	1	2	3	4
x	1	2	3	4
$f(x)$	1	8	27	64

Calculating the intervals:-

$x_2 - x_1 = 2 - 1 = 1$

$x_3 - x_2 = 3 - 2 = 1$

$x_4 - x_3 = 4 - 3 = 1$

Using lagrange's interpolation method:-

→ We have to find ~~for~~ $f(x)$ for $x = 2.5$.

So,
$$y(2.5) = \frac{(2.5-4)(2.5-2)(2.5-3)}{(1-4)(1-3)(1-2)} \times 1 + \frac{(2.5-4)(2.5-3)(2.5-1)}{(2-4)(2-3)(2-1)} \times 8 + \frac{(2.5-4)(2.5-2)(2.5-1)}{(3-4)(3-2)(3-1)} \times 27 + \frac{(2.5-3)(2.5-2)(2.5-1)}{(4-3)(4-2)(4-1)} \times 64$$

$= (-0.0625 \times 1) + (0.5625 \times 8) + (0.5625 \times \cancel{8}^{27}) + (-0.0625 \times 64)$

$= 15.625.$

Using newton's forward interpolation method:-

x	y	Δy_0	$\Delta^2 y$	$\Delta^3 y$
1	1			
		7		
2	8		12	6
		19		
3	27		18	
		37		
4	64			

$h = \frac{x - x_0}{1} = \frac{2.5 - 2}{1} = 0.5$

$f(2.5) = f(2) + (0.5 \times \cancel{8}^{19}) + \frac{(0.5)(0.5-1)}{2} \times 18 \cancel{+ (0.5)(0.5-1)}$

$\cancel{+ \frac{(0.5)(0.5-1)(0.5-2)}{6} \times 18}$

$\cancel{= 8 + 4 + (-2.375) + (}$

$= 8 + 9.5 - 2.25 = 15.25$

Using Newton's backward interpolation method:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1	1			
		7		
			12	
2	8			
				6
		19		
			18	
3	27			
		37		
4	64			

$$y = \cancel{f(2)}\ \frac{2.5-2}{2} = 0.5$$

$$y \Rightarrow f(2.5) = f(2) + (0.5 \times 8) + (0.5 \times 1.5 \times 7/2)$$

$$= 8 + 4 + 2.625 = 14.625.$$

By observation, we should use newton's forward interpolation method as the numbers in 'x' ~~have~~ are in very close intervals.