Exercise 3.1 Deductive validity

1. If tungsten is a metal, it conducts electricity. Tungsten does not conduct electricity. Therefore, tungsten is not a metal.

Valid (Modus Tollens)

1. If **A** then **B**
2. **B** is **not true**

Therefore:

C. **A** is **not true**

A: tungsten is a metal
B: tungsten conducts electricity

2. If Bill's fingerprints match those found at the crime-scene, he must be guilty. Bill's fingerprints do match those found at the crime-scene. So Bill must be guilty.

Valid (Modus Ponens)

1. If **A** then **B**
2. **A** is true

Therefore:

C. **B** is true

A: Bill's fingerprints match those found at the crime-scene
B: Bill is guilty

3. Either war will continue or peace will be negotiated. Peace will not be negotiated, so war will continue.

Valid (Disjunctive syllogism)

1. Either **A** or **B**
2. **B** is **not true**

Therefore:

C. **A** is true

A: War will continue
B: Peace will be negotiated

4. If Cathy goes to the party, her mother will be upset. Cathy's mother is upset. Therefore, Cathy went to the party.

Invalid. The form of the argument is:

1. If **A** then **B**
2. **B** is true
Therefore
C. **A** is true

A: Cathy goes to the party
B: Cathy's mother is upset

This is invalid because Cathy's mother might be upset for some other reason. Here is a counter-example.

1. If Alice lives in Melbourne then she lives in Victoria (If **A** then **B**)
2. Alice lives in Victoria (**B** is true)
Therefore
C. Alice lives in Melbourne (**A** is true)

But if Alice lives in Geelong or Bendigo, then the premises would be true, but the conclusion would be false. So this is not a valid form of reasoning – it allows us to go from true premises to a false conclusion.

5. All English playwrights write sonnets. William Shakespeare did not write sonnets. Therefore, William Shakespeare was not an English playwright.

Valid (Universal Modus Tollens)

1. All **A** are **B**
2. **x** is **not B**
Therefore
C. **x** is **not A**

A: English playwrights
B: people who write sonnets
x: William Shakespeare

6. Some of my students are members of the Monash sailing club and some are members of the debating society. So some of my students are members of both the Monash sailing club and the debating society.

Invalid. The form of the argument is as follows:

1. Some **S** are **C**
2. Some **S** are **D**

Therefore:

C. Some **S** are both **C and D**

S: my students
C: members of the Monash sailing club
D: members of the debating team

The following counter-example shows that this is not a valid form of reasoning – it allows us to go from true premises to a false conclusion.

1. Some animals are dogs (TRUE)
2. Some animals are cats (TRUE)

Therefore:

C. Some animals are both cats and dogs (FALSE)

7. If science is entirely objective, then the emotions and ambitions of scientists have nothing to do with the pursuit of their research. But the emotions and ambitions of scientists do have something to do with their pursuit of research. Therefore, science is not entirely objective.

Valid (Modus Tollens)

1. If **A** then **B**
2. **B** is **not true**

Therefore:

C. **A** is **not true**

A: science is entirely objective
B: the emotions and ambitions of scientists have nothing to do with the pursuit of their research

8. Punishment for crimes is justified if it actually deters people from committing them. But a great deal of carefully assembled and analyzed empirical data show clearly that punishment is *not* a deterrent. So punishment is never justified.

Invalid. The form of the argument is:

1. If **A** then **B**

2. **A** is **not** true
Therefore:
C. **B** is not true

A: punishment for crimes deters people from committing them
B: punishment for crimes is justified

The argument is invalid because the first premise does not rule out their being other considerations that might justify punishment for crimes. So even if punishment is not a deterrent, it still might be justified for those other reasons.

Here is a counter-example:

1. If it is raining today, then there's cloud in the sky (If **A** then **B**)
2. It is not raining today (**A** is not true)
Therefore:
C. There's no cloud in the sky today (**B** is not true)

Suppose today is a cloudy day, but it's not raining. Then the premises would be true and the conclusion false. So this is not a valid form of reasoning – it allows us to go from true premises to a false conclusion.

9. Does God exist? It would seem not, for if God exists, a perfectly good being exists. But if a perfectly good being exists, no evil could exist. And evil does exist.

This argument is **valid**. The first thing to note is that the conclusion of this argument is 'God does not exist'. The first part, 'Does God exist? It would seem not ....' states the conclusion. The claims that follow provide the reasoning for that conclusion (note the premise indicator 'for').

Here is the form of the argument:

1. If **A** then **B**
2. If **B** then **C**
3. **C** is **not** true
Therefore:
C. **A** is **not** true

A: God exists
B: A perfectly good being exists
C: No evil exists

This is a valid form of argument. The conclusion 'A is not true' follows from the premises. There are several ways of seeing this. One way is to notice that if you take premises 2 and 3 together, you can use *modus tollens* to deduce that B is not true. But premise 1 says that If A then B. So using modus Tollens again, we can deduce the final conclusion, 'A is not true'.

Alternatively, note that premises 1 and 2 imply that If A then C, by hypothetical syllogism. Combining that with premise 3, we can deduce 'A is not true' by *modus tollens*.

10. If there is a designer of the universe, then God exists. If the universe has no designer, inanimate things would not behave in an orderly and understandable way. But inanimate things do behave in an orderly and understandable way. So, God does exist.

This argument is **valid.** The form of this argument can be represented as follows:

1. If **A** then **B**
2. If **A** is **not** true then **C** is **not** true
3. **C** is true

Therefore

C. **B** is true

A: There is a designer of the universe
B: God exists
C: Inanimate things behave in an orderly and understandable way

As in question 9, the conclusion follows using a combination of several deductive patterns of argument. From premise 2 and 3, we can deduce that A must be true ('there is a designer of the universe') using *modus tollens*. Using that and premise 1 we can deduce the conclusion B using *modus ponens*.

11. Children will not do well at school unless their parents are interested in education. Sosuke's parents are interested in education, so he will do well at school.

Invalid.

One way to represent the form of the argument is as follows:

1. If **not A** then **not B** (If parents of a child are not interested in education, that child will not do well at school)
2. **x** is **A**

Therefore:

C. **x** is **B**

A: Child has parents who are interested in education
B: Child does well at school
x: Sosuke

Alternatively and equally correctly, we can represent the form of the argument like this:

1. If **A** then **B** (If a child does well at school, then their parents are interested in education)
2. **x** is **B**
Therefore
C. **x** is **A**

A: child does well at school
B: Child has parents who are interested in education
x: Sosuke

In either case the argument is invalid because the first premise does not mean that having parents with an interest in education is *sufficient* for a child to do well at school. It only means that such an interest is *necessary*.

Here is a counter-example to the first way of representing the form of the argument:

1. If an animal does not have four legs, then it is not a dog (If not A then not B)
2. Garfield has four legs (x is A)
Therefore
C. Garfield t is a dog (x is B)

A: animal has four legs B: dogs
x: Garfield

Since dogs have four legs, premise 1 is true. Now suppose Garfield is a four-legged cat. Then the second premise is also true, but the conclusion would be false. So this is not a valid form of reasoning – it allows us to move from true premises to a false conclusion.

Exercise 3.2 Lewis Carroll puzzles

1. That story of yours about your once meeting the sea-serpent, always sets me off yawning. I never yawn, unless when I'm listening to something totally devoid of interest.

That story of yours about once meeting the sea-serpent always makes me yawn.
If I yawn, I am listening to something totally devoid of interest.
Conclusion: That story of yours about your once meeting the sea-serpent is totally devoid of interest.

2. Everyone who is sane can do Logic. No lunatics are fit to serve on a jury. None of *your* friends can do Logic.

If someone is fit to serve on a jury, they are sane.
If someone is sane, they can do logic.
If someone can do logic, they are not one of your friends.
Conclusion: If someone is fit to serve on a jury, they are not one of your friends.
Alternatively: None of your friends is fit to serve on a jury.

3. Showy talkers think too much of themselves. No really well-informed people are bad company. People who think too much of themselves are not good company.

Anyone who is really well-informed is good company.
Anyone who is good company does not think too much of themselves.
Anyone who does not think too much of themselves is not a showy talker.
Conclusion: Anyone who is really well-informed is not a showy talker.
Alternatively: No really well-informed people are showy talkers.

4. No birds, except ostriches are 9 feet high. There are no birds in this aviary that belong to any one but me. No ostrich lives on mince pies. All my birds are 9 feet high.

All birds in this aviary belong to me.
All birds that belong to me are 9 feet high.
All birds that are 9 feet high are ostriches.
All ostriches, do not live on mince pies.
Conclusion: All birds in this aviary do not live on mince pies.
Alternatively: No birds in this aviary live on mince pies.

5. No kitten that loves fish is unteachable. No kitten without a tail will play with a gorilla. Kittens with whiskers always love fish. No teachable kitten has green eyes. No kittens have tails unless they have whiskers.

All kittens that love fish are teachable.
All kittens that will play with a gorilla have tails.
All kittens with whiskers love fish.
All teachable kittens do not have green eyes.
All kittens with tails have whiskers. Re-ordering:
All kittens that will play with a gorilla have tails. All kittens with tails have whiskers.
All kittens with whiskers love fish. All kittens that love fish are teachable.
All teachable kittens do not have green eyes.
Conclusion: All kittens that will play with a gorilla, do not have green eyes.
Alternatively: No kitten with green eyes will play with a gorilla.

6. No interesting poems are unpopular among people of real taste. No modern poetry is free from affectation. All your poems are on the subject of soap-bubbles. No affected poetry is popular among people of real taste. No ancient poetry is on the subject of soap-bubbles.

The premises in 'All A are B' format:

All interesting poems are popular among people of real taste.
All poetry which is free from affectation is not modern poetry.
All your poems are on the subject of soap-bubbles.
All poetry which is popular among people of real taste is not affected poetry.

All ancient poetry is not on the subject of soap-bubbles.

Reordering:

All interesting poems are popular among people of real taste.
All poetry which is popular among people of real taste is not affected poetry.
All poetry which is free from affectation is not modern poetry.
All ancient poetry is not on the subject of soap-bubbles.
All poems which are not on the subject of soap-bubbles are not your poems.
Conclusion: All interesting poems are not your poems.
Alternatively: None of your poems are interesting.