Tommy O'Shaughnessy
Prof. Feng Luo
Abstract Algebra I
12/05/2019

Chapter 1

Problem 9.1. The objects of **Rel** are sets, and an arrow $A \to B$ is a relation from A to B, that is, a subset $R \subseteq A \times B$. The equality relation $\{\langle a, a\rangle \in A \times A \mid a \in A\}$ is the identity arrow on a set A. Composition in Rel is to be given by

$$S \circ R = \{\langle a, c\rangle \in A \times C \mid \exists b(\langle a, b\rangle \in R \& \langle b, c\rangle \in S)\}$$

for $R \subseteq A \times B$ and $S \subseteq B \times C$.

Solution. (a) Show that Rel is a category.
Composability:
Identity:
Associativity:
Unit:

(b) Show also that there is a functor G : Sets $\to$ Rel taking objects to themselves and each function $f : A \to B$ to its graph,

$$G(f) = \{\langle a, f(a)\rangle \in A \times B \mid a \in A\}$$

(c) Finally, show that there is a functor C : $\mathbf{Rel}^{\mathrm{Op}} \to$ Rel taking each relation $R \subseteq A \times B$ to its converse $R^c \subseteq B \times A$, where,

$$\langle a, b\rangle \in R^c \Leftrightarrow \langle b, a\rangle \in R$$

□

Problem 9.2.

Solution. □