MONASH University

Information Technology

# FIT1006
# Business Information Analysis

## Lecture 18
## Hypothesis Testing (Part 2)

# Topics covered:

- p-value
- Type I and II errors
- The significance and power of a test.

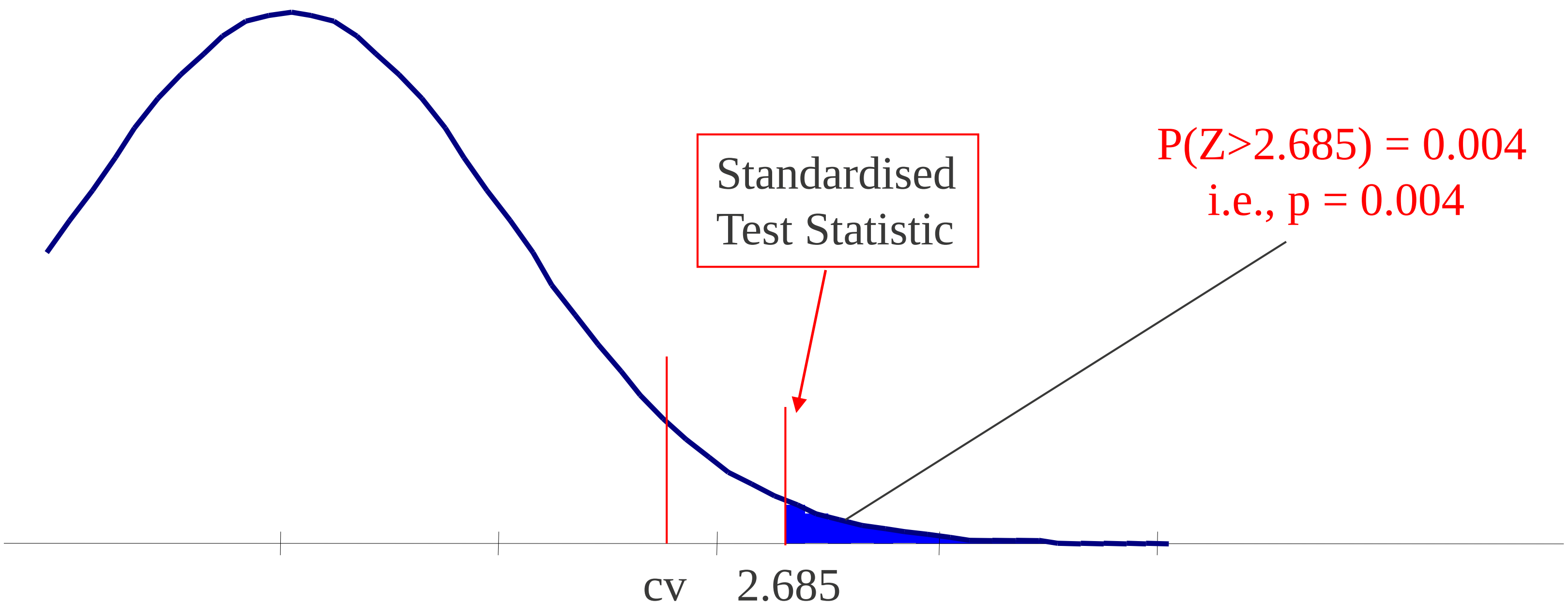
# The Steps of Hypothesis Testing

1. Decide on a null hypothesis $H_0$.
2. Decide on an alternative hypothesis $H_1$.
3. Decide on a significance level.
4. Calculate the appropriate test statistic.
5. Find from tables the corresponding tabulated test statistic.
6. Compare calculated and tabulated test statistics and decide whether to accept or reject the null hypothesis.
7. State the conclusion and assumptions of the test.

(Source: Rees, D.G. Essential Statistics, Chapman and Hall 1995.)

# *p-value*

- The p-value of a test is the smallest value of the critical region that leads to the rejection of $H_0$. For example, if a test had the following test statistic:



# Calculating the p-value

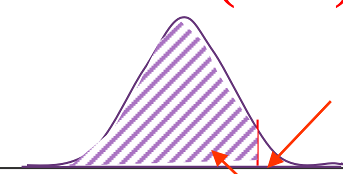
- Modified Question from last lecture:
- A hypothesis test for the population mean when the population variance is known. (Note: unrealistic situation)
- The Axle manufacturing company has been making axles for a long time and kept records for every axle produced.
- Population parameters are $\mu$ = 90mm and $\sigma$ = 2.5mm.
- A sample of 100 axles from new machine has mean = 90.5.
- Is the new machine making parts with same average length required (90mm) – or are they longer than this?
- What is the p-value of the test?

https://flux.qa (Feed code: SJ6KGV)

# Question 1

From the tables: $P(Z > 2)$ =

$P(Z>2) = 1 - 0.9772$
$= 0.0228$



This area = 0.9772; z = 2.0
(one tail)

A. 0.0114

✓ B. 0.0228

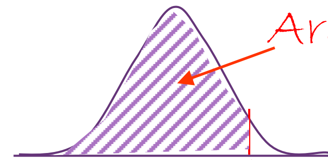
C. 0.4886

D. 0.9772

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Cumulative Probabilities for the Standard Normal Distribution | | | | | | | | | | |
| | | | | | | | | | | |
| Table gives P(Z<z) for Z = N(0,1) | | | | | | | | | | |
| | | | | | | | | | | |
| **z** | **0.00** | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |

Given in Slide 5:
$\mu$ = 90mm and $\sigma$ = 2.5mm

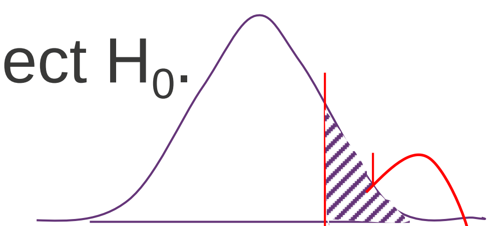
# Solution



$\alpha = 0.01$,
Area $= 1 - 0.01$
$= 0.99$

One tail critical value for $\alpha = 0.01$, from table: $z = 2.33$

1. $H_0$, $\mu$ = 90mm
2. $H_1$, $\mu >$ 90mm (a one-sided experiment)
3. Significance = 0.01.
4. The test statistic, $\bar{x}$ = 90.5. We calculate $Z_x$. *(standardising)* $Z = (90.5 - 90)/(2.5/\sqrt{100}) = 2$.
5. From tables the calculated critical value is 2.33.
6. We see that 2 < 2.33 and thus do not reject $H_0$.
7. Conclude …
8. *p-value* of the test is 0.0228 – this is the highest level of significance at which $H_0$ will be rejected.



2 2.33

$P(Z>2) = 1 - 0.9772$
$= 0.0228$

# … visually…

<u>Interpreting the p-value:</u>

- A small p-value indicates that there is ample evidence to support the alternative hypothesis.
- A big p-value indicates that there is little evidence to support the alternative hypothesis.

Since, $2 < 2.33$

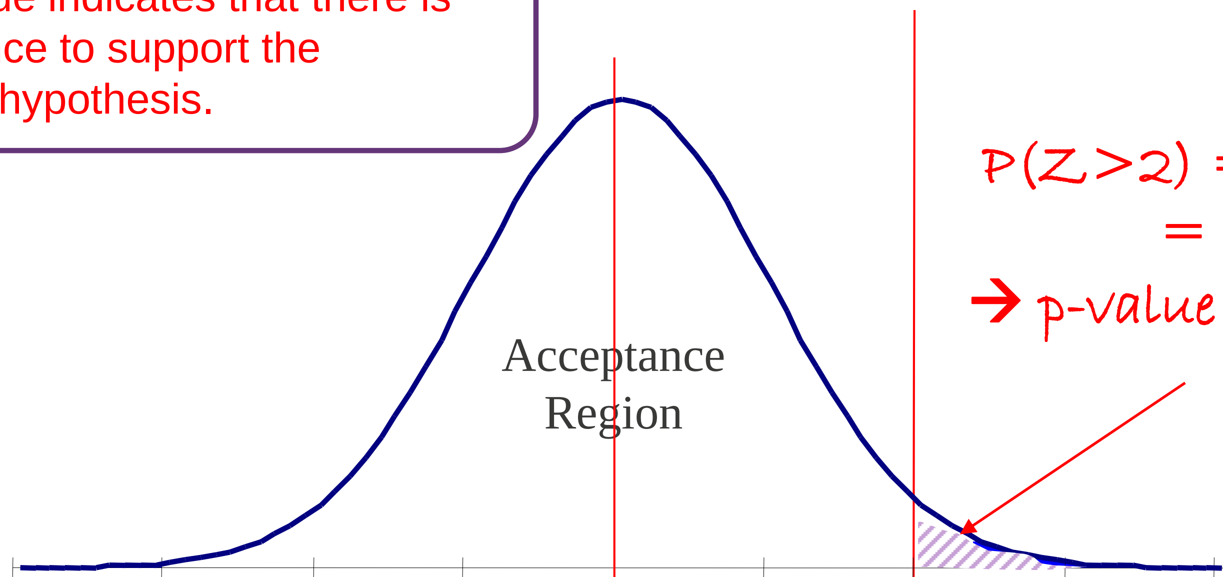
→ do not reject $H_0$

Test statistic = 90.5

Standardised test statistic = <u>2</u>

$P(Z>2) = 1 - 0.9772$
$= 0.0228$

→ p-value = 0.0228




$\mu = 90$

For $\alpha$ = 0.01, one tail;
critical value = <u>2.33</u>

# Errors in Hypothesis Testing

- We can make two errors when testing hypotheses
  - A Type I error occurs when the null hypothesis is correct, but is rejected.
  - A Type II error occurs when the null hypothesis is incorrect but is not rejected.

| | $H_0$ is Not Rejected | $H_0$ is Rejected |
|---|---|---|
| $H_0$ is Correct | Correct Decision<br>$(1 - \alpha)$ | Type I Error<br>$\alpha$ |
| $H_1$ is Correct | Type II Error<br>$\beta$ | Correct Decision<br>$(1 - \beta)$ |

## …

- The distinction between type I and II errors is easier to understand using a criminal law analogy.

| | Person Acquitted | Person Convicted |
|---|---|---|
| Person is Innocent | Correct Decision | Type I Error |
| Person is Guilty | Type II Error | Correct Decision |

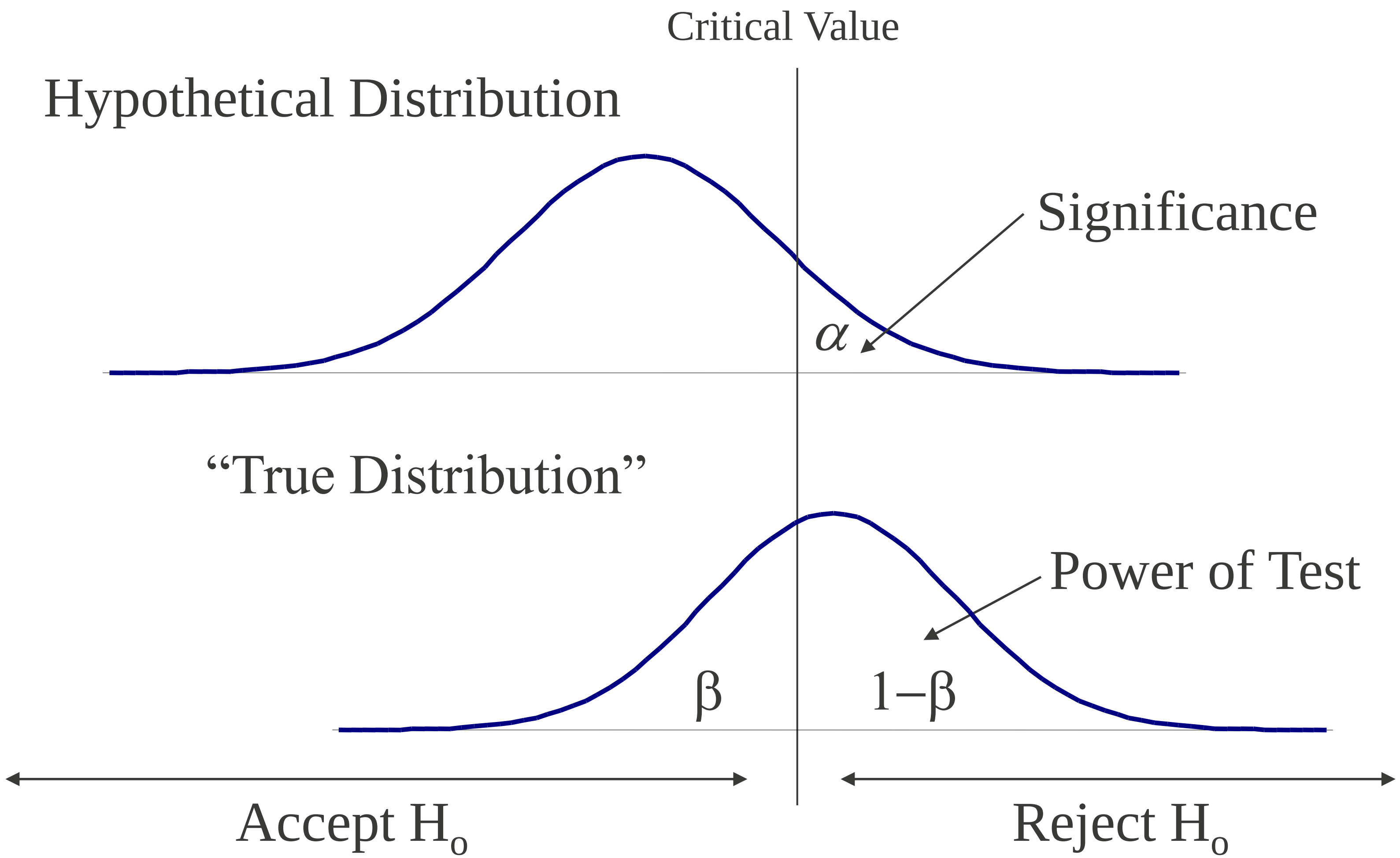
- When we attempt to minimize the type I error, the consequence is an increase in the type II error.

# The Power of the Test

Rejecting the Null Hypothesis when you should

- The probability of making a Type II error is β. If the null hypothesis is incorrect, the probability of avoiding a Type II error is (1-β), this is the power of the test.
- α and β are interrelated. Increasing α reduces β and *vice versa*. To reduce both, sample size needs to be increased.
- Only α or β (usually α) can be chosen. The true value of β also depends on the actual value of the population parameter – which is often unknown.

# The Power of the Test ctd..



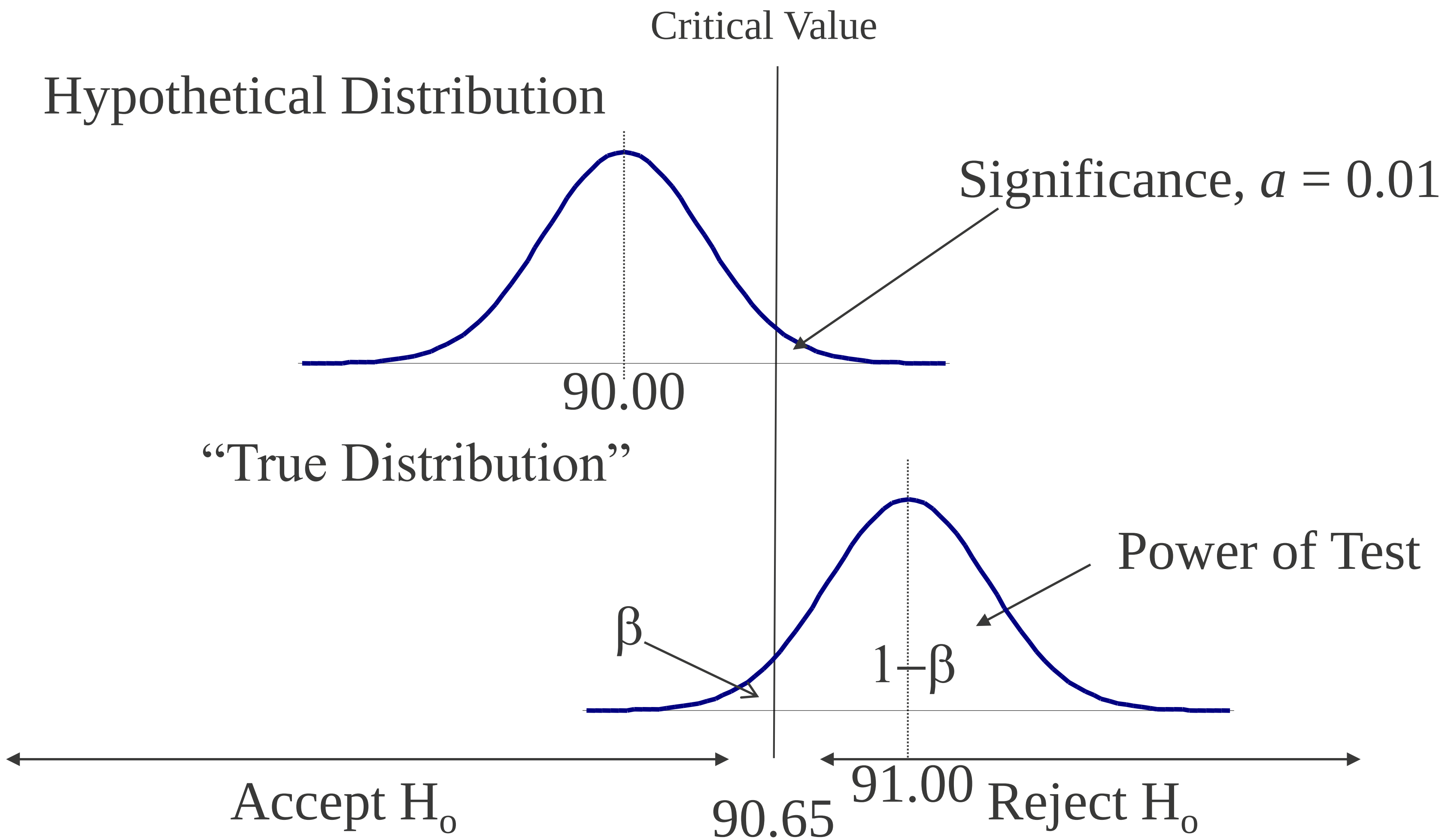
# Calculating β

- Using the same example:
  - The Axle manufacturing company has been making axles for a long time and kept records for every axle produced.
  - Population parameters are $\mu$ = 90mm and $\sigma$ = 2.5mm.
  - A sample of 100 axles from new machine has mean = 92.7.
  - Is the new machine making parts with same average length required (90mm)?
  - Assume a 1% significance.
  - What is the power of the test if it eventuates that the new machines produces axles with a mean length of 91mm?

# Solution

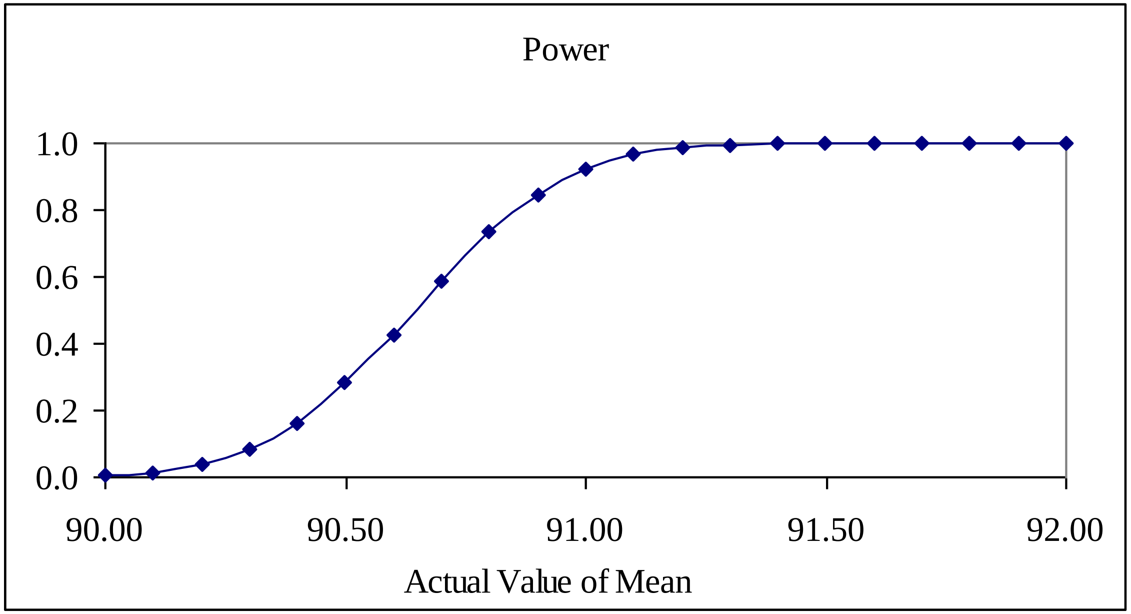
- From our previous calculations
- $H_0$, $\mu = 90$mm
- $H_1$, $\mu \neq 90$mm (a two tailed experiment)
- Significance = 0.01.
- From tables the critical values are $\pm 2.58\sigma_x$
- Use upper critical value since alternative is > $H_0$
- Calculate upper critical value = 90 + 2.58(0.25) = 90.65
- P(x < 90.65) when $X \sim N(91, 0.25^2)$ is P(z < -1.42) = 0.08
- This is the probability that we fail to reject the null hypothesis, when the population mean for the new axle is 91mm. The power of the test = 0.92

# The Power of the Test ctd..



# Power Curves

- To see how the power of a test changes as the value of the alternative hypothesis changes, we can calculate the power for a range of values and view the resulting curve.



# Example

- From last lecture...
- It is claimed that Melbourne families are spending more than $150 per week on food and grocery items on average. A sample of 15 families was conducted and the amount spent each week was recorded. Do these results support this thesis? (assume a 1% significance)
- Weekly food and grocery expenditure ($)
- 156, 234, 199, 78, 256, 189, 221, 49, 220, 178, 120, 290, 97, 177, 231.
- What is the power of the test if the average cost of groceries is $160?

# Summary Statistics from Excel

| | |
|---|---|
| Mean | 179.7 |
| Standard Error | 17.7 |
| Median | 189.0 |
| Mode | #N/A |
| Standard Deviation | 68.5 |
| Sample Variance | 4697.0 |
| Kurtosis | -0.5 |
| Skewness | -0.5 |
| Range | 241.0 |
| Minimum | 49.0 |
| Maximum | 290.0 |
| Sum | 2695.0 |
| Count | 15.0 |

## https://flux.qa (Feed code: SJ6KGV)

## Question 2

The power of the test is:

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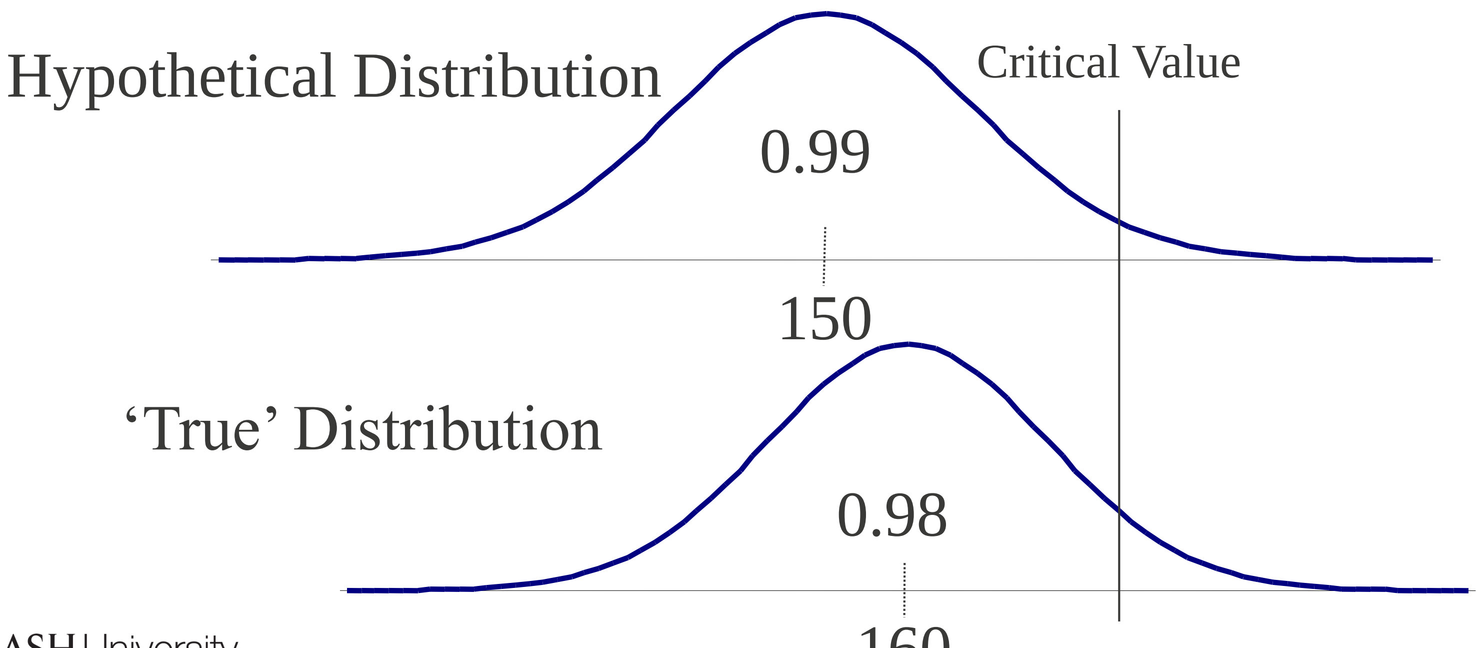
A. 0.99

B. 0.01

C. 0.98

✓ D. 0.02



## https://flux.qa (Feed code: SJ6KGV)

# Question 3

For a one-sided test and 15 observations the *t* Statistic for a 1% significance is:

A. 2.6025

B. 2.9467

✓ C. 2.6245

D. 2.9768

Critical Values of the t Distribution

Table gives upper critical values only

| n | 0.300 | 0.200 | 0.150 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
|---|---|---|---|---|---|---|---|---|
| | | | | *a* | | | | |
| 12 | 0.5386 | 0.8726 | 1.0832 | 1.3562 | 1.7823 | 2.1788 | 2.6810 | 3.0545 |
| 13 | 0.5375 | 0.8702 | 1.0795 | 1.3502 | 1.7709 | 2.1604 | 2.6503 | 3.0123 |
| 14 | 0.5366 | 0.8681 | 1.0763 | 1.3450 | 1.7613 | 2.1448 | 2.6245 | 2.9768 |
| 15 | 0.5357 | 0.8662 | 1.0735 | 1.3406 | 1.7531 | 2.1314 | 2.6025 | 2.9467 |
| 16 | 0.5350 | 0.8647 | 1.0711 | 1.3368 | 1.7459 | 2.1199 | 2.5835 | 2.9208 |
| 17 | 0.5344 | 0.8633 | 1.0690 | 1.3334 | 1.7396 | 2.1098 | 2.5669 | 2.8982 |
| 18 | 0.5338 | 0.8620 | 1.0672 | 1.3304 | 1.7341 | 2.1009 | 2.5524 | 2.8784 |

…

- $H_0$, $\mu$ = \$150
- $H_1$, $\mu >$ \$150 (a one tailed experiment)
- Significance = 0.01.
- From tables the critical value $T_{(0.01)(\nu=14)}$ is $2.624\sigma_x$
- Calculate upper critical value: 150 + 2.624(17.7) = 196.5
- Assume alternative distribution Normal, new $\mu$, original SE.
- $P(x < 196.5)$ when $X \sim N(160, 17.7^2)$ = $P(z < 2.061) = 0.98$
- This is $\beta$, the probability that we fail to reject the null hypothesis, when the population mean for expenditure is \$160.
- The power of the test is 0.02.

# Other Hypothesis Tests

- We have looked at hypothesis tests for the population mean and a population proportion.
- There are a range of other hypothesis tests which are regularly used, these include tests for:
  - The difference of means
  - Variance
  - Distribution Shape
  - Independence.

We're not going to cover this.

# Reading/Questions (Selvanathan)

- Reading: Hypothesis Testing
  - 7th Ed. Sections 12.3, 12.5, 12.6.

- Questions: Hypothesis Testing
  - 7th Ed. Questions 12.1, 12.19, 12.25, 12.26, 12.56, 12.59, 12.65, 12.66, 12.67, 12.70, 12.72, 12.74.