Principles of Mathematical Analysis Notes

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1 Addition in the Real Number Field

Proof. Let α and β be cuts, such that $\alpha \subset \beta$. Let $r \in \alpha$ and $s \in \beta$. The cut defined by $\alpha + \beta$ is thus the set of all $r + s$. Since $\alpha \in \beta$, by (II), $r - s \in \alpha$. Since $r = r - s + s$, we can say $(r - s) + (s) \in \alpha + \beta$ and therefore $r \in \alpha + \beta$. □

Proof. To verify that $\alpha + \beta$ satisfies (II), for some $r' \in \alpha$ such that $r < r'$ and $s' \in \beta$ such that $s < s'$. It follows that $r + s < r' + s' \in \alpha + \beta$. □

2 Chapter 1 Exercises

2.1 1.1

Proof. To prove (a) by contradiction, let $r + x = \frac{p}{q}$ for some $p, q \in \mathbb{Z}$. It follows that $x = \frac{p-rq}{q}$ which contradicts $x \in \mathbb{I}$. Thus, $r + x \in \mathbb{I}$.
Similarly, to prove (b) by contradiction, let $rx = \frac{p}{q}$. It follows that $x = \frac{p}{qr}$ which contradicts $x \in \mathbb{I}$. Thus, $rx \notin \mathbb{Q}$. □

2.2 1.2

Proof. To prove this by contradiction, assume $\frac{p^2}{q^2} = 12$. It follows that

$$p^2 = 12q^2 = 2^2 \cdot 3^1 \cdot q^2.$$

By the fundamental theorem of arithmetic, p^2 must factor into a product of primes of even multiplicity. By the same argument, q^2 must factor into a product of an even multiplicity of 3, contradicting the unique factorization of p. Therefore, the assumption is false and $\sqrt{12} \in \mathbb{I}$. $\square$