Comparison of Kalman algorithms for MEMS based pitch and roll angle estimation

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Abstract— *The scope of research is to develop a reliable measuring system for vehicular platform's actual states in real time. These states are platforms angle and angular rate. Estimations were done in two perpendicular directions. Three methods to detect platform's dynamical behavior in laboratory conditions are presented and compared. The test-bench design for laboratory tests and analysis of sensor platform's dynamical behavior in real operational conditions are based on authors' research analyzed earlier. Only the results got with this test-bench are shown. The research is based on earlier encouraging results got with preliminary measuring system and vehicular platform's dynamics analysis. The test bench caused inclination to vary from –9.1 to 9.4 degrees and the maximum angular rate varied from -13.5 to 12.3 deg/s in pitch direction. In roll direction the inclination varied from –9.8 to 8.6 degrees and angular rate form –12.7 to 12.3 deg/s. Inclination estimations presented are based on different sensor fusion methods where the basic model of stochastic Kalman filtering was modified. The analysis is an important part of development task of a 2-dimensional orthogonal stabilizer.*

Keywords: adaptation, angular rate, estimation, inclination, sensor fusion

I Introduction

In the starting point of the study the information of sensor platform's dynamics was achieved [1] and the basic estimation model was confirmed [1] to fit for the application. A development task of a 2-axis stabilizer for true operational conditions with possible applications on different types of vehicular platforms like ships and vehicles on wheels or on caterpillar chain [2] requires developing the measuring system to have adaptive characteristics for changing conditions in real operational environment to ensure reliable feedback signal for control. The measuring system integration to the selected application requires system design and tuning process in laboratory conditions. Thus, a test apparatus design and construction based on precise information about platform's dynamics [1] was completed before these real conditions could be simulated. The goal was to compare different estimation methods both in the laboratory by test-bench runs and by Matlab Simulink simulations.

In the field of inertial navigation, progress in micro electro mechanical systems (MEMS) has led to commercial cheap MEMS gyroscopes and inclinometers [3]. The drawbacks with these instruments are drifts and biases that force to use some sort of sensor fusion algorithms. Our experiments used a MEMS gyroscope and inclinometer [3] for sensor fusion because the actual measuring system should be suitable for low cost series production. Several estimation algorithms have been published [3] and in here basics of a recursive stationary Kalman filter algorithm [4] and adaptive Kalman filter algorithms [5] are presented and compared. Applicability of these three algorithms for the particular application is tested in laboratory conditions.

II Instrumentation

A coordinate system was set to define the measuring directions (Fig. 1). The attention was paid to inclination angle time histories according to roll and pitch directions, or according to θ and ψ axes.

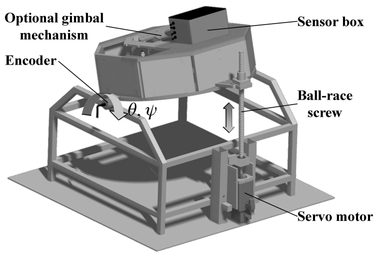

Fig. 1. Test bench's coordinate system

These were the variable magnitudes under interest in earlier study where the sensor platform's dynamics were measured [1] (Fig. 2), also. In Fig. 2 two coordinate systems are presented. One is the platform's own coordinate system. It follows platform's motion. Its *x*-axis was aligned with vehicle's direction of motion in front direction and the *y*-axis in the side direction. The third axis, namely the *z*-axis, was aligned orthogonally to these *x*- and *y*-directions in the left-handed system. Corresponding orthogonal main coordinate system's axes were set to be in the horizontal level. *Z*-axis was aligned with vertical level (Fig. 2).

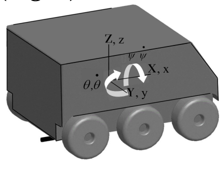

Fig. 2. Left-handed Euler's coordinate systems

The effects of backlash and friction were minimal, also. The path was based on earlier measurements [1]. Simple PC to servo drive connection is shown in Fig. 3.

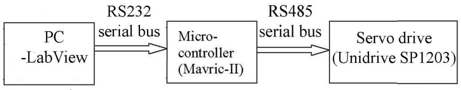

Fig. 3. Schematic view of servo control connections

The reference device in system monitoring was an incremental encoder. This encoder detected the real inclination angle and it was manufactured by Sick-Stegmann. The accuracy of the encoder was 10 000 lines per revolution or 0,036°. A personal computer with LabView 6.1 software collected gyroscope's and inclinometer's sensor data during test-bench runs (Fig 4). The MEMS sensors were used synchronously (Fig. 4) to collect data for inclination levels and rates for the sensor fusion algorithm. The sampling rate was 100 Hz. Inclinometers and gyroscopes used both internal temperature measurement and compensation. Instruments' specifications are summarized in [1].

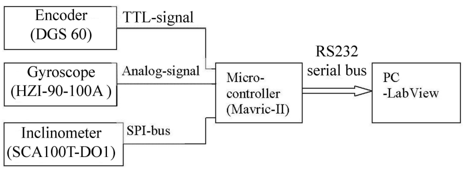

Figure 4. Schematic view of sensor fusion and reference encoder connections

Control Techniques manufactured the servo drive. The motor type was Unimotor 95UM A 30 0 CBCAA (Fig. 5).

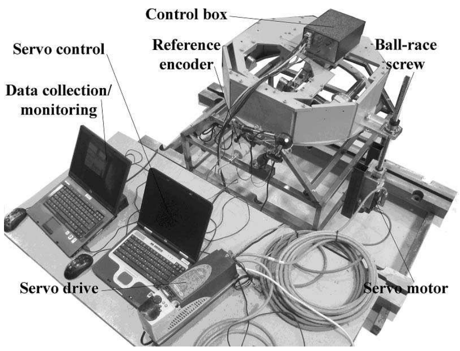

Fig. 5. The simulation environment included servo driven test bench and motion monitoring system.

The mobile and autonomous MEMS sensor box was instrumented with a low cost HZI-90-100A gyroscope and with one dual axis SCA100T-DO1 inclinometer (Fig. 6). Power supply was unregulated 12 VDC.

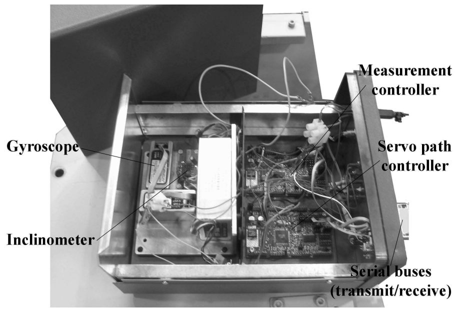

Figure 6. The control box included instruments used in estimation

III. Methods

In the beginning, the stationary Kalman filter algorithm (Fig. 7) was used to construct state estimations based on sensor fusion process to get the first idea of the platform's inclination time history for test-bench runs. Sensor data for this was collected in operational environment during platform's action. During one cycle the algorithm updates state estimations and generates 1-step state predictions which are based on the state space model and the real time measurements, *s*(k) [4]. In here, different methods to generate these estimations and predictions are considered.

The system's state space presentation here did not include any known input terms. Generally, values of the Kalman gain matrix (Fig. 4) are based on covariances of disturbances. In here, these tuning parameters, or the noise properties, of the estimation error's covariance matrix were unknown. Simulations found them experimentally. In theory, the stationary Kalman gain matrix is obtained with a stationary solution of optimal estimation error covariance by Riccati equation [6]. Now, this recursive estimation algorithm used is defined as a stochastic Kalman filter with a Kalman gain based on constant or adaptive error covariance matrix.

At the very first stage, the basic stationary Kalman algorithm was applied in order to estimate the platform's states in real conditions during one test run. When estimation method was developed further, this Kalman gain matrix, *K*, was constructed by disturbance matrices *Q* and *R* found experimentally according the algorithm presented earlier in [4] (Fig. 7):

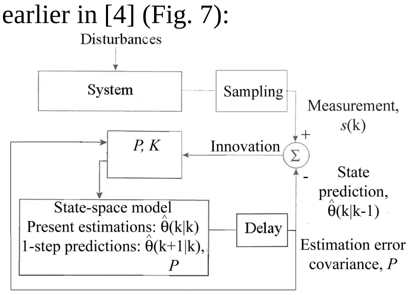

Figure 7. Block diagram of a stationary Kalman filter

In Fig. 7, the *R* represents the covariance matrix caused by measurement noise process and *Q* the covariance matrix caused by system's noise process. This approach helped to analyze the characteristics of unknown disturbances more in details than in the algorithm used in [1]. Matlab Simulink simulations could approximate these matrixes experimentally. The resulting inclination estimations based on these disturbance approximations were compared against the reference encoder in test-bench setup. The tuning was based on these residuals.

In the next stage (Fig. 8) the *K*-terms were still based on estimation error covariances, *P*-terms, but a constant weighting term, λ, was added to avoid *P*-terms to freeze on certain level [5]. The smaller the weighting function the faster the changing rate of estimation error covariance matrix, *P(k)*. In this recursive method the weight of one error covariance estimation decreases exponentially if weighting term is smaller than 1 [5]. Usually, when the square of the innovation increases the diagonal correction element, *D(k)*, is added to the estimation of error covariance matrix. This diagonal correction matrix is proportional to the square of the innovation. In here, these elements were used to build the estimations of error covariance of angular rate. The error covariance estimation of angular rate includes now a constant weighing term, λ_{rate}, and the varying correction element, $D_{rate}(k)$, both. The effect of λ_{rate} to the angle estimation is obvious because the angular rate has a direct influence to the angle prediction. These maneuvers helped the system to adapt to changing circumstances. The variant Kalman gain term is denoted by *K(k)* (Fig. 8).

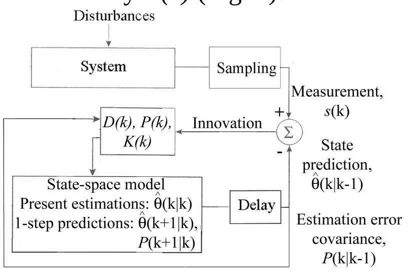

Fig. 8. Block diagram of an adaptive Kalman filter

The Simulink model is presented below (Fig. 9):

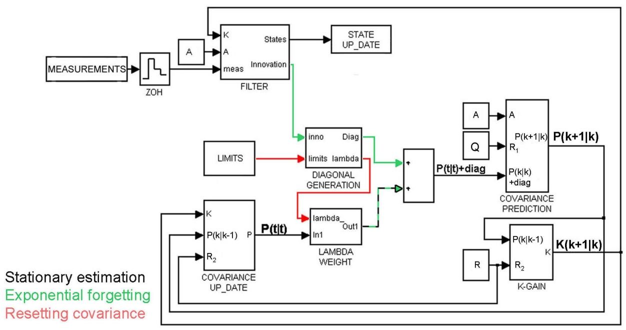

Figure 9: the three different Simulink models used

The model with black colour connections was used to define the disturbances' covariance matrices. The block connections described by black and green colour presents the exponential forgetting model and the model containing all the blocks presents the covariance resetting model. In here, the system and measurement matrices were:

$$A = \begin{vmatrix} 1 & t \\ 0 & 1 \end{vmatrix}, C = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

where sampling interval *t* was 0.01 s.

In some situations it is appropriate to reset the estimation error covariance matrix to the large one when parameters change abruptly occasionally [5]. In [5] the method is called covariance resetting. It is a suitable method if parameters to tune are assumed to be piecewise constant. In here, the time instant when parameters was assumed to change piecewisely is detected by monitoring innovations.

These three basic algorithms (Figs. 7 and 8) presented above were tested both by Matlab Simulink (Fig. 9) and test-bench runs. In order to define the optimal tuning parameters the data of low cost sensors and reference encoder was collected during test-bench runs. The path used in these test-bench runs was based on earlier measurements completed in real conditions [1]. The sensor and reference datasets collected during test-bench runs were used in Matlab Simulink simulations in order to find the optimum tuning parameters for estimation algorithms. After Matlab simulations these parameters found were applied to Kalman filter written in microcontroller (Fig. 6). The measuring system's performance could be monitored by test-bench setup (Fig. 5), again.

IV. Results

Fig. 10 presents the inclination time history during one test-bench run and servo drive's reference path data:

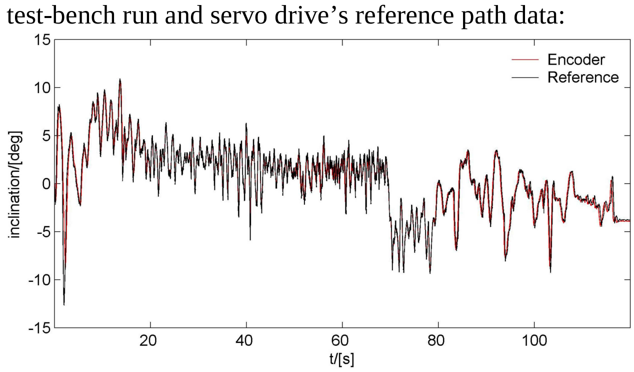

Fig. 10. Inclination time histories in the pitch direction for a reference and real motion during the test-bench run in pitch direction.

Datas in Fig. 10 are measured asynchronously. The servo driven test-bench (Fig. 5) is capable to simulate the inclination time history in real environment reliably even if there are some minor differences if compared to reference (Fig. 10). The path was generated by basic stationary estimation algorithm generated earlier [1] and is always an approximation of platform's real movements.

No reference device to measure the platform's exact movements was on hand. In this sense, the tracking is reliable to demonstrate platform's dynamics in operational conditions. The inclination angle varied from –9.1° to 9.4° and the maximum pitch rate during test-drive from -13.5 °/s to 16.7 °/s in pitch direction. In roll direction these varied form -12.7° to 12.3° and from –9.8 °/s to 8.6 °/s. In Figs. 11 and 12 two different inclinometer's signals are presented. The signals are measured in real conditions and in one test-bench:

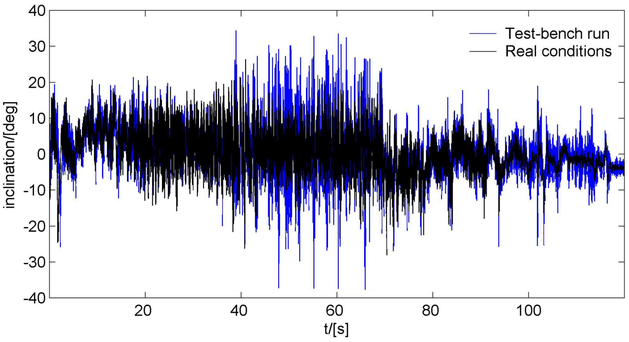

Fig. 11. Rough surveys of inclinometer during test-bench run and in real conditions in pitch direction.

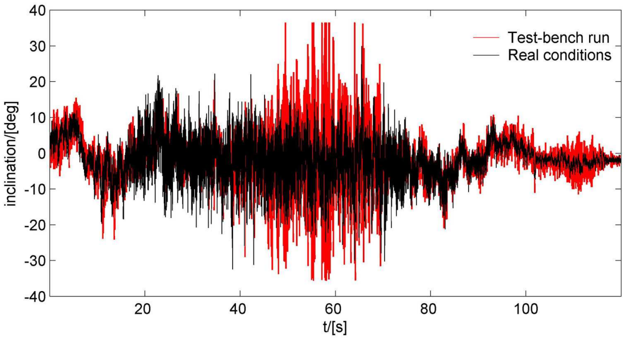

Figure 12: Rough surveys of inclinometer during test-bench run and in real conditions in roll direction

The disturbance levels in these signals are significant (Figs. 11 and 12). In Figs. 13 and 14 the rough surveys for gyroscopes angular rates are presented. Signals in Figs. 11-14 are measured synchronously in each direction:

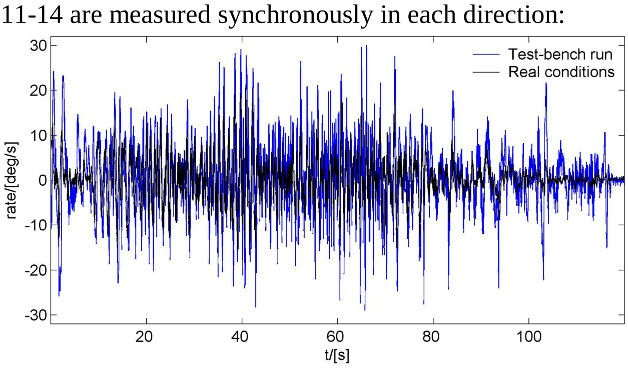

Fig. 13. Gyroscope's rough survey during test-bench run and in real conditions

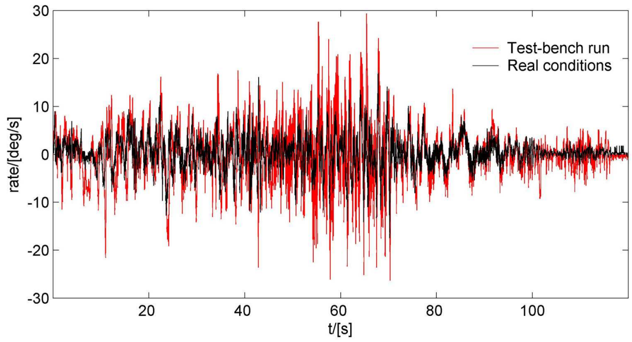

Figure 14: Gyroscope's rough surveys during test-bench run and in real conditions in roll direction

The disturbance levels in gyroscopes' and inclinometers' signals during test-bench runs are rather higher than lower than the disturbance levels in real environment. It is obvious that different tuning parameters are needed in different directions, too.

At the first stage, the parameters for stationary algorithm (Fig. 7) found experimentally by Matlab Simulink simulations. The sensor datasets (Figs. 11 and 13) used in these simulations (Figs. 15 and 16) were collected during test-bench run. The tuning parameters, Q and R terms, in here were slightly different in different directions. The same method was applied when seeking the parameters for the algorithm using exponential forgetting (Figs. 8 12, 14, 15 and 16) and for the resetting covariance algorithm (Figs. 15 and 16). The disturbance parameters, in matrices R and Q, found earlier by stationary model were now kept fixed. The adaptive characteristic was applied to gyroscope's signal processing like explained above. The residuals of these algorithms monitored in Matlab Simulink are presented in Figs. 15 and 16. The third residual in Figs. 15 and 16 has got by algorithm using covariance resetting.

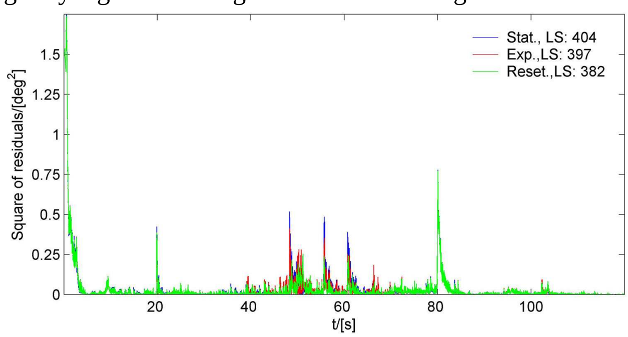

Figure 15. The square of the residuals for the stationary, the exponential forgetting and the covariance resetting models achieved by Simulink simulations in pitch direction.

The results in roll direction using the same methods above are shown below:

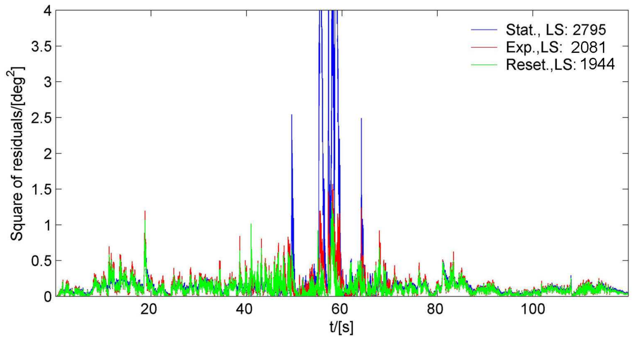

Figure 16: The square of the residuals for the stationary, the exponential forgetting and the covariance resetting models achieved by Simulink simulations in roll direction.

The parameters found were then written to control box's microcontroller used in test-bench construction (Fig. 6). In Figs. 17 and 18 are shown the residuals of different models during test-bench runs. Each of the residual signals in Figs. 17 and 18 are constructed by encoder signal and estimation value sent to LabView data collection system (Fig. 4).

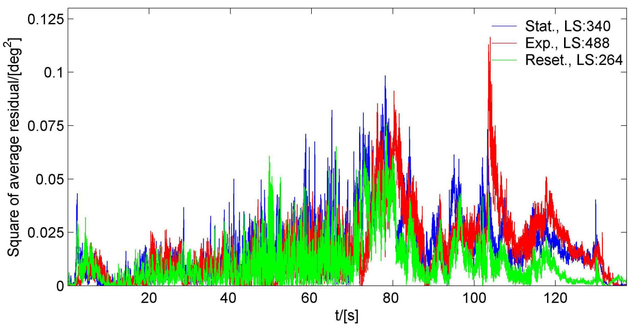

Fig. 17. The square of average residuals for the stationary, the exponential forgetting and the covariance resetting models achieved by test-bench runs in the pitch direction, ten data sets/algorithm.

The results in roll direction using the same methods above are shown below:

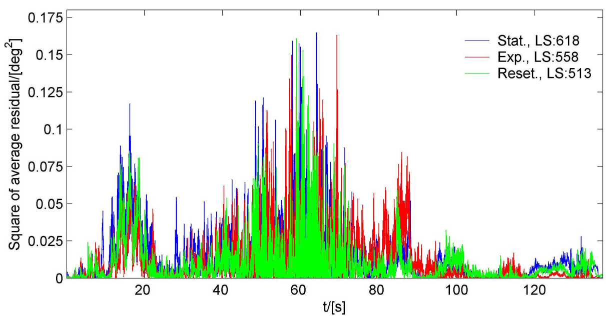

Figure 18: The square of average residuals for the stationary, the exponential forgetting and the covariance resetting models achieved by test-bench runs in the roll direction, ten data sets/algorithm.

V. Discussion

The basic behaviour of measuring algorithms can be seen in the Simulink simulations (Figs. 15 and 16) and in the test-bench runs (Figs. 17 and 18). If the Simulink simulations are compared (Figs. 15 and 16) the residual of method using resetting covariance is the closest to 0-level in both of the directions. During the first 50 seconds the residuals of all the estimation methods are approximately equal but when innovation increases over the threshold value the resetting covariance method is more effective and no strong peak values appear. After this heavy driving period between 50 to 70 seconds the innovation of angular rate decreases under the threshold value. The residuals of different methods are equal (Figs. 15 and 16), again. Note the least square error values, too.

If the residuals got with test-bench runs (Figs. 17 and 18) are compared the difference of these three methods is more obvious than with Simulink simulations (Figs. 15 and 16) and the residual differences are bigger, too. During these test-bench runs residuals monitored show that the exponential forgetting and stationary methods contain more peaks but the residual of the resetting covariance method stays smooth. In here, the parameter values of the resetting covariance method was set be like in the stationary method if the innovation was under the threshold value and above this threshold the parameters were set to be like used in exponential forgetting method. The smooth residual behaviour of resetting covariance method justifies abrupt behaviour in parameter changes.

Thus, in heavy conditions method resetting covariance is the most effective. If the common tuning parameters are kept fixed the residual of the exponential forgetting method is detected to be only a little lower than the residual got with stationary method (Figs. 15- 18).

So, the estimation value of error covariance of angular rate was increased when the increasing innovation caused by increasing changing speed in angular rate, or increasing angular acceleration, was detected. The corresponding term in $K(k)$ matrix was increased in order to make the system react faster to the influence of angular acceleration. When this corresponding $K(k)$ term was increased the system's sensitivity to gyroscope's disturbances will increase, as well. If no high angular acceleration was detected the innovation started to decreases and the $P(k)$ term was decreased, too. Now, the system was less sensitive to disturbances in gyroscope's signal (Figs. 15-18). If the innovation of angular rate increased the more weight should be put on real measurement instead of filtered state-space model's value (Fig. 8). This maneuver made the system react faster to higher angular acceleration magnitudes (Figs. 13 and 14). The more reliable angular rate estimation achieved improved inclination estimation via better prediction. This is because the inclination prediction is based on state-space model and old inclination and rate estimates, both. Naturally, the better predictions improve the estimates (Figs. 15-18). In test-bench simulation, the lowest residual level was got by algorithm using covariance resetting (Figs. 15 and 16). In these simulations, the forgetting factor of angular rate estimation error covariance was not changed dramatically but it helped the system to adapt to changing circumstances. When using covariance resetting the values of λ_{rate} and $D_{rate}(k)$ were changed when the threshold value of innovation was exceeded.

These results gave an ensurance to the intuitive picture that platform's (Fig. 2) characteristics ghance abruptly according to driving speed level and the current gear used during the test-drive made in real operational conditions (Figs. 11-14). The most effective method to estimate current angle for this kind of vehicular platform is the method using resetting covariance (Figs. 15-18). The stationary system requires less computational capacity for the microcontroller used (Fig. 6) but the adaptive system selected improves the inclination estimation result and made the residual to be smoother.

Finally, according to stationary method's tuning process in pitch and roll directions, the parameters describing measurement disturbances were lower in pitch direction than in roll direction. In pitch direction these were:

$$R = \begin{vmatrix} 2.1 & 0 \\ 0 & 0.05 \end{vmatrix}$$

And in the roll direction:

$$R = \begin{vmatrix} 3.5 & 0 \\ 0 & 0.1 \end{vmatrix}$$

The system disturbance's tuning parameter stayed at the same level in both directions. This matrix was:

$$Q = \begin{vmatrix} 0.0005 & 0.006 \\ 0 & 0.13 \end{vmatrix}$$

The tuning for resetting covariance method has to be performed separately for different directions, too.

VI. Conclusion

It is now obvious that the tuning parameters have to be defined separately for different directions. This because of different dynamical behavior in these pitch and roll directions [1].

With the presented instrumentation and methods three basic algorithms for angle estimation were compared for pitch and roll directions. Angle estimation was based on MEMS sensors' data. The test-bench construction was needed to simulate real conditions measured earlier [1] in laboratory conditions. The basic stationary Kalman filter algorithm presented [4] was used to construct the 1-step recursive state estimator. This algorithm was expanded to include adaptive characteristics [5]. The requirements set to application's performance in real conditions forced to use adaptive algorithms and time variant parameter tuning. The comparison of these different estimation algorithms was based on Matlab Simulink simulations (Figs. 15 and 16) and test-bench runs (Figs. 17 and 18). The abrupt parameter change behavior was justified and a proper adaptive method was selected for parameter estimation.

When platform's movements occur on higher frequency level than where the stationary algorithm is optimized in least square sense more weight must be put on gyroscope's real measurement. This reduced the change of system's estimation reliability during real operation according to simulations and test-bench runs.

The next step is to construct a servo controlled 2-axis stabilizer and ensure the stabilizer's performance and capability to meet the boundary conditions set earlier in the development process in a realistic field environment [7]. The state estimation achieved by presented methods could be used as a feedback signal in this stabilizer control purpose. Monitoring the object to be stabilized could ensure the performance of the stabilizer. If the object to be stabilized is e.g. distance sensor for sea depth detection in hard conditions the performance of the measuring system could be ensured by reference measurements done in calm conditions. After this the system is reliable to use in these rough conditions defined.

References

[1] Mattila S. et al., *Analysis of sensor platform dynamics in ground vehicle*, Electrical and aerospace systems, IEEE, 2006, 5 p.

[2] Kuosmanen P. et al., Universal radar antenna stabilizer for vehicular platforms, Espoo, URSI/IEEE XXIX convention on radar science, 2004, 3 p.

[3] Titterton D. et al., Strapdown inertial navigation technology, 2nd edition, USA, The institution of electrical engineers, 2004, 558 p., ISBN 0 86341 358 7

[4] Haid M. et al., Low cost inertial orientation tracking with Kalman filter, Germany, Applied mathematics and computing, Elsevier, inc., 2003, 9 p.

[5] Åström, K. J. et al., Adaptive control, second edition, Addison-Wesley publishing company, inc., Lund institue of technology, 1995, 573 p., ISBN 0-201-55866-1

[6] Glad T. et al., Control theory multivariable and nonlinear methods, London, Taylor & Francis, 2000, 467 p., ISBN 0-7484-0878-9 (pbk)

[7] Eppinger, S. D. et al., Product design and development, international edition, USA, David K. Burke, 2000, 358 p., ISBN 0-07-116993-8

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