 61]
          [190 192 227]
          [ 55 160 179]
          [ 26 190 160]
```



```
In [67]:
         main()
            26 190 160]
             5
                 4 28]
            49 143 250]
                49 187]
            67
            19
                21
                    67]
          [133 131 178]
          247
                88
                    41]
                    92]
           [103
                51
          [131 250
                   74]
           [148 108 141]
            49
                65 212]
           210
                94 64]
          [ 60 27 71]
            25 119 234]
          [244 174 116]]
         Compressed image (63 colors, Project 1: Color Compression)
         ⟨Figure size 432x288 with 0 Axes⟩
```



3. Comments

- When we increase the value of K, there's a noticeable improvement in showing the image.
- The larger the value of K is, the clearer and sharper the image is shown and the longer it takes to finish the program.





- 4. Compare with KMeans algorithm in scikit-learn library
- Executing KMeans algorithm in scikit-learn library:

```
# compare results with KMeans in scikit-learn library
n colors = 3
# Load the Summer Palace photo
pic_name = input("Enter image's name: ")
print("Filename: " + pic_name)
image = img.open(pic_name)
# Convert to floats instead of the default 8 bits integer coding. Dividing by
# 255 is important so that plt.imshow behaves works well on float data (need to
# be in the range [0-1])
image = np.array(image, dtype=np.float64) / 255
# Load Image and transform to a 2D numpy array.
w, h, d = original_shape = tuple(image.shape)
assert d == 3
image_array = np.reshape(image, (w * h, d))
print("Fitting model on a small sub-sample of the data")
t0 = time()
image array sample = shuffle(image array, random_state=0, n_samples=1_000)
km = KMeans(n clusters=n colors, random state=0).fit(image array sample)
print(f"done in {time() - t0:0.3f}s.")
# Get labels for all points
print("Predicting color indices on the full image (k-means)")
t0 = time()
labels = km.predict(image_array)
print(f"done in {time() - t0:0.3f}s.")
def recreate_image(codebook, labels, w, h):
    """Recreate the (compressed) image from the code book & labels"""
    return codebook[labels].reshape(w, h, -1)
print(f'Centroids: \n {km.cluster_centers_ * 255}')
plt.figure(2)
plt.clf()
plt.axis("off")
plt.title(f"Compressed image ({n_colors} colors, K-Means)")
plt.imshow(recreate_image(km.cluster_centers_, labels, w, h))
```

- K = 3:



Enter image's name: 1.jpg

Filename: 1.jpg

Fitting model on a small sub-sample of the data

done in 0.143s.

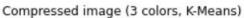
Predicting color indices on the full image (k-means)

done in 0.207s.

Centroids:

[[114.87359551 90.10674157 133.38483146] [29.00877193 24.1754386 64.66666667] [196.92384106 154.59933775 168.01655629]]

Out[41]: <matplotlib.image.AxesImage at 0x25404d47df0>





- K = 5:





Enter image's name: 1.jpg

Filename: 1.jpg

Fitting model on a small sub-sample of the data

done in 0.243s.

Predicting color indices on the full image (k-means)

done in 0.082s.

Centroids:

[[91.80811808 79.29151292 128.6199262]

[154.68269231 134.79326923 172.19711538]

[227.33018868 192.20754717 190.0754717]

[196.4444444 99.53703704 102.52777778]]

Out[44]: <matplotlib.image.AxesImage at 0x25405be2d40>

Compressed image (5 colors, K-Means)



- K = 7:





Enter image's name: 1.jpg Filename: 1.jpg Fitting model on a small sub-sample of the data done in 0.163s. Predicting color indices on the full image (k-means) done in 0.191s. Centroids: [[13.81987578 9.80745342 41.30434783] [150.40935673 137.00584795 176.64912281] [135.42372881 56.52542373 84.61016949] [203.54621849 114.67226891 119.04201681] 227.22 194.82 192.61 [96.64903846 88.54807692 138.80288462] [40.13736264 39.26923077 88.98351648]] Out[34]: <matplotlib.image.AxesImage at 0x1198b5dbb80> Compressed image (7 colors, K-Means)

- K = 13:





```
Enter image's name: 1.jpg
         Filename: 1.jpg
         Fitting model on a small sub-sample of the data
         done in 0.184s.
         Predicting color indices on the full image (k-means)
         done in 0.085s.
         Centroids:
          [[191.08641975 134.82716049 155.43209877]
          [ 87.07142857 26.96428571 61.35714286]
          [ 90.64473684 87.07894737 139.34868421]
           22.46527778 23.73611111 70.90972222
          [157.25862069 159.86206897 201.93103448]
          [131.91954023 126.01149425 172.01149425]
          [139.22916667 62.60416667 91.33333333]
                          4.95918367 28.51020408]
           11.70408163
          [237.46031746 186.3015873 166.77777778]
          [ 48.12765957 53.85106383 106.03191489]
          [218.51923077 105.59615385 93.42307692]
          [224.07407407 219.62962963 240.40740741]
          [147.54411765 99.39705882 131.82352941]]
Out[36]: <matplotlib.image.AxesImage at 0x1198c220b20>
```

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- K = 27:





```
Enter image's name: 1.jpg
Filename: 1.jpg
Fitting model on a small sub-sample of the data
done in 0.218s.
Predicting color indices on the full image (k-means)
done in 0.100s.
Centroids:
 [ 67.58064516 36.5483871
                             79.80645161
 [155.93023256 131.65116279 164.76744186]
 [192.38888889 195.7222222 231.88888889]
 71.24615385 77.18461538 130.
 [ 10.93939394 12.39393939 51.31818182]
              104.3125
                           120.09375
 [179.4375
 [229.09090909 166.
                           161.15151515]
 98.0483871
               99.19354839 155.80645161]
 [150.3902439 156.80487805 203.97560976]
 [124.48484848 61.78787879 99.54545455]
               91,9375
 [226.8125
                            58.875
 [193.63333333 153.56666667 173.16666667]
               50.57352941 103.94117647]
 41.25
               18.90909091 43.27272727
 [109.72727273
 43,17647059
                5.41176471 18.76470588
              232.33333333 135.55555556]
 [253.
 [231.6
              139.9
                           101.4
 [137.10869565 97.89130435 135.34782609]
 [126.78846154 126.98076923 176.51923077]
 [240.30769231 234.53846154 247.84615385]
 [191.23809524 127.78571429 148.47619048]
 [218.72222222 101.27777778 112.22222222]
 [166.6875]
               63.75
                            75.
   3.12244898
                1.7755102
                            21.36734694]
 [ 99.62745098 81.88235294 129.21568627]
 [243.54166667 199.54166667 191.29166667]]
```





- K = 63:





```
Enter image's name: 1.jpg
Filename: 1.jpg
Fitting model on a small sub-sample of the data
done in 0.375s.
Predicting color indices on the full image (k-means)
done in 0.149s.
Centroids:
 [[141.6
               146.9
                           193.35
 [ 60.11764706 29.47058824 73.58823529]
 [205.46666667 139.26666667 155.4
 96.
               93.875
                          153.29166667]
   3.46153846
                1.26923077 12.38461538]
 [167.2]
               84.
                           81.2
 [249.4444444 240.
                          246.77777778]
 [ 49.
               59.43478261 114.47826087
 [241.41176471 198.88235294 198.88235294]
 [195.64705882 109.70588235 117.94117647]
 [138.66666667
              99.55555556 136.333333333
 [117.05555556 62.94444444 103.22222222]
 [229.5
               64.25
                           71.
 [253.25
              243.25
                          128.5
 [247.28571429 193.
                          171.57142857]
 73.75
               79.25
                          133,208333331
 [129.
              129.48
                          181.68
 [212.63636364 212.36363636 244.45454545]
 166.52
              141.64
                          170.36
                3.27272727
                            7.54545455]
 45.
               75.04166667 123.29166667]
 93.375
              173.5
                          199.1
 [172.7
              116.4
                          133.
 [188.63636364 130.68181818 150.72727273]
  94.11111111
               22.4444444
                           52.77777778]
   2.93939394
                           35.
                3.42424242
 [126.52631579 89.36842105 128.42105263]
  76.38461538 47.76923077 90.23076923]
```





```
2.93939394
                3.42424242
                            35.
[126.52631579
              89.36842105 128.42105263]
[ 76.38461538 47.76923077 90.23076923]
             159.3333333 107.66666667]
[143.63157895 130.68421053 168.21052632]
248.
             132.
                            9.
[221.63636364 124.
                            92.818181821
              42.5
[157.5]
                            51.5
219.375
              89.5
                           109.625
              49.53571429 102.53571429]
41.75
              12.5
[131.25]
                            32.5
[150.68421053 159.10526316 210.57894737]
[198.57142857 155.92857143 179.92857143]
             189.9
                          228.
  9.75757576 13.81818182 57.84848485]
[226.39130435 169.82608696 166.69565217]
[212.71428571 100.57142857 63.42857143]
[102.89285714 105.21428571 158.92857143]
               10.125
41.
                           43.375
[234.44444444 154.55555556 146.
[153.7222222 114.33333333 148.88888889]
[169.78571429 94.
                          119.92857143
[ 17.82222222 21.77777778 70.02222222]
[252.
              205.33333333 126.
[151.09090909 69.18181818 101.54545455]
              86.66666667 44.
247.
[106.31578947
              87.47368421 131.94736842]
32.09677419 39.4516129
                           90.41935484]
[253.66666667 246.66666667 169.
83.33333333 88.13333333 142.2
[184.
              58.66666667 65.
[116.85]
             117.55
                           167.7
[ 63.04347826 70.52173913 122.82608696]
125.625
              52.375
                           89.5
[169.4]
             114.8
                           125.
[177.30769231 115.53846154 144.53846154]]
```





- Comment:

- ➤ Like our kmeans function, KMeans algorithm in scikit-learn library improve the quality of the image when we keep increasing the value of K aka the number of clusters.
- ➤ Using only numpy library take us more time to handle the large matrices instead of using the others libraries (such as Scipy, OpenCV...).
- The result (the value of centroids) of our kmeans is quite similar to scikit-learn one.
- ➤ The larger the value of K is, the more difference in the two results of those algorithms you get.
- As you can see increasing the value of K causes the value differential between the centroids return by our kmeans function and scikit-learn library's KMeans.
- ➤ This happens because of rounding the result returned by numpy.mean() in optimize_centroid function and the value of max_iter in kmeans function it limits the number of times that optimize_centroid is executed which means when K is larger, executing optimize_centroid max_iter times is not enough to converge or to optimize centroids.



IV. References

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