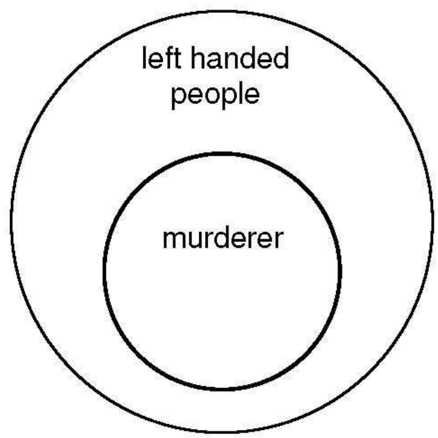
3. Deductive support

A detective is investigating a murder and discovers that the guilty person must be left handed. The detective has two suspects, Mr. Brown and Miss Green. Miss Green is left handed. Should the detective conclude that Miss Green is the murderer? Mr. Brown is not left handed. Should the detective conclude the he is innocent? Let's investigate.

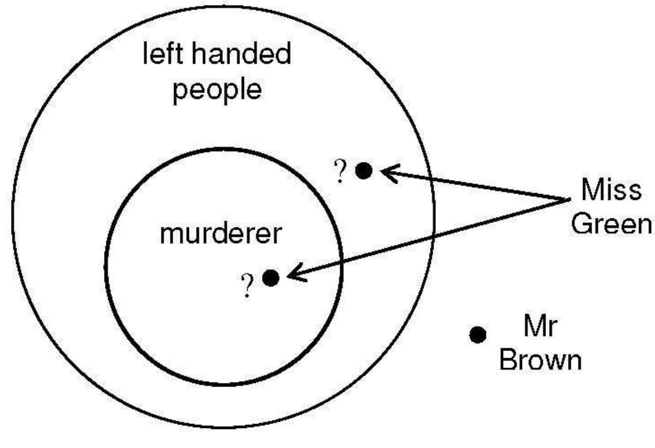
We can represent the evidence available to the detective in a simple diagram (called an 'Euler diagram' after the Dutch mathematician Leonhard Euler, who invented them).



The diagram represents the fact that the murderer must be a left handed person. Notice that if a point is inside the smaller circle, labelled 'murderer', then it is also inside the larger circle, labelled 'left handed people'. So the diagram represents the following *conditional* statement:

> If a person is the murderer, then they are left handed.

The other items of evidence available to the detective are that Miss Green is left handed and that Mr. Brown is not. Let's add that information to our diagram.



To represent the information that Mr. Brown is *not* left handed, we place a dot representing him *outside* the larger circle labelled 'left handed people'. Where do we put Miss Green? We know she *is* left handed, so she has to go somewhere *inside* the larger circle. But as you can see from the diagram, there are two different places where we could put a dot representing Miss Green. Miss Green must be inside the larger circle, but she could be either inside or outside the smaller circle, labelled 'murderer'. The information we have about Miss Green is consistent with *both* possibilities.

You can see from the diagram then that Mr. Brown is definitely not the murderer, since his dot is outside the larger circle and so must be outside the inner circle too. But we cannot draw any valid conclusion about Miss Green – she may or may not be the murderer.

We can represent the arguments concerning Miss Green and Mr. Brown as follows.

> **1.** If a person is the murderer, then they are left handed.
> **2.** Miss Green is left handed.
> **Therefore:**
> **C**. Miss Green is the murderer.

> **1.** If a person is the murderer, then they are left handed.
> **2.** Mr. Brown is not left handed.
> **Therefore:**
> **C.** Mr. Brown is not the murderer

We have just seen that the conclusion of the first argument does not necessarily follow from the premises, while the conclusion of the second argument certainly does. The second argument is *deductively valid*. Given that both premises are true, the conclusion must be true too. But in the first argument, this is not so. The diagram shows that the premises could both be true, while the conclusion is false. From the information we have, it could be that Miss Green is left handed, but *not* the murderer.

Deductively valid arguments

An argument like the one about Mr. Brown, in which it is *impossible* for the premises to be true and the conclusion false is called a *deductively valid* argument. In a deductively valid argument, if the premises are true then the conclusion *must* be true too (it is impossible for the conclusion not to be true). So a deductively valid argument provides a kind of maximum support for its conclusion – the premises, if true, would conclusively establish that the conclusion is true.

Of course, this does not mean the premises themselves are true. The detective in our example might have made a mistake: Mr. Brown might really be left handed. But this does not alter the fact that the argument about Mr. Brown is deductively valid. It is still the case that if the premises were both true, the conclusion would have to be true too.

> A set of premises provide **deductive support** for a conclusion when the following condition is satisfied:
>
> *If* all the premises were true, *then* the conclusion **must** be true.
>
> We can also say that in this case the argument is **deductively valid**.

How can you decide whether an argument is deductively valid or not? Here is a test you can use:

A test for deductive validity

Ask yourself the following question. Supposing that all the premises of the argument are true, is it *possible* for the conclusion to be false?

- If the answer is **yes**, then the argument is **deductively invalid.**
- If the answer is n**o** (it is *impossible* for the premises to be true and the conclusion false), then argument is **deductively valid.**

This test requires you to ask whether it is *possible* for all the premises to be true and the conclusion false. In some cases, the answer will be 'definitely not', as in the argument about Mr. Brown. In these cases, the premises support the conclusion deductively – the argument is deductively valid.

In other cases, the answer to the question will be clearly 'yes' – it *is* possible for the premises to be true and the conclusion false. This is the way things are in the argument about Miss Green. Supposing that the murderer is left handed and that Miss Green is left handed, it is still possible that Miss Green is not the murderer, as the diagram shows. So the argument fails our test. It is not deductively valid.

Deductively valid forms

Have a look at the following argument about three sisters, Alice, Bethany and Catherine.

1. Alice is older than Bethany.
2. Bethany is older than Catherine.
Therefore:
C. Alice is older than Catherine.

Most people have no trouble recognizing that this is a deductively valid argument: it's easy to see that if premise 1 and 2 are true, the conclusion must be true too. And notice that you can recognize this fact, without knowing anything about the relative ages of the three sisters! That is, you can tell that the argument is invalid without knowing whether any of the premises are true or false.

The reason you can do this is that the argument has a certain *form* or *pattern*. We can exhibit the form of the argument by replacing the particular names 'Alice', 'Bethany' and 'Catherine' with arbitrary labels such as A, B and C:

1. A is older than **B**
2. B is older than **C**
Therefore:
C. A is older than **C**

This represents a pattern or form which a wide variety of different arguments could take. For example, here is a different argument, which has the same form:

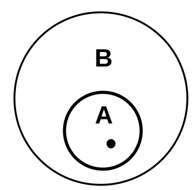
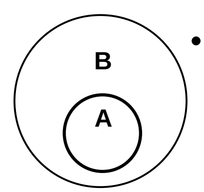
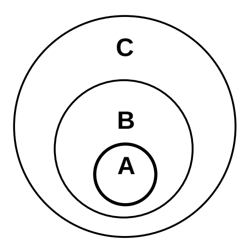
1. John is older than Jill
2. Jill is older than Frank
Therefore:
C. John is older than Frank

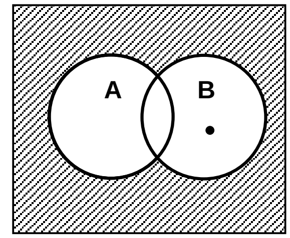
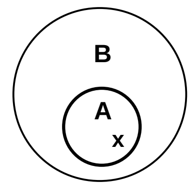
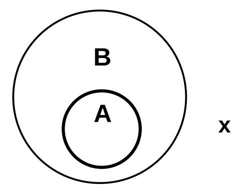
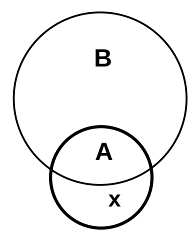
Of course this argument is deductively valid too. In fact any argument which conforms to the pattern will be deductively valid. No matter what names or descriptions we put in place of A, B and C, we will *never* get an argument with true premises and a false conclusion. Whenever the premises of an argument of that form are true, the conclusion will also be true. So we can say that this is a **deductively valid form** of argument.

> A **deductively valid form** is an argument form with the following property:
>
> *Every* instance of the form with true premises also has a true conclusion.
>
> Equally, we can say that in a deductively valid form
>
> There are *no* instances of the from with true premises and a false conclusion.

Some common valid forms of argument

Here are some further examples of commonly used deductively valid forms of argument. There is an example of each form of argument, along with a diagram intended to illustrate why the form is valid.

Argument form	Examples	Diagram
1. If **A** then **B** **2. A** **Therefore:** **C**. **B** **Modus ponens**	**1.** If Miss Scarlet is guilty, she is left handed. **2.** Miss Scarlet is guilty. **Therefore:** **C.** Miss Scarlet is left handed.	
1. If **A** then **B** **2. B** is not true **Therefore** **C. A** is not true **Modus tollens**	**1.** If Miss Scarlet is guilty, she is left handed. **2.** Miss Scarlet is not left handed. **Therefore:** **C.** Miss Scarlet is not guilty.	
1. If **A** then **B** **2.** If **B** then **C** **Therefore:** **C.** If **A** then **C** **Hypothetical syllogism**	**1**. If it's raining tomorrow, the party will be cancelled. **2.** If the party is cancelled I won't get to see Jill this week. **Therefore:** **C**. If it's raining tomorrow, I won't get to see Jill this week.	

1. Either **A** or **B** **2. A** is not true **Therefore:** **3. B** is true **Disjunctive syllogism**	**1.** The murderer is either Mr. Brown or Mr. Orange. **2.** The murderer is not Mr. Brown. **Therefore:** **C.** The murderer is Mr. Orange.	
1. All **A** are **B** **2. x** is **A** **Therefore:** **C**. **x** is **B** **Universal modus ponens**	**1.** All critical thinkers know how to spot a valid argument. **2.** Sarah is a critical thinker. **Therefore:** **C.** Sarah knows how to spot a valid argument.	
1. All **A** are **B** **2. x** is not **B** **Therefore:** **C**. **x** is not **A** **Universal modus tollens**	**1.** All critical thinkers know how to spot a valid argument. **2.** Brendan does not know how to spot a valid argument. **Therefore:** **C.** Brendan is not a critical thinker.	
1. x is **A** **2. x** is not **B** **Therefore:** **C**. Not all **A** are **B** **Counter-example**	**1.** Dogs are mammals. **2.** Dogs are not herbivores. **Therefore:** **C.** Not all mammals are herbivores.	

A second test for deductive validity: the counter example method

We can use the idea of a deductively valid form of argument as the basis for a second test for deductive validity. Look again at the argument from the last chapter:

> **1.** Anything that has an engine need oil.
> **2.** Cars need oil.
> **Therefore**
> **C.** Cars have engines.

Although the conclusion is true, it does not follow from the premises. Why not? WE can represent the argument as having the following form:

> **1.** All **A** are **B**
> **2.** All **C** are **B**
> **Therefore**
> **C.** All **C** are **A**

where A stands for 'things with engines', B stands for 'things that need oil' and C stands for 'cars'. Is this kind of argument *reliable*? Does it always lead us from true premises to a true conclusion? The answer is no. There are many arguments of this form which have true premises and a *false* conclusion. Here is one. It is the argument we get if we put 'plants' for A, 'needs water' for B and 'animals' for C in the form above. You can probably think of many more.

> **1.** All **plants** need **water**
> **2.** All **animals** need **water**
> **Therefore**
> **C.** All **animals** are **plants**

We started with an argument about cars and engines and showed that it was not a deductively valid argument by thinking of *another* argument like the first one, but with obviously true premises and a false conclusion. The second argument provides a *counterexample* to the validity of the argument form; it shows that there are examples of the form which have true premises and a false conclusion. This means that the form is not valid and so the inference used in the first argument is not reliable – it can lead from true premises to a false conclusion.

A **counter-example** to a **form of argument** is an example of the form which has *true* premises and a *false* conclusion.

What we call the **counter-example method** of testing arguments for deductive validity is then following procedure:

The counter-example method

1. Isolate the form of the argument. Leave in 'logical words' such as 'al'', 'some', 'not', 'if .. then' and so on. Replace other terms consistently with letters, **A**, **B**, **C** ...
2. Try to find a **counter-example** – an argument with the same form having true premises and a false conclusion. If you can find one, the argument is not deductively valid.
3. If no counter-examples are possible, the argument is deductively valid. Compare the argument form to the list of valid forms given in the previous section. If it matches one of those, the argument is deductively valid.

4. You could also try to construct a diagram like the Euler circle diagram used at the beginning of this chapter, to see if the conclusion of the argument must be true if the premises are.

The counter-example method in action

Is the following argument deductively valid or not?

> **1.** All quokkas are herbivores
> **2.** Some quokkas can live for weeks without drinking
> **Therefore:**
> **C.** Some herbivores can live for weeks without drinking

You might not know that the quokka is a small Australian marsupial, like a small kangaroo. Even if you did know that, you still might not know whether these animals are herbivores or whether some of them can live for weeks without drinking. So you wouldn't know whether the premises are true or not. But to discover whether the conclusion *follows* from the premises you do not *need* to know. Let's use the counter-example method to find out.

First we identify the form of the argument:

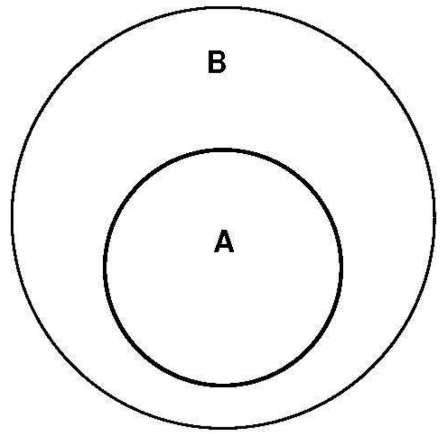
> **1.** All **A** are **B**
> **2.** Some **A** are **C**
> **Therefore:**
> **C.** Some **B** are **C**

Here the letter **A** represents 'quokkas', **B** represents 'herbivores' and **C** represents 'things that can live for weeks without drinking'. Can we construct a counter-example to this form of argument – an example with true premises and a false conclusion? Let's start by trying to think of a false conclusion. Suppose we now let B stand for 'plants' and C for 'animals'. Then the conclusion would become, 'Some plants are animals' which is false. Putting 'plants' for C and 'animals' for B in the premises we get the following partially complete argument:

> **1.** All **A** are **plants**
> **2.** Some **A** are **animals**
> **Therefore:**
> **C.** Some **plants** are **animals**

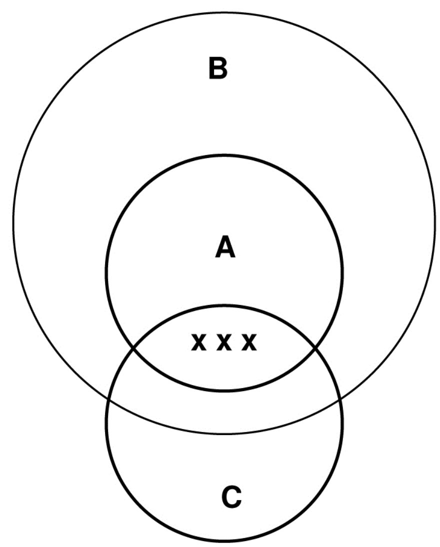
We now need to find to put for A to make the premises both true. If we put 'flowers' for A the first premise would be true (All flowers are plants). But the second premise would then become 'Some flowers are animals', which is false. If we let A stand for 'living things' then the second premise would be true: 'some living things are animals'. But then the first premise would be false: 'All living things are plants' is false.

This attempt to find a counter-example to the argument form does not seem to be working. We might try a few other replacements for A, B and C, but let's now consider the possibility that the argument is in fact deductively valid. The argument form is not among the 'common valid forms' in the previous section, so that list won't help here. So let's draw a diagram to represent the information in the premises. The first premise tells us that all **A** are **B**. We can represent that with two circles, like this:



Any point inside the smaller circle labelled A is also inside the bigger circle labelled B. So any A is B, which is what the diagram is supposed to represent.

The second premise tells us that some **A** are **C**. To represent this we can add a second circle for the **C**s:



We have placed some **X**s in the region of the **C** circle that overlaps the **A**s to represent the information that *some* (not necessarily all) of the **A**s are **C**.

The conclusion of our argument is that some **B** are **C**. Is this necessarily true, looking at the diagram? You should be able to see that the answer is yes. The **X**s we put in for the second premise are inside the **C** circle and also inside the **B** circle. So some things which are **B** are also **C**.

So the conclusion of the argument must be true if the premises are. The argument is deductively valid.

A second example

Let's try one more example. Is this argument about quokkas valid?

1. All quokkas are herbivores
2. Some herbivores are nocturnal
Therefore:
C. Some quokkas are nocturnal

The form of the argument is:

1. All **A** are **B**
2. Some **B** are **C**
Therefore:
C. Some **A** are **C**

Once again, this form of argument doesn't match any of those on the list of valid forms in the previous section. Let's see then if we can construct a counter-example. We want to find an example of this form of argument with true premises and a false conclusion. Start with the conclusion. We could make the conclusion false by putting 'animals' for **A** and 'plants' for **C**. Doing the same in the premises, we would get the following partial argument:

1. All **animals** are **B**
2. Some **B** are **plants**
Therefore:
C. Some **animals** are **plants**

Can you think of something to put for **B** that would make both premises true? One choice that works is the following (you might be able to think of a few more).

1. All **animals** need water to survive
2. Some things that need water to survive are **plants**
Therefore:
C. Some **animals** are **plants**

We have found a counter-example an argument of the same form as the first argument but with true premises and a false conclusion. This shows that the argument form is not deductively valid. So the inference involved in the first argument is not reliable. It does not provide us with a good reason to accept the conclusion as true, even if the premises are true.

Further Reading

The example from the beginning of this chapter has been adapted from Harold Jacob's beautiful high school textbook on geometry, which contains an excellent simple introduction to deductive reasoning.

Harold R. Jacobs, *Geometry*. W.H. Freeman and Company. Chapter 1, 'The Nature of Deductive Reasoning'.

See also:

Tracy Bowell and Gary Kemp, *Critical Thinking: A concise guide*. Chapter 3. Logic: Deductive validity.

Alec Fisher, *Critical Thinking. An introduction*. Chapter 8, pp. 107-123.

The 'Euler diagrams' mentioned here were later developed by John Venn into a systematic method for testing argument forms for validity. These are the three-circle 'Venn diagrams' you might be familiar with. For a good exposition of the method of Venn diagrams see:

Walter Sinott-Armstrong and Robert Fogelin, *Understanding arguments: an introduction to informal logic. 9th edition.* Cengage Advantage, 2015. Chapter 7 'Categorical Logic', pp. 151-176.

There are also a great many good online explanations, tutorials and exercises on using Venn diagrams to test arguments for validity. These come and go so I will not list any here; you should be able to find some by searching for "Venn diagrams validity test".

Exercise 3.1 Deductive validity

For each of the following arguments, say whether the argument is deductively valid or not. If you think it is valid, does it conform to any of the common valid forms of argument listed in Table 1 above? If you think the argument is invalid, can you find a counter-example?

1 If tungsten is a metal, it conducts electricity. Tungsten does not conduct electricity. Therefore, tungsten is not a metal.

2 If Bill's fingerprints match those found at the crime-scene, he must be guilty. Bill's fingerprints do match those found at the crime-scene. So Bill must be guilty.

3 Either war will continue or peace will be negotiated. Peace will not be negotiated, so war will continue.

4 If Cathy goes to the party, her mother will be upset. Cathy's mother is upset. Therefore, Cathy went to the party.

5 All English playwrights write sonnets. William Shakespeare did not write sonnets. Therefore, William Shakespeare was not an English playwright.

6 Some of my students are members of the Monash sailing club and some are members of the debating society. So some of my students are members of both the Monash sailing club and the debating society.

7 If science is entirely objective, then the emotions and ambitions of scientists have nothing to do with the pursuit of their research. But the emotions and ambitions of scientists do have something to do with their pursuit of research, Therefore, science is not entirely objective.

8 Punishment for crimes is justified if it actually deters people from committing them. But a great deal of carefully assembled and analyzed empirical data show clearly that punishment is *not* a deterrent. So punishment is never justified.

9 Does God exist? It would seem not, for if God exists, a perfectly good being exists. But if a perfectly good being exists, no evil could exist. And evil does exist.

10 If there is a designer of the universe, then God exists. If the universe has no designer, inanimate things would not behave in an orderly and understandable way. But inanimate things do behave in an orderly and understandable way. So, God does exist.

11 Children will not do well at school unless their parents are interested in education. Sosuke's parents are interested in education, so he will do well at school.

Exercise 3.2 Lewis Carroll puzzles

The following logic puzzles were devised by Lewis Carroll, the author of *Alice in Wonderland* and *Through the Looking Glass*. Not many people know that Lewis Carroll's real name was Charles Dodgson and that he taught logic and mathematics at Oxford University. These puzzles are taken from one of his logic books, written for children.

In the puzzles you are given some premises and must work out what conclusion follows from them. Let's look at an example.

> Babies are illogical. Nobody is despised who can manage a crocodile. Illogical persons are despised.

What conclusion follows from all three of these statements? It helps to begin by restating the premises in 'if .. then ...' form:

> If someone is a baby, they are illogical.
> If someone is despised, they cannot manage a crocodile.
> If someone is illogical, they are despised.

Changing the order of the premises makes things clearer:

> If someone is a baby, they are illogical.
> If someone is illogical, they are despised.
> If someone is despised, they cannot manage a crocodile.

It should now be clear that the conclusion we can validly draw from these three premises is:

> If someone is a baby, they cannot manage a crocodile

in other words

> No baby can manage a crocodile.

Now try the following puzzles for yourself.

1 That story of yours about your once meeting the sea sea-serpent, always sets me off yawning. I never yawn, unless when I'm listening to something totally devoid of interest.

2 Everyone who is sane can do Logic. No lunatics are fit to serve on a jury. None of *your* friends can do Logic.

3 Showy talkers think too much of themselves. No really well-informed people are bad company. People who think too much of themselves are not good company.

4 No birds, except ostriches are 9 feet high. There are no birds in this aviary that belong to any one but me. No ostrich lives on mince pies. All my birds are 9 feet high.

5 No kitten that loves fish is unteachable. No kitten without a tail will play with a gorilla. Kittens with whiskers always love fish. No teachable kitten has green eyes. No kittens have tails unless they have whiskers.

6 No interesting poems are unpopular among people of real taste. No modern poetry is free from affectation. All your poems are on the subject of soap-bubbles. No affected poetry is popular among people of real taste. No ancient poetry is on the subject of soap-bubbles.