a) The variable d_i is not stochastic, Whether a person ~~takes~~ takes diet ~~could~~ could be correlated to other omitted variables, for example, the motivation of following diet or exposure to a ~~region~~ region which diet was advatised.

b) i) $\frac{1}{n}Z^T\varepsilon \to 0$, means "whether or not a person is living in a region where the diet was advertised" is not correlated with the error term ε.

ii) $\frac{1}{n}Z^TX \to Q \neq 0$, Z and X are correlated

c) No, ~~but~~ we also need know whether Z is multicollinear, which is $\frac{1}{n}Z^TZ \to Q_{ZZ}$, and Q_{ZZ} invertible.

d) We have 2SLS estimators $= (Z^T X)^{-1} Z^T y = ①$

and
$$Z = \begin{pmatrix} 1 & z_1 \\ \vdots & \vdots \\ 1 & z_i \\ \vdots & \vdots \\ 1 & z_n \end{pmatrix} \quad X = \begin{pmatrix} 1 & d_1 \\ \vdots & \vdots \\ & d_i \\ \vdots & \vdots \\ 1 & d_n \end{pmatrix} \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{pmatrix}$$

$$\rightarrow ① = \begin{pmatrix} n & \Sigma d_i \\ \Sigma z_i & \Sigma d_i z_i \end{pmatrix}^{-1} \begin{pmatrix} \Sigma y_i \\ \Sigma z_i y_i \end{pmatrix} = ②$$

As $A^{-1} = \frac{adj(A)}{det(A)}$

$$\rightarrow ② = \frac{1}{n \Sigma d_i z_i - \Sigma z_i \Sigma d_i} \begin{pmatrix} \Sigma d_i z_i & -\Sigma d_i \\ -\Sigma z_i & n \end{pmatrix} \begin{pmatrix} \Sigma y_i \\ \Sigma z_i y_i \end{pmatrix}$$

$$\rightarrow \hat{\beta} = \frac{n \Sigma z_i y_i - \Sigma z_i \Sigma y_i}{n \Sigma d_i z_i - \Sigma z_i \Sigma d_i}$$

$$= \frac{\Sigma z_i y_i - \frac{1}{n} \Sigma z_i \Sigma y_i}{\Sigma d_i z_i - \frac{1}{n} \Sigma z_i \Sigma d_i}$$

$$= \frac{\frac{1}{\Sigma z_i} \Sigma z_i y_i - \frac{1}{n} \Sigma y_i}{\frac{1}{\Sigma z_i} \Sigma d_i z_i - \frac{1}{n} \Sigma d_i}$$

$$= \frac{\overline{\Delta'} - \overline{\Delta}}{\overline{d'} - \overline{d}}$$