MONASH University

Information Technology

FIT1006
Business Information Analysis

Lecture 17
Hypothesis Testing

Topics covered:

- The Null and Alternative Hypothesis
- The test procedure
- The hypothesis test for a population mean and proportion.

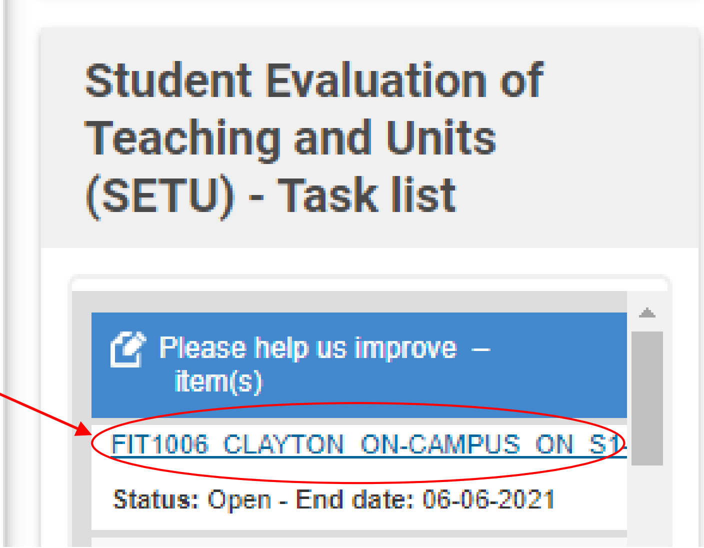
Student Evaluation of Teaching

- SETU, Please take 5 minutes to evaluate this unit.
- Your feedback is confidential and used to improve teaching and learning across all units.
- There is a $5,000 prize draw on offer to students who complete their evaluations this semester (100 * $50 gift vouchers)
- Two options to access SETU:

1. Use the SETU link on the right hand panel in the Moodle homepage.

Or:

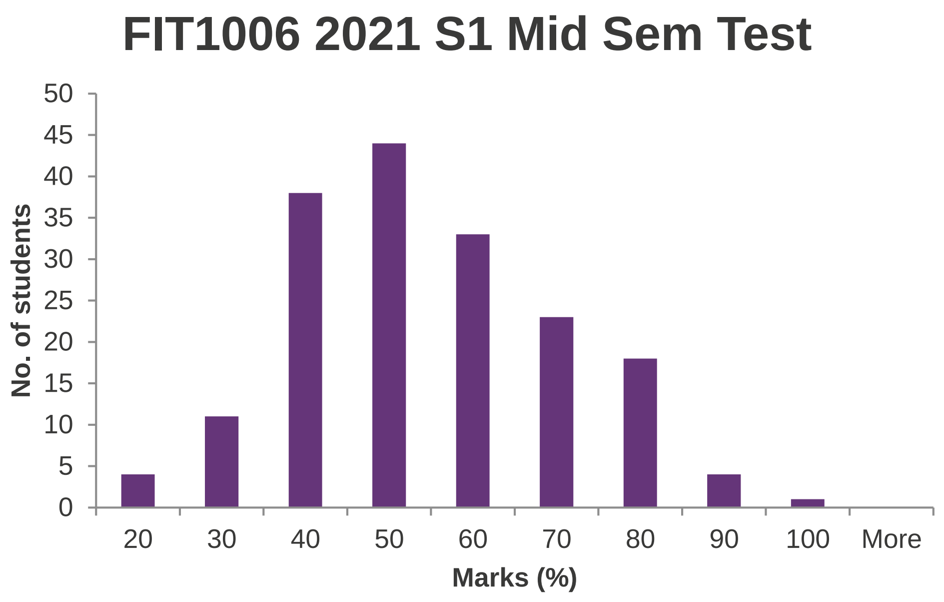
2. At the bottom of 'Week 10' content page



Mid-semester Test Results

- Well done! Test marks released this week. (Sorry for the delay as we need to cater for the special consideration cases who sat for the test on Monday)

Test Scores (100%)	
Mean	50.3
Standard Error	1.2
Median	50.0
Mode	50.2
Standard Deviation	15.8
Sample Variance	250.2
Kurtosis	-0.4
Skewness	0.2
Range	73.3
Minimum	17.5
Maximum	90.8
Sum	8816.5
Count	176



Motivating Problem

- Would Labor win if we have an election today?
- The Australian Newspoll had the two-party preferred vote at: ALP 51% vs Coalition (Liberal) 49% from a sample of 1,160 people chosen at random (taken on 25 April 2021).
- Hint: Find a 95% CI for the expected Liberal-NP vote.
- Ref: http://www.theaustralian.com.au/national-affairs/newspoll

Statistical Inference

- Statistical inference is concerned with the way we draw conclusions about the population using a sample.
- There are two approaches to statistical inference:
  - confidence intervals for population parameters,
  - hypothesis testing, in which we test an assumption, or point of view about a population parameter.
- The two approaches are closely related.

Chance Difference?

- When we are hypothesis testing, we are attempting to determine whether the sample statistic could plausibly have come from a population having a certain parameter value, or whether it just occurred by chance.
- For example, if we take a sample of 100 student test results and find the sample mean is 37. We would be more accepting of the hypothesis that the population mean is 40 rather than 60, as it is unlikely that a population with mean 60 would yield a sample of 100 having a mean of 37.
- But at what value would your acceptance change?

What is an Hypothesis?

- An hypothesis is an assumption, or statement about a population parameter. Some possible hypotheses are:
  - The population mean = 50.
  - The proportion of left handed people in the class is 0.1.

- We then test the hypothesis in a systematic way and state a conclusion based on the test.

The Null and Alternate Hypothesis

- When we test the hypothesis, we test against an alternative. For example, the hypothesis:
  - H_0: The population mean = 50
- Could be tested against the alternative hypothesis:
  - H_1: The population mean > 50.
- We use H_0: to denote the null hypothesis – this is the hypothesis that our population parameter has no difference from a particular value.
- The alternative hypothesis is denoted H_1.

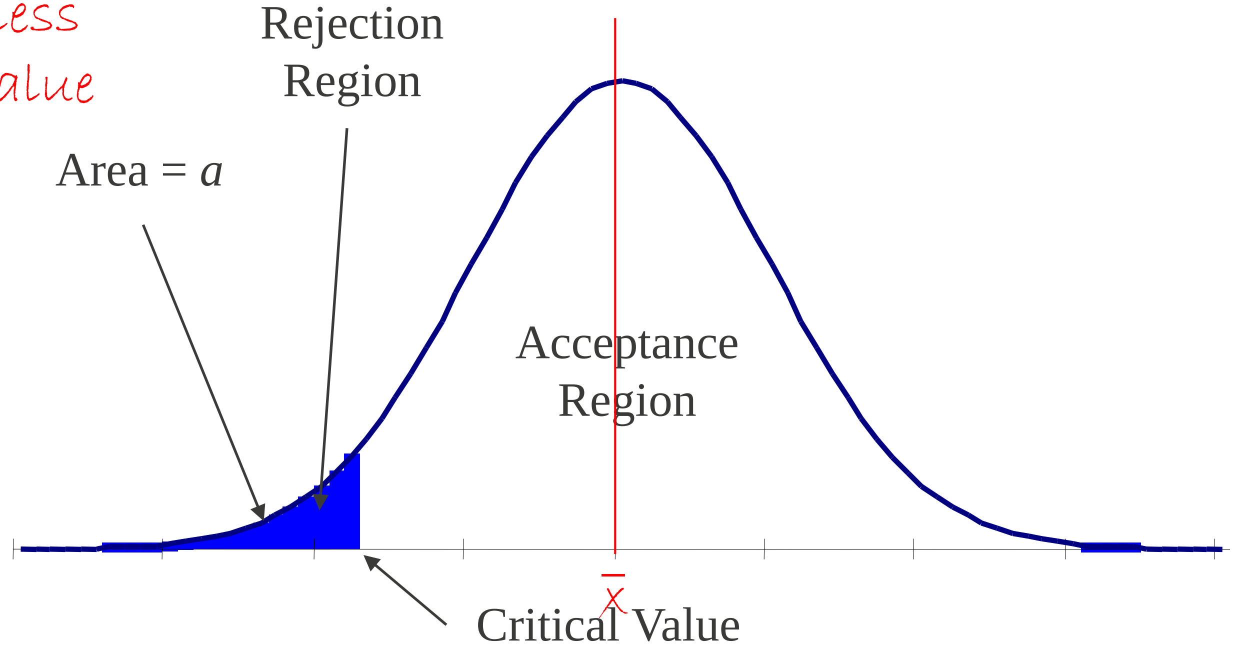
One and Two Sided Tests

- H_0: The population mean = 50
- Could be tested against **one sided** hypothesis:
  - H_1: The population mean > 50.
  - *Reject the hypothesis for high values of the sample statistic.*
- Alternatively H_0 could be tested against the **two sided** hypothesis:
  - H_1: The population mean $\neq$ 50.
  - *Reject the hypothesis for high **or** low values of the sample statistic.*

One Sided Test: rejection region

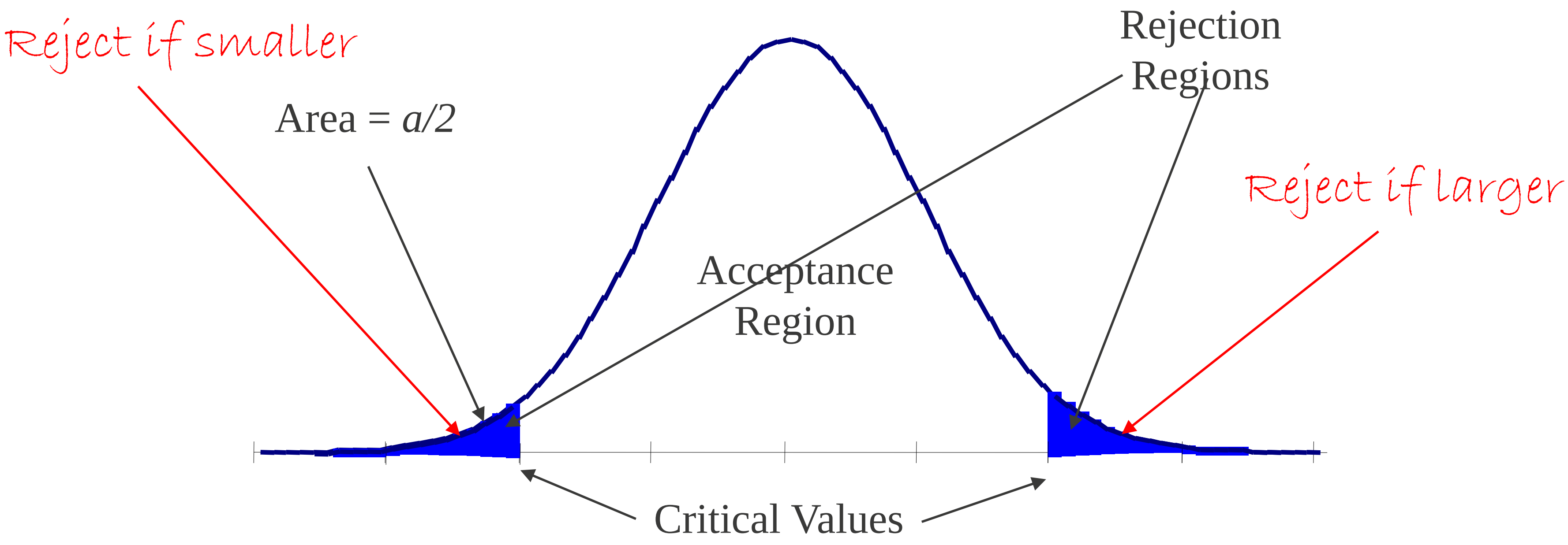
- For a one sided test there is an upper or lower rejection region. Reject H_0 if the statistic lies in the rejection region as defined by our test.

Reject if values less than a certain value

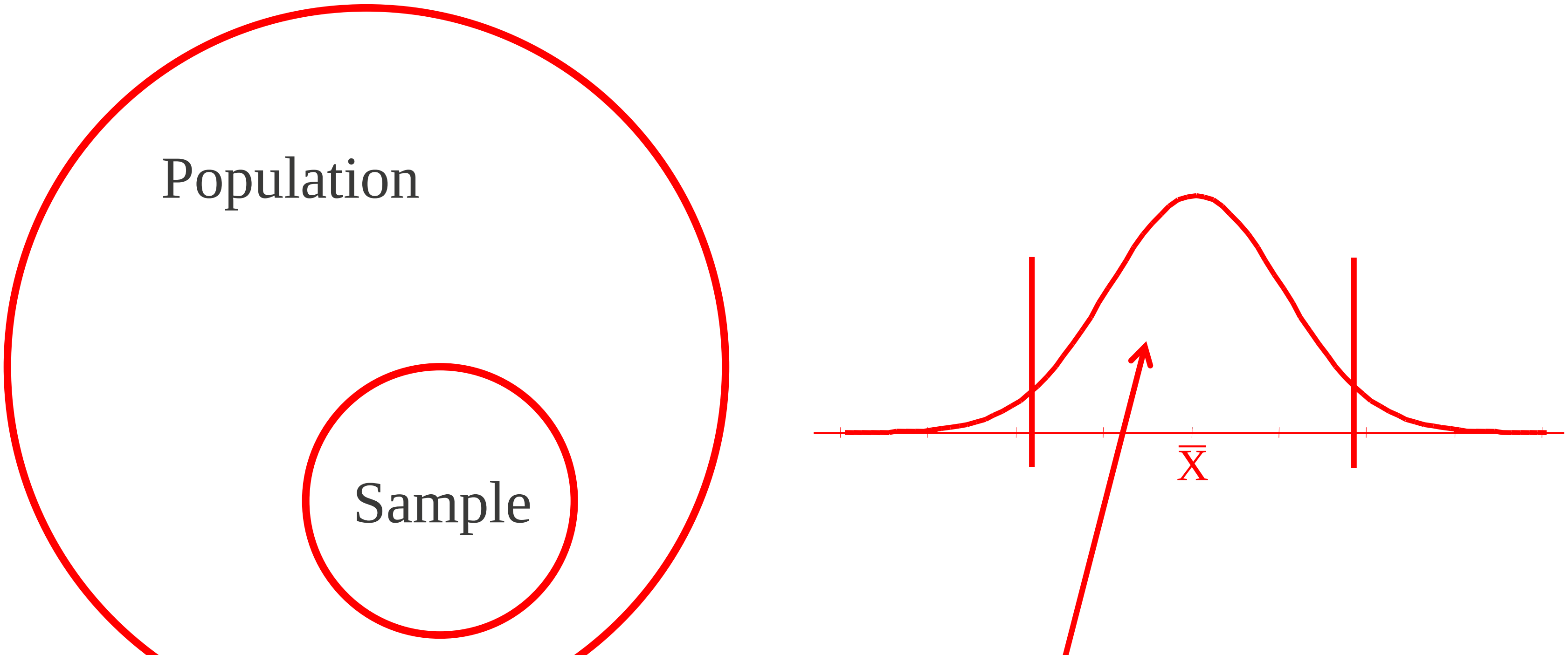


Two Sided Test: rejection region

- For a two sided test there is an upper and lower rejection region. Reject H_0 if the statistic lies in either region.

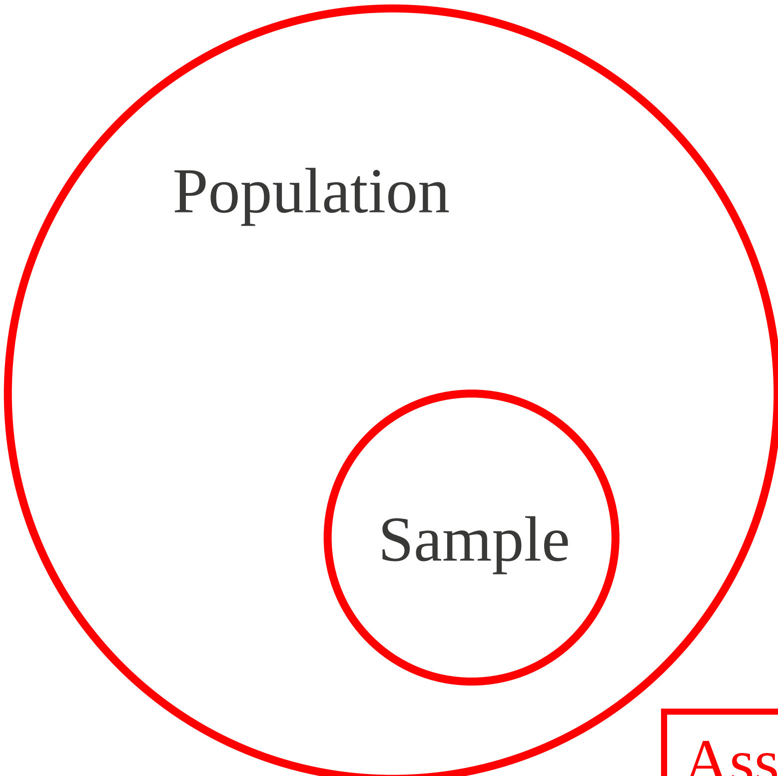


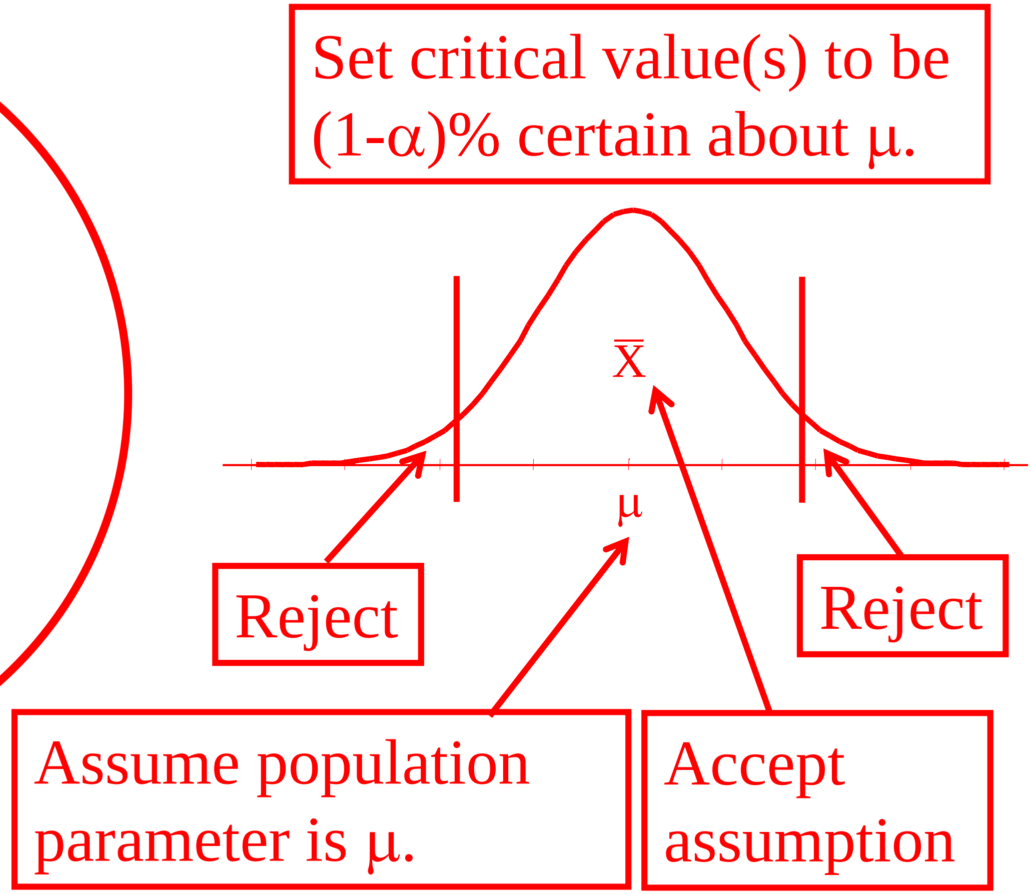
Estimation



We are (1-α)% sure the population parameter falls within this range

Hypothesis testing





H_0

The Steps of Hypothesis Testing

1. Decide on a null hypothesis H_0.
2. Decide on an alternative hypothesis H_1.
3. Decide on a significance level.
4. Calculate the appropriate test statistic.
5. Find from tables the corresponding tabulated test statistic.
6. Compare calculated and tabulated test statistics and decide whether to accept or reject the null hypothesis.
7. State the conclusion and assumptions of the test.

(Source: Rees, D.G. Essential Statistics, Chapman and Hall 1995.)

Example 1

- A hypothesis test for the population mean when the population variance is known.

(Note: unrealistic situation that lets us use the Normal distribution)

- The Axle manufacturing company has been making axles for a long time and kept records for every axle produced.
  - Population parameters are μ = 90mm and σ = 2.5mm.
  - A sample of 100 axles from new machine has mean = 92.7.
  - Is the new machine making parts with same average length required (90mm)?
  - Assume a 1% significance.

https://flux.qa (Feed code: SJ6KGV)

Question 1

From previous slide: The company is using a new machine... a sample of 100 yielded a mean length of 92.7mm. Which hypothesis test do we perform to test whether the new machine is producing parts with the same average length (of 90mm) as required?

A. H_0: $\mu = 90$, H_1 $\mu > 90$.

✓ B. H_0: $\mu = 90$, H_1 $\mu \neq 90$.

C. H_0: $\mu = 92.7$, H_1 $\mu > 92.7$.

D. H_0: $\mu = 92.7$, H_1 $\mu \neq 92.7$.

https://flux.qa (Feed code: SJ6KGV)

Question 2

For a 1% significance, 2, sided test, the *z* statistic is:

A. 1.645

B. 1.96

C. 2.33

✓ D. 2.58

α = significance = 0.01

This area = $\alpha/2 = 0.005$

This area = 0.995; $z = 2.58$

Cumulative Probabilities for the Standard Normal Distribution

Table gives P(Z<z) for Z = N(0,1)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952

Given in Slide 16:
$\mu = 90$mm and $\sigma = 2.5$mm

Solution

$$\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{92.7 - 90}{2.5/\sqrt{100}}$$
$$= 10.8$$

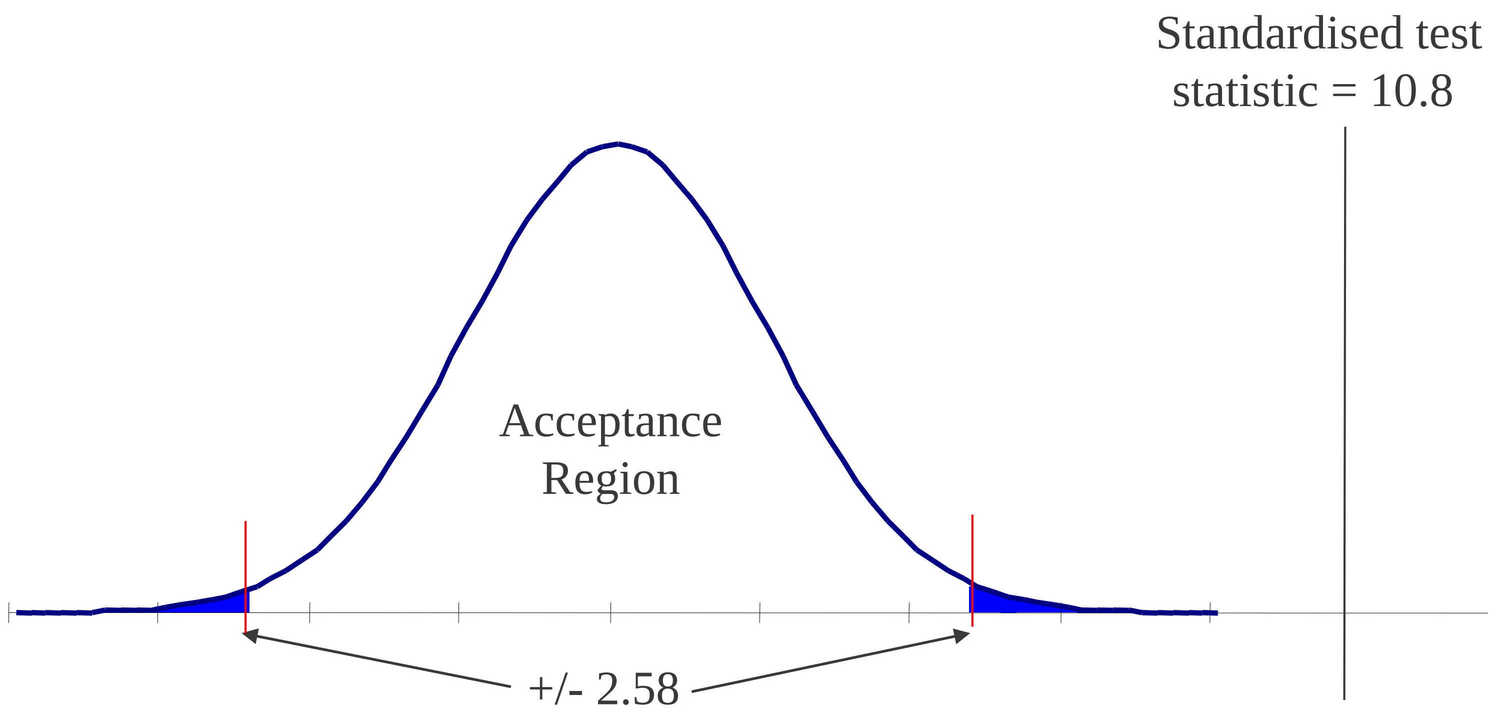
1. H_0, $\mu = 90$mm
2. H_1, $\mu \neq 90$mm (a two sided experiment)
3. Significance = 0.01.
4. The test statistic, $\bar{x} = 92.7$. We calculate Z_x.
   (standardising) $Z = (92.7 - 90)/(2.5/\sqrt{100}) = 10.8$

From the previous slide: Two tail critical value is 2.58

5. From tables the calculated critical values are ± 2.58
6. We see that $10.8 > 2.58$ and thus we reject H_0.
7. Thus we conclude that the axles produced by the new machine have a mean significantly different from 90mm (1% level), assuming that axle length is normally distributed.

…visually, this is what it means…

10.8 > 2.58 and thus we reject H_0



Example 2

- A hypothesis test for the population mean when the population variance is not known.
- It is claimed that Melbourne families are spending more than $150 per week on food and grocery items on average. A sample of 15 families was surveyed and the amount spent each week was recorded. Do these results support this thesis?
- Weekly food and grocery expenditure ($):
- 156, 234, 199, 78, 256, 189, 221, 49, 220, 178, 120, 290, 97, 177, 231.

Summary Statistics from Excel

$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{68.5}{\sqrt{15}}$

$= 17.7$

Mean	179.7
Standard Error	17.7
~~Median~~	~~189.0~~
~~Mode~~	~~#N/A~~
~~Standard Deviation~~	~~68.5~~
~~Sample Variance~~	~~4697.0~~
~~Kurtosis~~	~~-0.5~~
~~Skewness~~	~~-0.5~~
~~Range~~	~~241.0~~
~~Minimum~~	~~49.0~~
~~Maximum~~	~~290.0~~
~~Sum~~	~~2695.0~~
~~Count~~	~~15.0~~

https://flux.qa (Feed code: SJ6KGV)

Question 3

For a one sided test and 15 observations the *t* statistic for a 1% significance is:

A. 2.6025

B. 2.9467

✓ C. 2.6245

D. 2.9768

α = significance = 0.01
(since one tail)
$v = 15 - 1 = 14$

Critical Values of the t Distribution

Table gives upper critical values only

				α				
n	**0.300**	**0.200**	**0.150**	**0.100**	**0.050**	**0.025**	**0.010**	**0.005**
12	0.5386	0.8726	1.0832	1.3562	1.7823	2.1788	2.6810	3.0545
13	0.5375	0.8702	1.0795	1.3502	1.7709	2.1604	2.6503	3.0123
14	0.5366	0.8681	1.0763	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.5357	0.8662	1.0735	1.3406	1.7531	2.1314	2.6025	2.9467
16	0.5350	0.8647	1.0711	1.3368	1.7459	2.1199	2.5835	2.9208
17	0.5344	0.8633	1.0690	1.3334	1.7396	2.1098	2.5669	2.8982
18	0.5338	0.8620	1.0672	1.3304	1.7341	2.1009	2.5524	2.8784

Given in Slide 22:
$\mu = 179.7$ and $\sigma = 68.5$

Solution

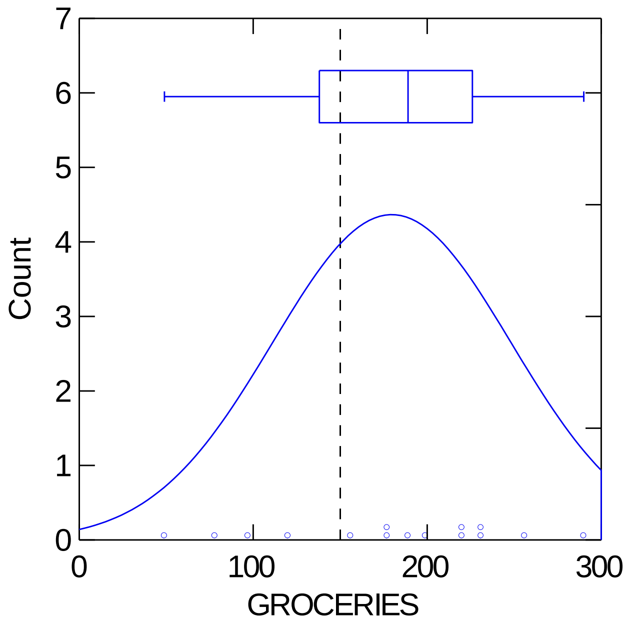
$\frac{\bar{x} - \mu}{\sigma_x} = \frac{179.7 - 150}{17.7}$
$= 1.67$

From the previous slide: One tail at $\alpha = 0.01$ and $v = 14$, critical value $= 2.625$

1. H_0, $\mu = \$150$
2. H_1, $\mu > \$150$ (a one tailed experiment)
3. Significance = 0.01.
4. The test statistic, $\bar{x} = 179.7$. We calculate T_x. *(standardising)* $T = (179.7 - 150) / (68.5/\sqrt{15}) = 1.67$
5. From tables our calculated critical value $T_{(0.01)(\nu=14)}$ is 2.625
6. We see that $1.67 < 2.625$ and thus we do not reject H_0.
7. Thus we conclude that the mean expenditure is not significantly greater than \$150, assuming that expenditure is normally distributed.

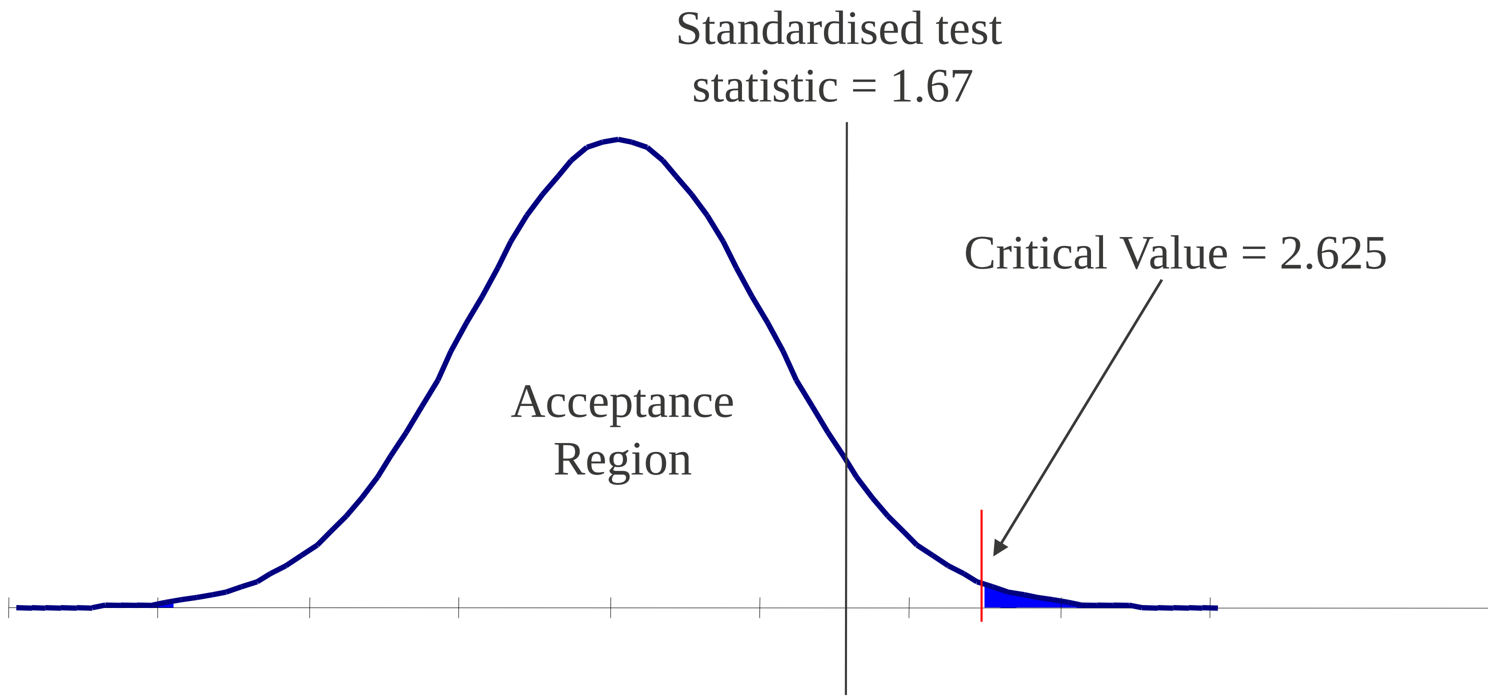
SYSTAT Solution

- One-sample t-test of GROCERIES with 15 cases
- Ho: Mean = 150.000 against Alternative = 'greater than'
-
- Mean = 179.667
- SD = 68.534
- t = 1.677
- df = 14
- p-value = 0.058



…visually, this is what it means…

1.67 < 2.625 and thus we do not reject H_0



Example 3

- A hypothesis test for a population proportion.

- In a contest of two political parties, a party will win if it gets more than 50% of the vote. In a contest between the Liberal Party and the Australian Labor Party for a particular electorate as survey of 237 voters, 180 indicated that they would vote for the ALP in the next election. Test the hypothesis that the ALP will win the next election.

$p = \frac{180}{237}$

Example 3 continued

- We assume that the rules of binomial probabilities hold (that is, *n* independent trials with probability π of success, *np > 5* and *n(1-p)>5*) and test the hypothesis that the population parameter $\pi > 0.5$.
- The standard error of the sample is determined using the assumed proportion in the hypothetical distribution, H_0 (π).

$$p = \frac{180}{237} = 0.7594$$

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.5(1-0.5)}{237}} = 0.0325$$

https://flux.qa (Feed code: SJ6K

Remember, 99% confident for one-sided test, Z = 2.33

Question 4

For a one sided test and 1% significance, z =

A. 1.645

B. 1.96

C. 2.33

D. 2.58

α = significance = 0.01 (since one tail)

Cumulative Probabilities for the Standard Normal Distribution

Table gives P(Z<z) for Z = N(0,1)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
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2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952

Given in Slide 28:
$p = 0.7594$ and $\sigma_p = 0.0325$

Solution

$\frac{p - \pi}{\sigma_p} = \frac{0.7594 - 0.5}{0.0325}$

$= 7.98$

1. H_0, $\pi = 0.5$
2. H_1, $\pi > 0.5$ (a one tailed experiment)
3. Significance = 0.01.
4. The test statistic, p = *0.7594*. We calculate the Z_x. *(standardising)* Z = *(0.7594 - 0.5)/ 0.0325* $= 7.98$
5. From tables our calculated critical value $Z_{(0.01)}$ is 2.33
6. We see that 7.98 > 2.33 and thus we reject H_0.
7. Thus we conclude that proportion of voters intending to vote ALP is greater than 50% at the 1% level. Assuming the rules for a binomial probability hold.

From the previous slide: One tail at α=0.01 critical value = 2.33

Motivating Problem

- Would Labor win if we have an election today?
- The Australian Newspoll had the two-party preferred vote at: ALP 51% vs Coalition (Liberal) 49% from a sample of 1,160 people chosen at random (taken on 25 April 2021).
- Ref: http://www.theaustralian.com.au/national-affairs/newspoll

https://flux.qa (Feed code: SJ6KGV)

Question 5

Would Labor have won a Federal Election if one is held today? Newspoll had the two-party preferred vote at: Labor 51% Liberal-NP 49% (taken on 25th April 2021). For this question, the null and alternate hypothesis is:

✓ A. H_0: $\pi = 0.50$, H_1 $\pi > 0.50$.

B. H_0: $\pi = 0.50$, H_1 $\pi \neq 0.50$.

C. H_0: $\pi = 0.51$, H_1 $\pi > 0.51$.

D. H_0: $\pi = 0.51$, H_1 $\pi \neq 0.51$.

Motivating Problem in Groups

(a) Is Labor going to win the next election based on these data? (Assume 1% Significance)

$p = 0.51$

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.5(1-0.5)}{1160}} = 0.0147$$

$$Z_{Calc} = \frac{0.52 - 0.50}{0.0147} = 0.68$$

$$Z_{Tables} = 2.33$$

Since $0.68 < 2.33$ we do not reject H_0

https://flux.qa (Feed code: SJ6KGV)

Question 6

The Newspoll (25 April 2021) gave the two-party preferences at 51% Labor and 49% Coalition, for a sample size of 1160 (*Calc z = 0.68*). Based on this information, if an election had been held at the time:

A. Labor would have won (*99% confidence*)

B. Coalition would have won (*99% confidence*)

✓ C. Too close to call (1% sig)

D. Just don't know…

Reading/Questions (Selvanathan)

- Reading: Hypothesis Testing
    - 7th Ed. Sections 12.3, 12.5, 12.6.

- Questions: Hypothesis Testing
    - 7th Ed. Questions 12.1, 12.19, 12.25, 12.26, 12.56, 12.59, 12.65, 12.66, 12.67, 12.70, 12.72, 12.74.