caseproject-assignment

September 18, 2023

[1]:
```
import pandas as pd
import numpy as np
import math
import matplotlib.pyplot as plt
import statsmodels.api as sm
from patsy import dmatrices
from scipy import stats
```

0.0.1 Retrieve Data

[2]:
```
gdp_data = pd.read_excel('GDP.xls')
gdp_data['GDP_L1'] = gdp_data['GDP'].shift(1)
gdp_data['li1_L1'] = gdp_data['li1'].shift(1)
gdp_data['li2_L1'] = gdp_data['li2'].shift(1)
gdp_data['li1_L2'] = gdp_data['li1'].shift(2)
gdp_data['li2_L2'] = gdp_data['li2'].shift(2)
gdp_data['GrowthRate_L1'] = gdp_data['GrowthRate'].shift(1)
gdp_data.head()
```

[2]:
```
     Date      GDP  GDPIMPR    LOGGDP  GrowthRate  li1  li2  T  GDP_L1  \
0  1950Q1   94.300      NaN  4.546481         NaN    0    0  0     NaN
1  1950Q2   95.200      1.0  4.555980    0.009499    0    0  1  94.300
2  1950Q3   97.663      1.0  4.581523    0.025543    3    1  2  95.200
3  1950Q4   99.728      1.0  4.602446    0.020924    4    2  3  97.663
4  1951Q1  100.445      1.0  4.609610    0.007164    2    1  4  99.728

   li1_L1  li2_L1  li1_L2  li2_L2  GrowthRate_L1
0     NaN     NaN     NaN     NaN            NaN
1     0.0     0.0     NaN     NaN            NaN
2     0.0     0.0     0.0     0.0       0.009499
3     3.0     1.0     0.0     0.0       0.025543
4     4.0     2.0     3.0     1.0       0.020924
```

0.1 Part A

[3]:
```
#Setting Start and End For model data
start = '1951Q1'
end = '2010Q4'
```

[4]:
```
data_a = gdp_data[['Date', 'li1_L1', 'li2_L1', 'GDPIMPR']]
data_a = data_a[(data_a['Date']>=start)&(data_a['Date']<=end)]
```

[5]:
```
yA_full, xA_full = dmatrices('GDPIMPR~li1_L1+li2_L1', data_a)
```

[6]:
```
model_A_full = sm.Logit(endog=yA_full, exog=xA_full).fit()
model_A_full.summary2()
```

```
Optimization terminated successfully.
         Current function value: 0.559076
         Iterations 5
```

[6]:
```
<class 'statsmodels.iolib.summary2.Summary'>
"""
                         Results: Logit
=================================================================
Model:              Logit            Pseudo R-squared: 0.122
Dependent Variable: GDPIMPR          AIC:              274.3565
Date:               2023-09-17 21:26 BIC:              284.7984
No. Observations:   240              Log-Likelihood:   -134.18
Df Model:           2                LL-Null:          -152.76
Df Residuals:       237              LLR p-value:      8.4833e-09
Converged:          1.0000           Scale:            1.0000
No. Iterations:     5.0000
------------------------------------------------------------------
               Coef.  Std.Err.    z    P>|z|   [0.025   0.975]
------------------------------------------------------------------
Intercept     0.7288    0.1536  4.7454 0.0000  0.4278   1.0298
li1_L1       -0.3719    0.0727 -5.1176 0.0000 -0.5143  -0.2294
li2_L1       -0.1203    0.0377 -3.1936 0.0014 -0.1941  -0.0465
=================================================================

"""
```

[7]:
```
yA_li1, xA_li1 = dmatrices('GDPIMPR~li1_L1', data_a)
yA_li2, xA_li2 = dmatrices('GDPIMPR~li2_L1', data_a)
```

[8]:
```
model_A_li1 = sm.Logit(endog=yA_li1, exog=xA_li1).fit()
model_A_li2 = sm.Logit(endog=yA_li2, exog=xA_li2).fit()
```

```
Optimization terminated successfully.
         Current function value: 0.582277
```

```
         Iterations 5
Optimization terminated successfully.
         Current function value: 0.623002
         Iterations 5
```

[9]:
```
model_A_li1.summary2()
```

[9]:
```
<class 'statsmodels.iolib.summary2.Summary'>
"""
                         Results: Logit
=================================================================
Model:              Logit            Pseudo R-squared: 0.085
Dependent Variable: GDPIMPR          AIC:              283.4932
Date:               2023-09-17 21:26 BIC:              290.4544
No. Observations:   240              Log-Likelihood:   -139.75
Df Model:           1                LL-Null:          -152.76
Df Residuals:       238              LLR p-value:      3.3552e-07
Converged:          1.0000           Scale:            1.0000
No. Iterations:     5.0000
------------------------------------------------------------------
               Coef.   Std.Err.     z     P>|z|    [0.025   0.975]
------------------------------------------------------------------
Intercept      0.8120    0.1509   5.3831  0.0000   0.5164   1.1077
li1_L1        -0.3404    0.0718  -4.7387  0.0000  -0.4812  -0.1996
=================================================================

"""
```

[10]:
```
model_A_li2.summary2()
```

[10]:
```
<class 'statsmodels.iolib.summary2.Summary'>
"""
                        Results: Logit
===============================================================
Model:              Logit            Pseudo R-squared: 0.021
Dependent Variable: GDPIMPR          AIC:              303.0409
Date:               2023-09-17 21:26 BIC:              310.0022
No. Observations:   240              Log-Likelihood:   -149.52
Df Model:           1                LL-Null:          -152.76
Df Residuals:       238              LLR p-value:      0.010874
Converged:          1.0000           Scale:            1.0000
No. Iterations:     5.0000
----------------------------------------------------------------
             Coef.   Std.Err.     z     P>|z|    [0.025   0.975]
----------------------------------------------------------------
Intercept    0.6361    0.1395   4.5613  0.0000   0.3628   0.9094
li2_L1      -0.0865    0.0349  -2.4759  0.0133  -0.1550  -0.0180
```

```
=================================================================

"""
```

[11]:
```python
# This is in the question table as well.
const_ll = model_A_full.llnull
full_ll = model_A_full.llf
li1_ll = model_A_li1.llf
li2_ll = model_A_li2.llf
```

[12]:
```python
def likelihood_ratio(logL_b0, llgL_b1):
    return 2.0*(logL_b0- llgL_b1)
```

[13]:
```python
logL_ratio_li1 = likelihood_ratio(full_ll,li1_ll)
logL_ratio_li2 = likelihood_ratio(full_ll,li2_ll)
logL_ratio_const = likelihood_ratio(full_ll, const_ll)
```

[14]:
```python
logL_ratio_li1, logL_ratio_li2, logL_ratio_const
```

[14]:
```
(11.136692691751932, 30.68446592235142, 37.17033387318395)
```

[15]:
```python
#The chi-squared probability of getting a log-likelihood ratio statistic␣
↪greater than llr.
#llr has a chi-squared distribution with degrees of freedom df_model.
model_A_li1.llr_pvalue, model_A_li2.llr_pvalue,  model_A_full.llr_pvalue
```

[15]:
```
(3.355197087485361e-07, 0.01087354332878751, 8.483294700541989e-09)
```

0.1.1 Summary Answer A

[16]:
```python
print("Answer A:")
print(f"Likelihood Ratio of li1 is {round(logL_ratio_li1, 3)}, p_value of the␣
↪model with li1 is {round(model_A_li1.llr_pvalue, 3)}")
print(f"Likelihood Ratio of li2 is {round(logL_ratio_li2,3)}, p_value of the␣
↪model with li2 is {round(model_A_li2.llr_pvalue, 3)}")
print(f"Likelihood Ratio of constant is {round(logL_ratio_const , 3)}")
print(f"p_value of the full model with li1 and li2 is {round(model_A_full.
↪llr_pvalue, 3)}")
print(f"Based on the p-vlaues, full model and li1 model are significant at 1%␣
↪level and li2 model is significant at 5% level.")
```

```
Answer A:
Likelihood Ratio of li1 is 11.137, p_value of the model with li1 is 0.0
Likelihood Ratio of li2 is 30.684, p_value of the model with li2 is 0.011
Likelihood Ratio of constant is 37.17
p_value of the full model with li1 and li2 is 0.0
```

Based on the p-vlaues, full model and li1 model are significant at 1% level and li2 model is significant at 5% level.

0.2 Part B

[17]:
```
# From the question we can get the likelihood of each model
const_ll = -152.763
model1_ll = -134.178
model2_ll = -134.126
model3_ll = -130.346
model4_ll = -130.461
```

[18]:
```
def McFadden_R2(logL_b, logL_b1):
    return 1 - logL_b/logL_b1
```

[19]:
```
Model_McFadden_R2 = []
for i in [model1_ll, model2_ll, model3_ll, model4_ll]:
    Model_McFadden_R2.append(McFadden_R2(i, const_ll))
```

[20]:
```
Model_MFR2_df = pd.DataFrame(np.array([Model_McFadden_R2]), columns =
 ↪["model1", "model2", "model3", "model4"])
```

[21]:
```
Model_MFR2_df
```

[21]:

	model1	model2	model3	model4
0	0.121659	0.121999	0.146744	0.145991

0.2.1 Summary Answer B

[22]:
```
print("Based on the McFadden_R2, the optimal model is model 3, which uses
 ↪constatnt + li1(-2) + li2(-1).")
```

Based on the McFadden_R2, the optimal model is model 3, which uses constatnt + li1(-2) + li2(-1).

0.3 Part C

[23]:
```
data_c = gdp_data[['Date', 'li1_L2', 'li2_L1', 'GDPIMPR']]
train_c = data_c[(data_c['Date']>=start)&(data_c['Date']<=end)]
predict_c = data_c[data_c['Date']>end]
```

[24]:
```
yC_train, xC_train = dmatrices('GDPIMPR~li1_L2+li2_L1', train_c)
yC_predict, xC_predict = dmatrices('GDPIMPR~li1_L2+li2_L1', predict_c)
```

[25]:
```
model_C = sm.Logit(endog=yC_train, exog=xC_train).fit()
model_C.summary2()
```

```
Optimization terminated successfully.
         Current function value: 0.543106
         Iterations 6
```

[25]:

```
<class 'statsmodels.iolib.summary2.Summary'>
"""
                         Results: Logit
=================================================================
Model:              Logit            Pseudo R-squared: 0.147
Dependent Variable: GDPIMPR          AIC:              266.6909
Date:               2023-09-17 21:26 BIC:              277.1328
No. Observations:   240              Log-Likelihood:   -130.35
Df Model:           2                LL-Null:          -152.76
Df Residuals:       237              LLR p-value:      1.8366e-10
Converged:          1.0000           Scale:            1.0000
No. Iterations:     6.0000
------------------------------------------------------------------
               Coef.   Std.Err.     z     P>|z|    [0.025   0.975]
------------------------------------------------------------------
Intercept      0.7457    0.1573   4.7397  0.0000   0.4373   1.0540
li1_L2        -0.4287    0.0763  -5.6175  0.0000  -0.5783  -0.2791
li2_L1        -0.1312    0.0386  -3.3994  0.0007  -0.2068  -0.0556
=================================================================

"""
```

[26]:

```
prediction_C = model_C.predict(xC_predict)
pred_table_C = model_C.pred_table(threshold=0.5) # This is for logistic␣
 ↪regrestion threshold to determie 0 or 1.
```

[27]:

```
pred_ratio = pred_table_C/pred_table_C.sum()
r_sum = np.sum(pred_table_C, axis=1)
```

[28]:

```
pred_rtable = pd.DataFrame(pred_ratio, columns=["y_hat=0", "y_hat=1"])
pred_rtable["Sum"] = r_sum
pred_rtable.index = ['y=0', 'y=1']
hit_rate = pred_ratio[0][0] + pred_ratio[1][1]
```

0.3.1 Summary Answer C

[29]:

```
print("Prediction Realization Model.")
print(pred_rtable)
print(f"hit rate is {round(hit_rate, 5)}")
```

```
Prediction Realization Model.
      y_hat=0  y_hat=1   Sum
y=0  0.133333   0.2000  80.0
```

```
y=1  0.104167   0.5625  160.0
hit rate is 0.69583
```

0.4 Part D

[30]:
```python
data_d = gdp_data[['Date', 'LOGGDP']]
xD = data_d[(data_d['Date']>=start)&(data_d['Date']<=end)]
```

[31]:
```python
# https://www.statsmodels.org/dev/generated/statsmodels.tsa.stattools.adfuller.
↪html
loggdp_ADF = sm.tsa.stattools.adfuller(xD['LOGGDP'], maxlag=1, autolag=None,␣
↪regression='ct', regresults=True)
```

[32]:
```python
adf_statistic = loggdp_ADF[0]
p_value = loggdp_ADF[1]
Critical_values = loggdp_ADF[2]
loggdp_model = loggdp_ADF[3].resols
adf_summary = loggdp_ADF[3].resols.summary2(xname=['LOGGDP Lag=1', 'Delta␣
↪LOGGDP Lag=1', 'Constant', 'Trend'])
```

[33]:
```python
loggdp_params = pd.DataFrame(np.array([loggdp_model.params]), columns=['LOGGDP␣
↪Lag=1', 'Delta LOGGDP Lag=1', 'Constant', 'Trend'])
```

[34]:
```python
adf_summary
```

[34]:
```
<class 'statsmodels.iolib.summary2.Summary'>
"""
                  Results: Ordinary least squares
===================================================================
Model:              OLS              Adj. R-squared:     0.393
Dependent Variable: y                AIC:                -1888.3395
Date:               2023-09-17 21:26 BIC:                -1874.4504
No. Observations:   238              Log-Likelihood:     948.17
Df Model:           3                F-statistic:        52.20
Df Residuals:       234              Prob (F-statistic): 7.23e-26
R-squared:          0.401            Scale:              2.0630e-05
-------------------------------------------------------------------
                     Coef.  Std.Err.    t    P>|t|   [0.025  0.975]
-------------------------------------------------------------------
LOGGDP Lag=1        -0.0204   0.0081 -2.5182 0.0125 -0.0364 -0.0044
Delta LOGGDP Lag=1   0.6325   0.0509 12.4338 0.0000  0.5323  0.7328
Constant             0.0956   0.0375  2.5512 0.0114  0.0218  0.1695
Trend                0.0001   0.0000  2.4979 0.0132  0.0000  0.0001
-------------------------------------------------------------------
Omnibus:              23.009       Durbin-Watson:          2.012
Prob(Omnibus):        0.000        Jarque-Bera (JB):       36.479
Skew:                 0.578        Prob(JB):               0.000
```

```
Kurtosis:                   4.530   Condition No.:              23965
===================================================================
* The condition number is large (2e+04). This might indicate
strong multicollinearity or other numerical problems.
"""
```

0.4.1 Summary Answer D

[35]:
```
print('ADF Test:')
print(f"The coefficients are shown in table")
print(loggdp_params)
print(f"The test statistic is {round(adf_statistic, 5)}")
print(f"Confidence Level is {Critical_values}.")
print(f"P-Value is {round(p_value, 5)}, which is away from 10% level of␣
↪significance.")
```

```
ADF Test:
The coefficients are shown in table
   LOGGDP Lag=1  Delta LOGGDP Lag=1  Constant     Trend
0     -0.020406            0.632529  0.095629  0.000064
The test statistic is -2.51821
Confidence Level is {'1%': -3.9973200578432064, '5%': -3.4290999471622556,
'10%': -3.1379848180498104}.
P-Value is 0.31892, which is away from 10% level of significance.
```

[36]:
```
gdp_data.head()
```

[36]:
```
     Date      GDP  GDPIMPR    LOGGDP  GrowthRate  li1  li2  T  GDP_L1  \
0  1950Q1   94.300      NaN  4.546481         NaN    0    0  0     NaN
1  1950Q2   95.200      1.0  4.555980    0.009499    0    0  1  94.300
2  1950Q3   97.663      1.0  4.581523    0.025543    3    1  2  95.200
3  1950Q4   99.728      1.0  4.602446    0.020924    4    2  3  97.663
4  1951Q1  100.445      1.0  4.609610    0.007164    2    1  4  99.728

   li1_L1  li2_L1  li1_L2  li2_L2  GrowthRate_L1
0     NaN     NaN     NaN     NaN            NaN
1     0.0     0.0     NaN     NaN            NaN
2     0.0     0.0     0.0     0.0       0.009499
3     3.0     1.0     0.0     0.0       0.025543
4     4.0     2.0     3.0     1.0       0.020924
```

0.5 Part E

[37]:
```
data_e = gdp_data[['Date', 'GrowthRate', 'GrowthRate_L1', 'li1_L1', 'li1_L2',␣
↪'li2_L1', 'li2_L2']]
data_e= data_e[(data_e['Date']>=start)&(data_e['Date']<=end)]
```

[38]:
```
yE_v1, xE_v1 = dmatrices("GrowthRate ~ GrowthRate_L1 + li1_L1 + li2_L1", data_e)
yE_v2, xE_v2 = dmatrices("GrowthRate ~ GrowthRate_L1 + li1_L2 + li2_L1", data_e)
yE_v3, xE_v3 = dmatrices("GrowthRate ~ GrowthRate_L1 + li1_L1 + li2_L2", data_e)
yE_v4, xE_v4 = dmatrices("GrowthRate ~ GrowthRate_L1 + li1_L2 + li2_L2", data_e)
```

[39]:
```
model_e_v1 = sm.OLS(yE_v1, xE_v1).fit()
model_e_v2 = sm.OLS(yE_v2, xE_v2).fit()
model_e_v3 = sm.OLS(yE_v3, xE_v3).fit()
model_e_v4 = sm.OLS(yE_v4, xE_v4).fit()
```

[40]:
```
model_e_rsqured = [model_e_v1.rsquared, model_e_v1.rsquared, model_e_v1.
 ↪rsquared, model_e_v1.rsquared]
model_e_df = pd.DataFrame(np.array([model_e_rsqured]),
        columns=["li1_Lag1 + li2_Lag1", "li1_Lag2 + li2_Lag1", "li1_Lag1 +␣
 ↪li2_Lag2", "li1_Lag2 + li2_Lag2"])
model_e_df.index = ["R_Square"]
model_e_v1_params = pd.DataFrame(np.array([np.around(model_e_v1.params, 6)]),␣
 ↪columns = ["Constant", "GrowthRate_L1", "li1_L1", "li2_L1"])
```

0.5.1 Summary Answer E

[41]:
```
print("R Square of 4 models are shown as below:")
print(model_e_df)
print("We can see model with li1 Lag = 1 and li2 Lag =1 has the largest r␣
 ↪square.")
print("The Coefficients of this model are:")
print(model_e_v1_params)
```

```
R Square of 4 models are shown as below:
          li1_Lag1 + li2_Lag1  li1_Lag2 + li2_Lag1  li1_Lag1 + li2_Lag2  \
R_Square             0.507975             0.507975             0.507975

          li1_Lag2 + li2_Lag2
R_Square             0.507975
We can see model with li1 Lag = 1 and li2 Lag =1 has the largest r square.
The Coefficients of this model are:
   Constant  GrowthRate_L1    li1_L1    li2_L1
0  0.001737       0.461579 -0.001023 -0.000149
```

0.6 Part F

0.6.1 Breusch-Godfrey

- Test if residual is autocorrelated or white noise.
- https://www.statsmodels.org/dev/generated/statsmodels.stats.diagnostic.acorr_breusch_godfrey.html

[42]:
```
model_BG = sm.stats.diagnostic.acorr_breusch_godfrey(model_e_v1, nlags=1,␣
 ↪store=True)
```

[43]:
```
BG_params = model_BG[:4]
BG_df = pd.DataFrame(np.array([BG_params]), columns=["BG Statistic", "BG␣
↪P-value", "BG F-value", "P-Value of f-test"])
BG_df.index = ["BG Test Result"]
```

[44]:
```
BG_df
```

[44]:

	BG Statistic	BG P-value	BG F-value	P-Value of f-test
BG Test Result	0.230366	0.631253	0.225783	0.63511

0.6.2 Summary Answer F

[45]:
```
print(f"Based on Breuch-Godfrey test: ")
print("given high p-value, we cannot reject null hypothesis that there is no␣
↪autocorrelation of residual.")
print("Therefore, there is no misspecification.")
```

```
Based on Breuch-Godfrey test:
given high p-value, we cannot reject null hypothesis that there is no
autocorrelation of residual.
Therefore, there is no misspecification.
```

0.7 Part G

[46]:
```
data_g = gdp_data[['Date', 'GrowthRate', 'GrowthRate_L1', 'li1_L1', 'li2_L1']]
train_g= data_g[(data_g['Date']>=start)&(data_g['Date']<=end)]
test_g = data_g[data_g['Date']>end]
test_g.set_index("Date", inplace=True)
```

[47]:
```
def rmse(pred, target):
    diff_sq = (np.array(pred)-np.array(target))**2
    ret = np.sqrt(diff_sq.mean())
    return ret
```

[48]:
```
yG_target, xG_test = dmatrices("GrowthRate ~ GrowthRate_L1 + li1_L1 + li2_L1",␣
↪test_g)
```

[49]:
```
yG_pred = model_e_v1.predict(xG_test)
```

[50]:
```
model_rmse = rmse(yG_pred, yG_target)
```

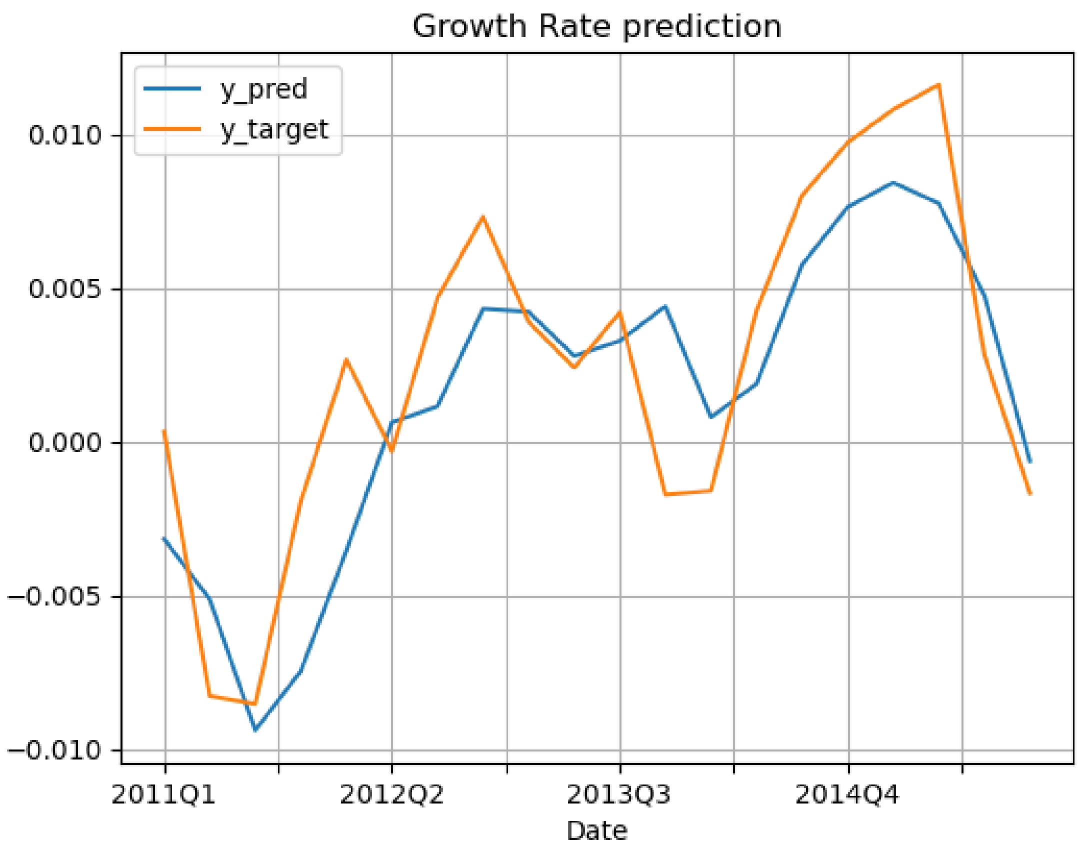
[51]:
```
pred_df = pd.DataFrame(yG_pred, columns = ["y_pred"], index = test_g.index)
```

[52]:
```
pred_df["y_target"] = yG_target
```

0.7.1 Summary Answer G

[53]:
```python
print(f"root mean square error for these forcast is {round(model_rmse, 6)}.")
print("Graph is shown in below:")
fig, ax = plt.subplots(1,1)
pred_df.plot(ax=ax, title="Growth Rate prediction")
ax.grid()
```

```
root mean square error for these forcast is 0.007383.
Graph is shown in below:
```



[ ]: