a) $AIC_0 = \log S_0^2 + \frac{2p_0}{n}$, $AIC_1 = \log S_1^2 + \frac{2p_1}{n}$

$$\frac{S_0^2}{S_1^2} < e^{\frac{2}{n}(p_1 - p_0)}$$

$$\log \frac{S_0^2}{S_1^2} < \frac{2}{n}(p_1 - p_0)$$

$$\log S_0^2 - \log S_1^2 < \frac{2p_1}{n} - \frac{2p_0}{n}$$

$$\log S_0^2 + \frac{2p_0}{n} < \log S_1^2 + \frac{2p_1}{n}$$

$$AIC_0 < AIC_1$$

Therefore, smallest model with smallest AIC is prefered

As $\lim_{x \to \infty} e^x = 1 + x$

b) $$\lim_{n \to \infty} e^{\frac{2}{n}(p_1 - p_0)} = 1 + \frac{2}{n}(p_1 - p_0)$$

then, $$\frac{S_0^2}{S_1^2} < 1 + \frac{2}{n}(p_1 - p_0)$$

$$\frac{S_0^2}{S_1^2} - 1 < \frac{2}{n}(p_1 - p_0)$$

$$\frac{S_0^2 - S_1^2}{S_1^2} < \frac{2}{n}(p_1 - p_0)$$

c) As $S_0^2 = e_R^T e_R$, $S_1^2 = e_U^T e_U$

then $$\frac{S_0^2 - S_1^2}{S_1^2} = \frac{e_R^T e_R - e_U^T e_U}{e_U^T e_U} < \frac{2}{n}(p_1 - p_0)$$

d) As $F = \dfrac{(e_R^T e_R - e_U^T e_U)/g}{e_U^T e_U/(n-k)}$

and $F < 2$, $g = p_1 - p_0$, $k = p_1$, n is larger number

then $\dfrac{(e_R^T e_R - e_U^T e_U)/(p_1 - p_0)}{(e_U^T e_U)/(n-p_1)} < 2$

$$\frac{e_R^T e_R - e_U^T e_U}{e_U^T e_U} < \frac{2}{n-p_1}(p_1 - p_0) \approx \frac{2}{n}(p_1 - p_0)$$

therefore, inequality from c) is approximately equivalent to F-test with critical value 2.