MONASH University

Information Technology

FIT1006
Business Information Analysis

Lecture 19
Time Series Analysis and Forecasting

Topics covered:

- Time series data
- Components of a time series
- Smoothing with moving averages and medians
- Exponential smoothing.

Lectures 19/20 Motivating problem

- Given the value of building work (quarterly) from Sep 1974 – Dec 2018 .
- Model time series.
- Use historical data to forecast demand for 2019 and 2020.
- Source: ABS.

http://www.abs.gov.au

(File: FIT1006 Lecture 19 and 20.xlsx)

Quarter/Year	Value of Building Work (all sectors) $'Bil
Sep-1974	11.53
Dec-1974	11.06
Mar-1975	9.64
Jun-1975	10.41
Sep-1975	11.15
Dec-1975	10.65
Mar-1976	10.18
Jun-1976	11.37
Sep-1976	11.63
Dec-1976	11.37
Mar-1977	10.14
Jun-1977	11.12
Sep-1977	11.07
Dec-1977	10.57
:	
Mar-2017	27.75
Jun-2017	30.59
Sep-2017	31.52
Dec-2017	31.86
Mar-2018	29.26
Jun-2018	32.84
Sep-2018	32.99
Dec-2018	32.69

Cont.

- If the actual value of building work in 2019 & 2020 is now known (as shown in the table), calculate the accuracy of the forecast.

Quarter/Year	Value of Building Work (All sectors) $'Bil
Mar-2019	29.74
Jun-2019	31.08
Sep-2019	32.17
Dec-2019	30.83
Mar-2020	28.35
Jun-2020	30.14
Sep-2020	30.24
Dec-2020	30.14

Time Series

- A Time Series describes a set of observations made over a period of time. Daily maximum temperatures, hourly share prices, annual population counts, weekly sales figures are all examples of time series.

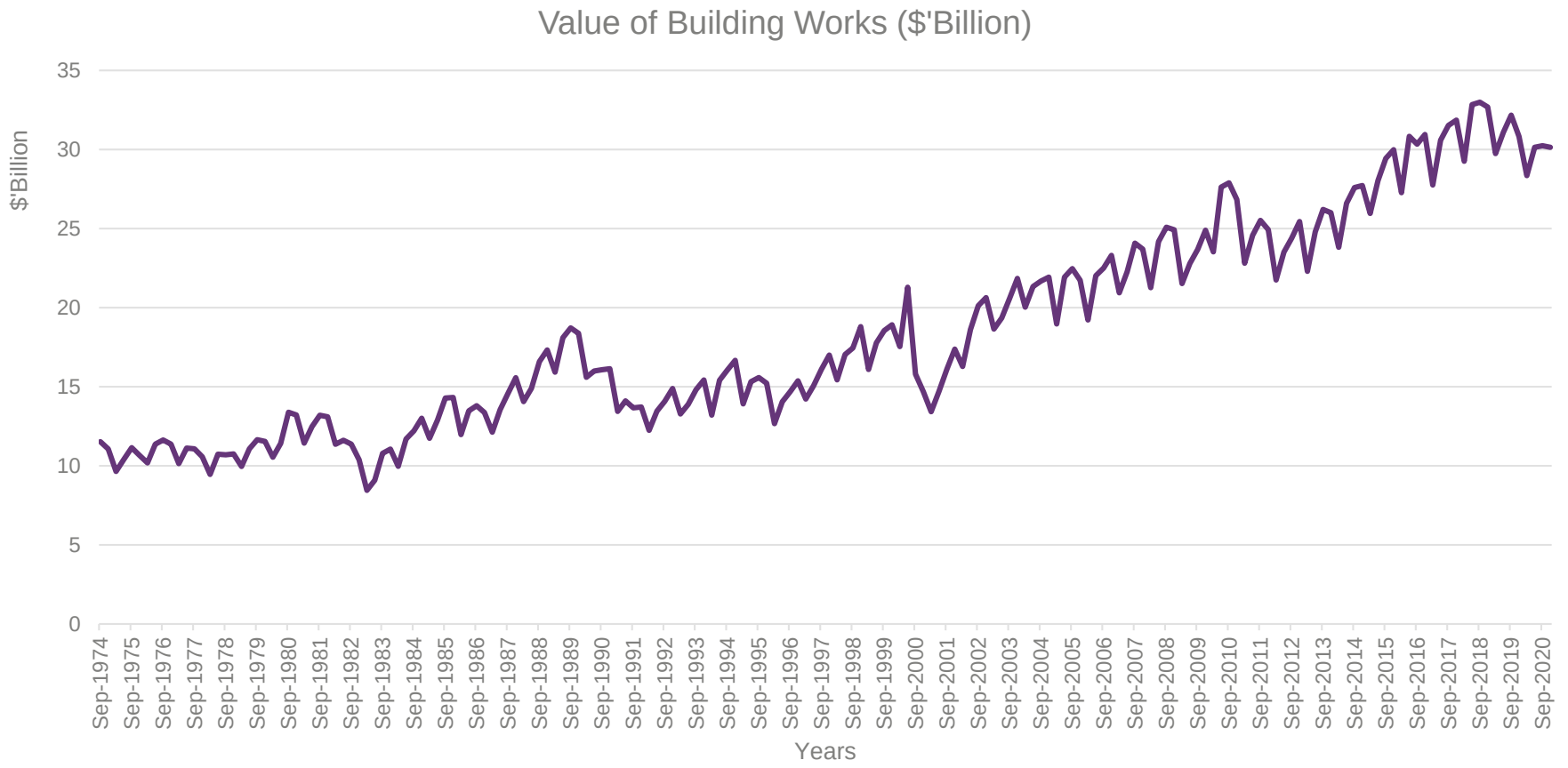
- It is usual, but not strictly necessary, that the observations are recorded at equal intervals.

- Some examples of time series follow:

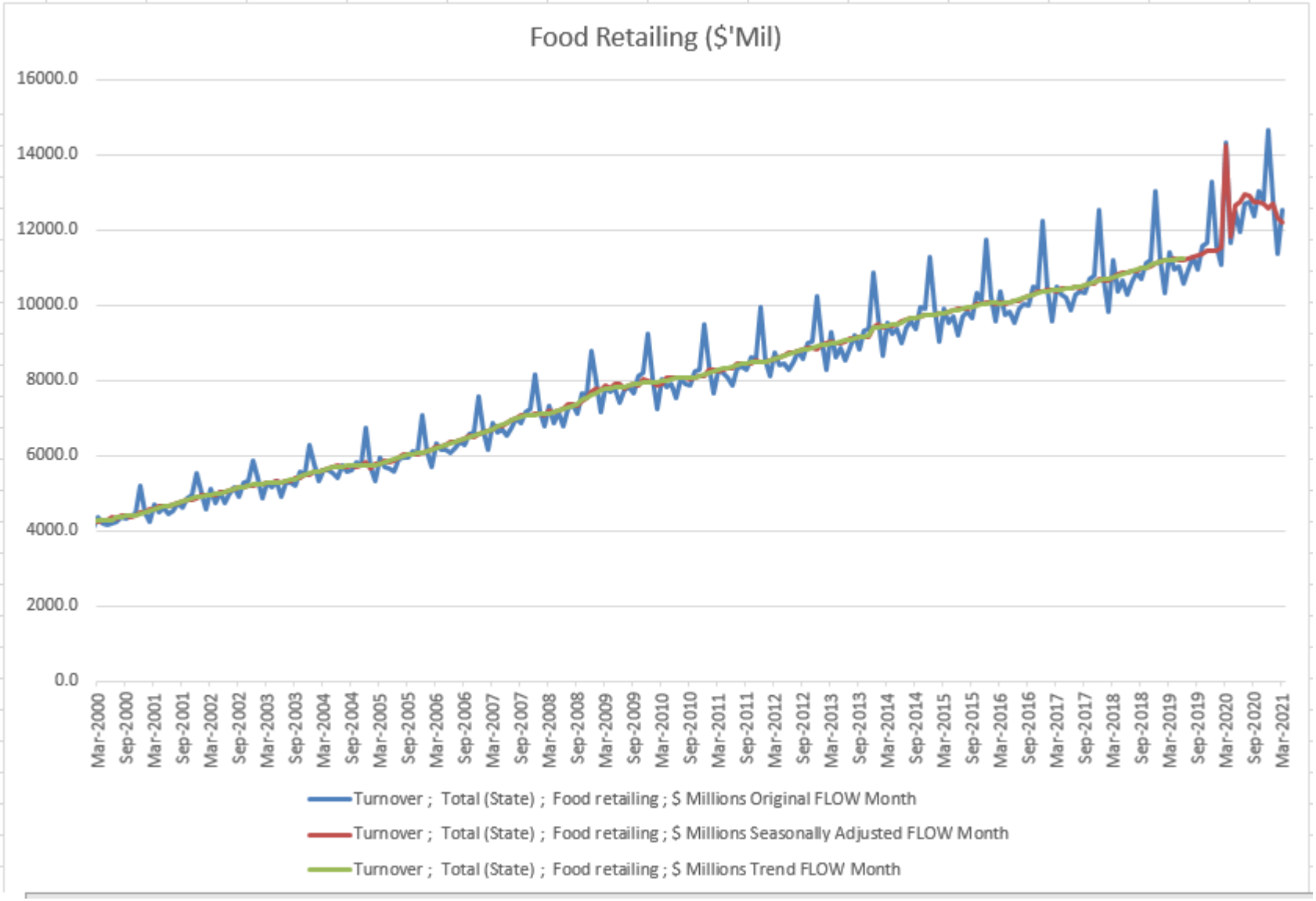
Australian All Ordinaries

Source: https://markets.businessinsider.com/index/asx

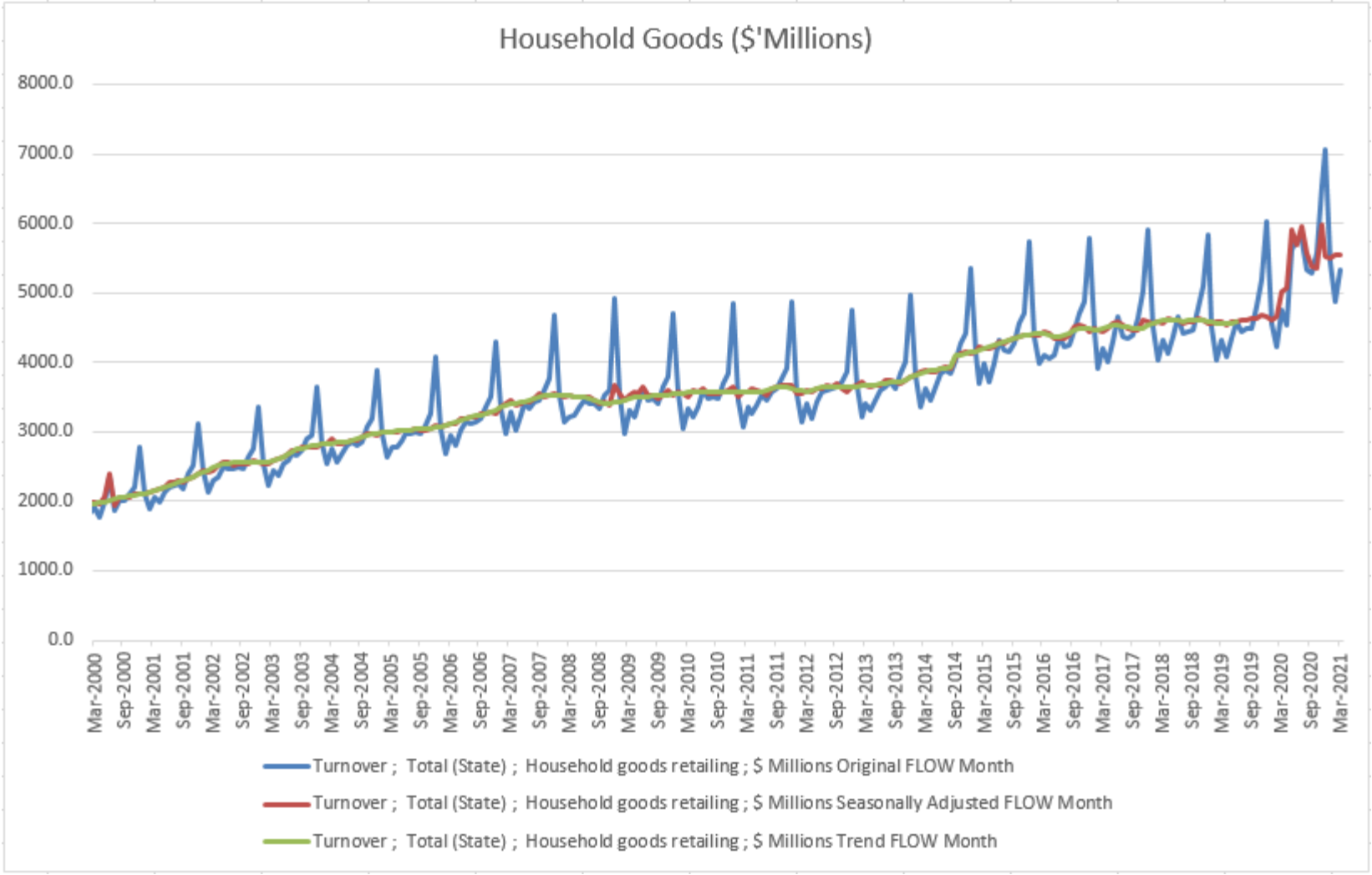
Value of Building Work (All Sectors)



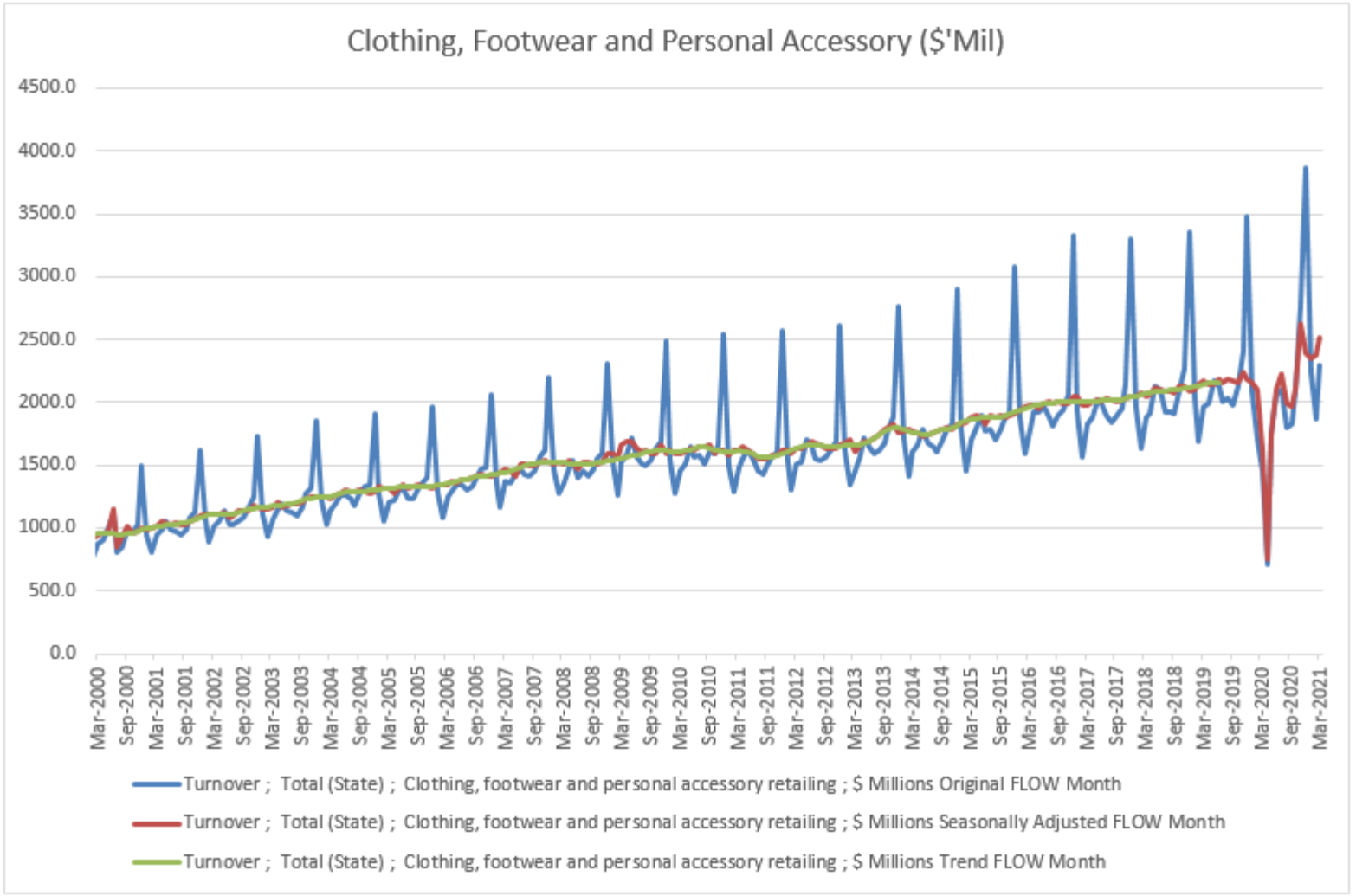
Monthly Food Retail Sales



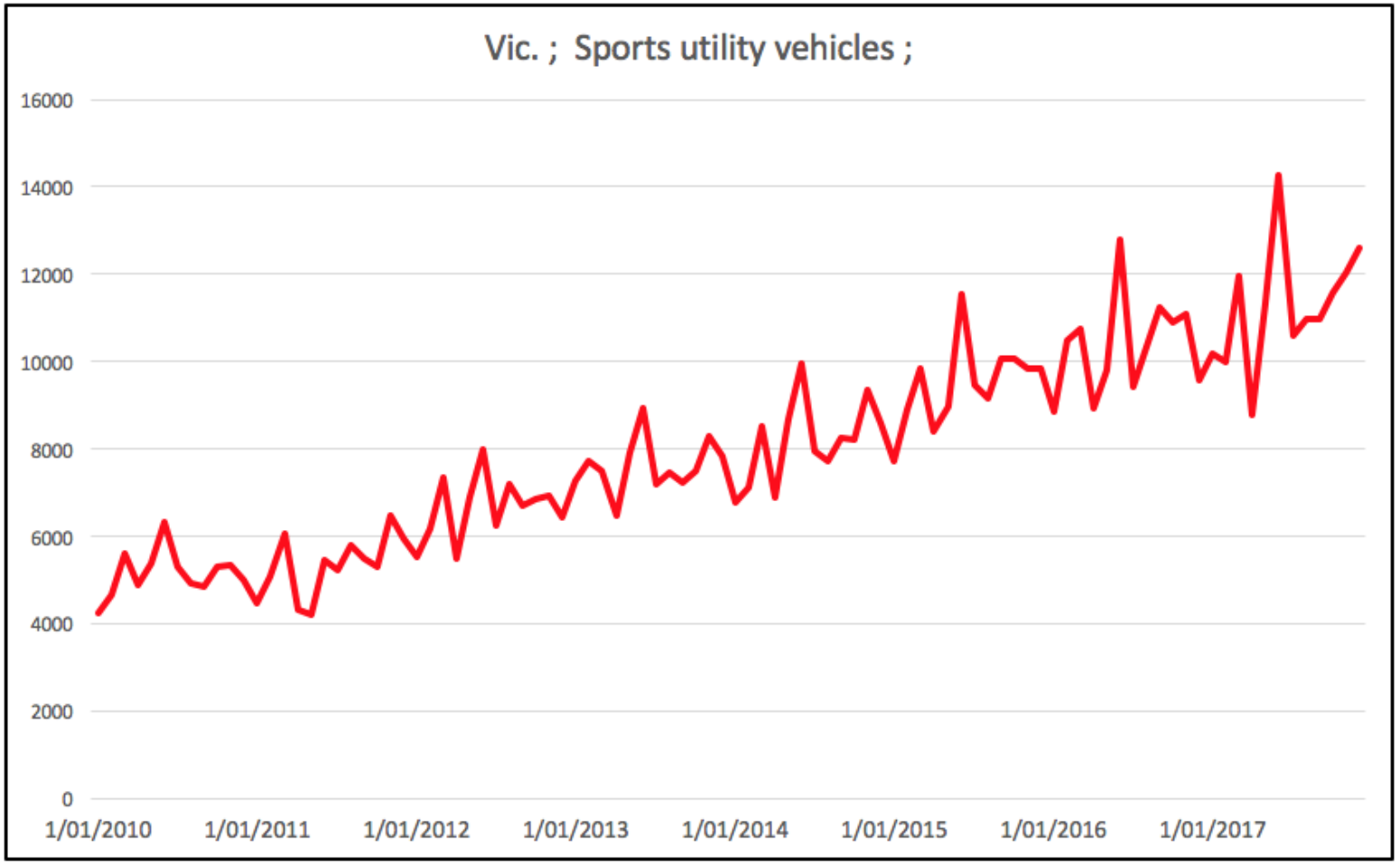
Monthly Household Goods Sales



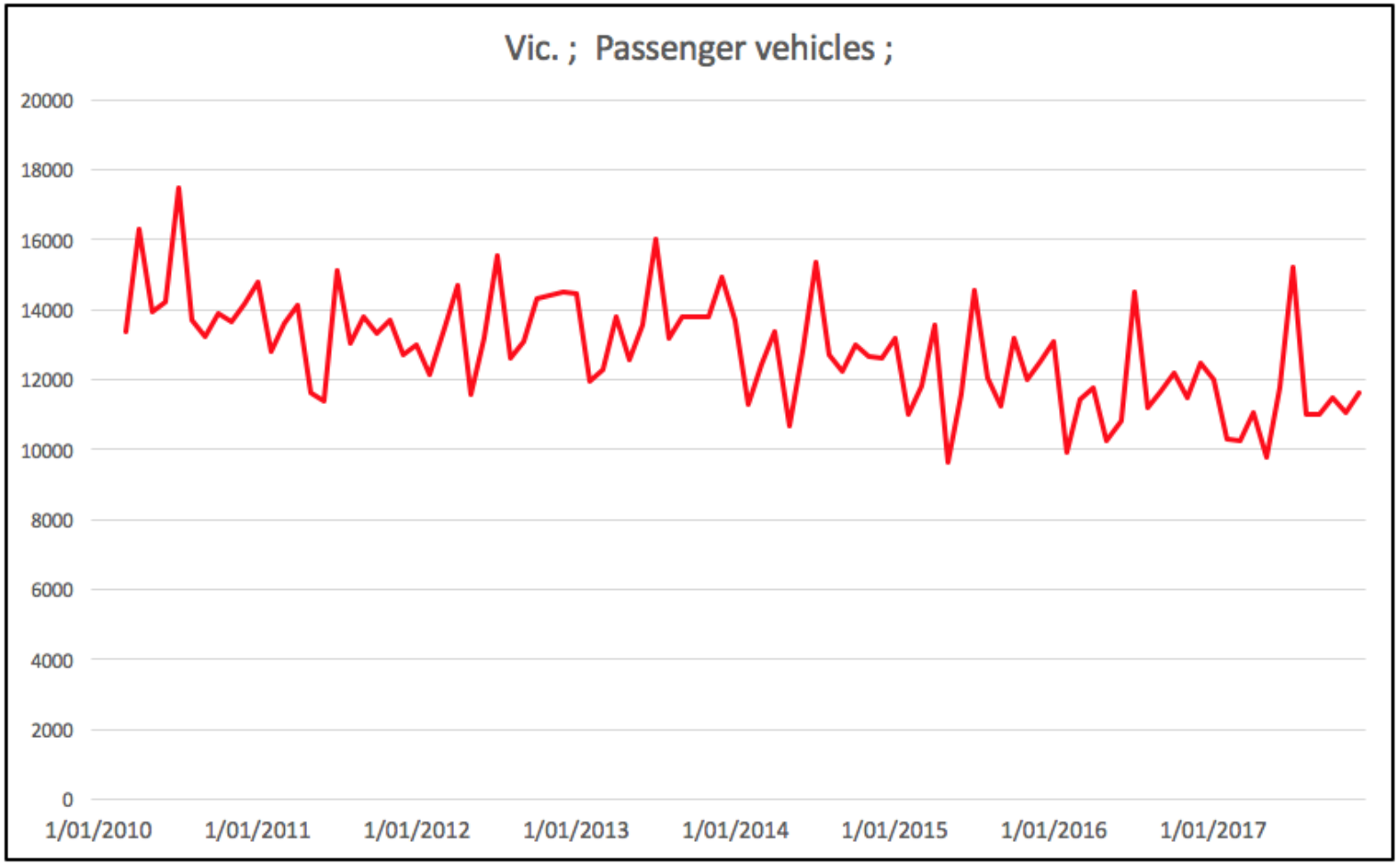
Monthly Clothing & Footwear Sales



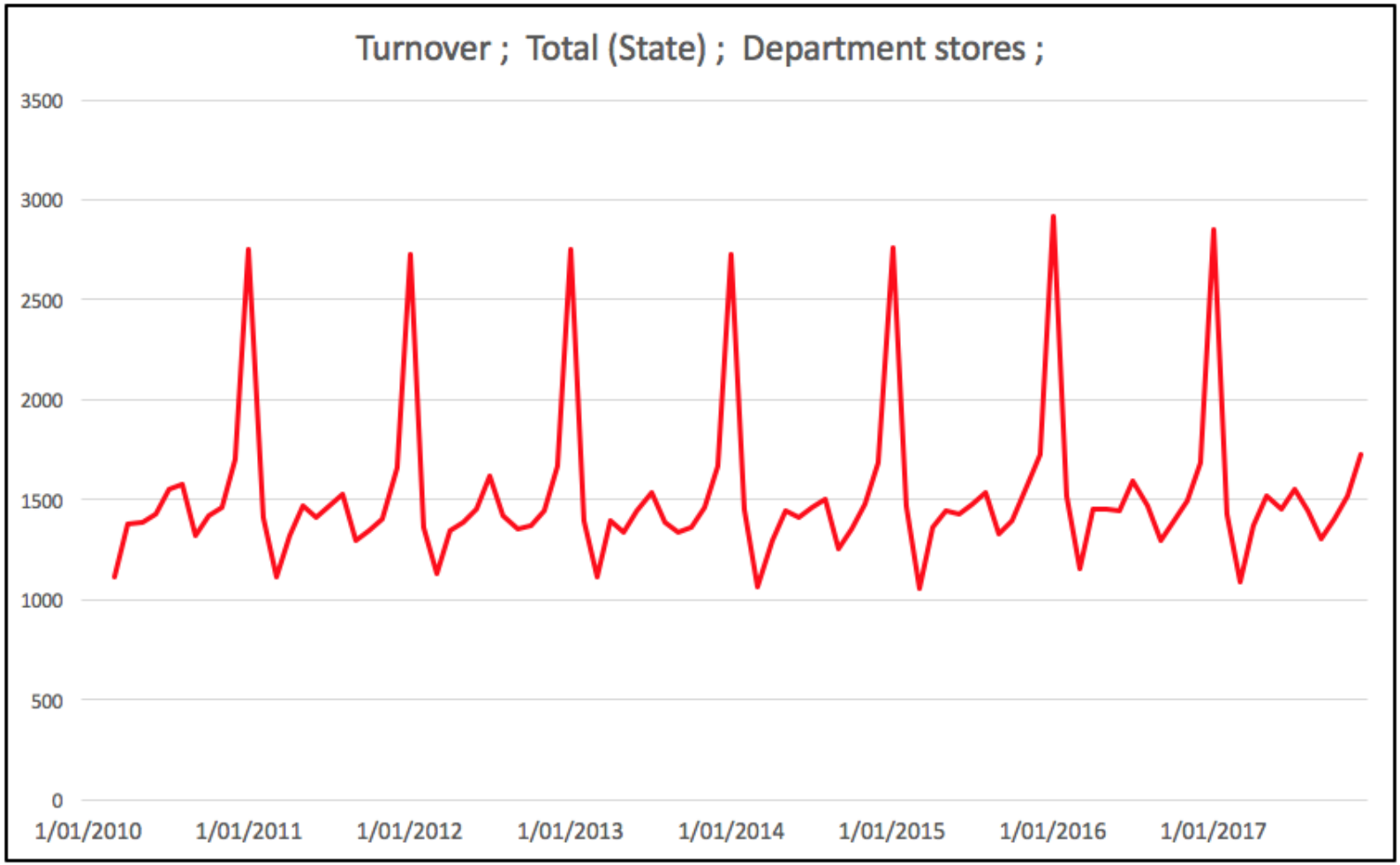
SUV sales, Monthly



Passenger vehicle sales, Monthly



Retail turnover, department stores



TS Analysis vs Forecasting

- Time series describes a set of observations made over a period of time – example, the daily maximum temperatures, hourly share prices, annual population counts, weekly sales figures , etc.
- Time series analysis is the description and modelling of a time series. For instance we might attempt to describe patterns in the data with a mathematical model.
- Forecasting is the method of attempting to predict the value of future observations from past data.
- Forecasting from past data is of great interest to business, for example in retailing and the financial sector.

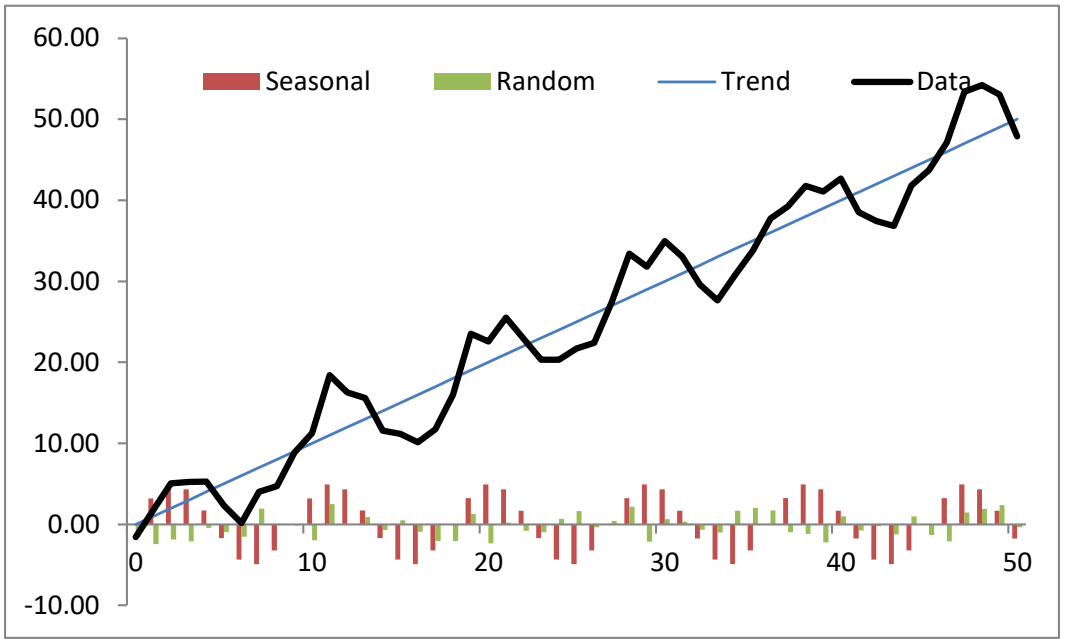
Components of a Time Series

- Time series can be thought of as being composed of three elements:
  - *Trend*, (absence of trend is ‘stationary’)
  - *Seasonal* or *cyclic* element, and
  - a *Random* component.

Additive Model

- Data has an additive model when it is reasonable to assume that the observed time series can be explained as:

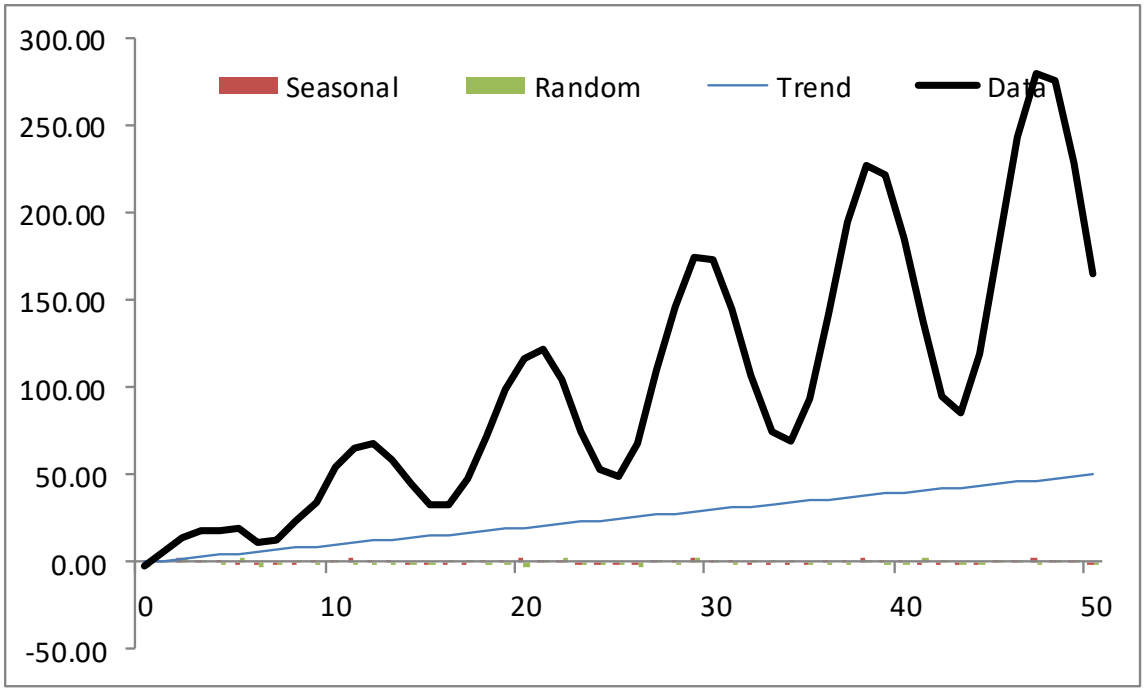
  Data = Trend + Seasonal Variation + Random Variation



Multiplicative Model

- Data has a multiplicative model when it is reasonable to assume that the observed time series can be explained as:

  *Data = Trend * Seasonal Variation * Random Variation*

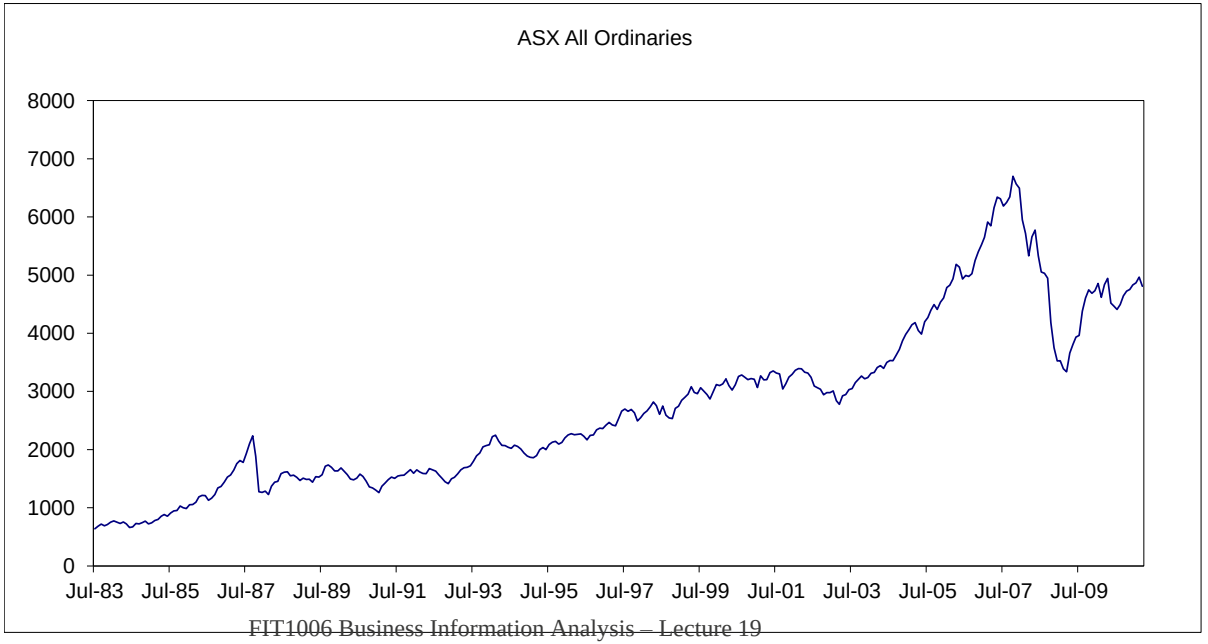


https://flux.qa (Feed code: SJ6KGV)

Question 1

The main features of this time series are:

- ✓ A. trend & random
- B. seasonal & random
- C. trend & seasonal
- D. random only



https://flux.qa (Feed code: SJ6KGV)

Question 2

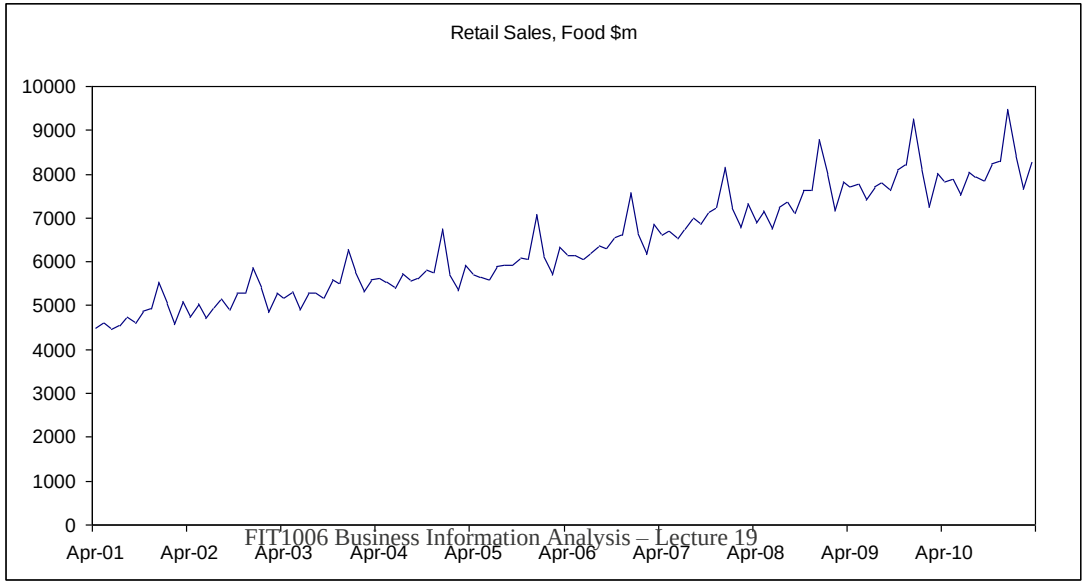
The main features of this time series are:

A. trend & random

B. seasonal & random

C. trend & seasonal

✓ D. trend & random & seasonal



https://flux.qa (Feed code: SJ6KGV)

Question 3

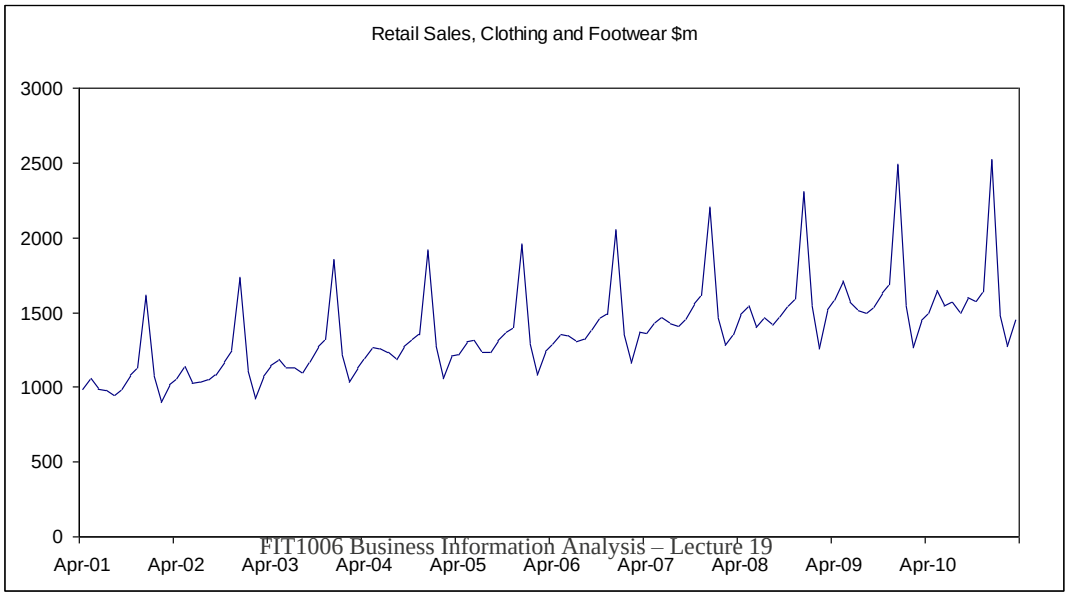
The main features of this time series are:

A. trend & random

B. seasonal & random

C. trend & seasonal

✓ D. trend & random & seasonal



https://flux.qa (Feed code: SJ6KGV)

Question 4

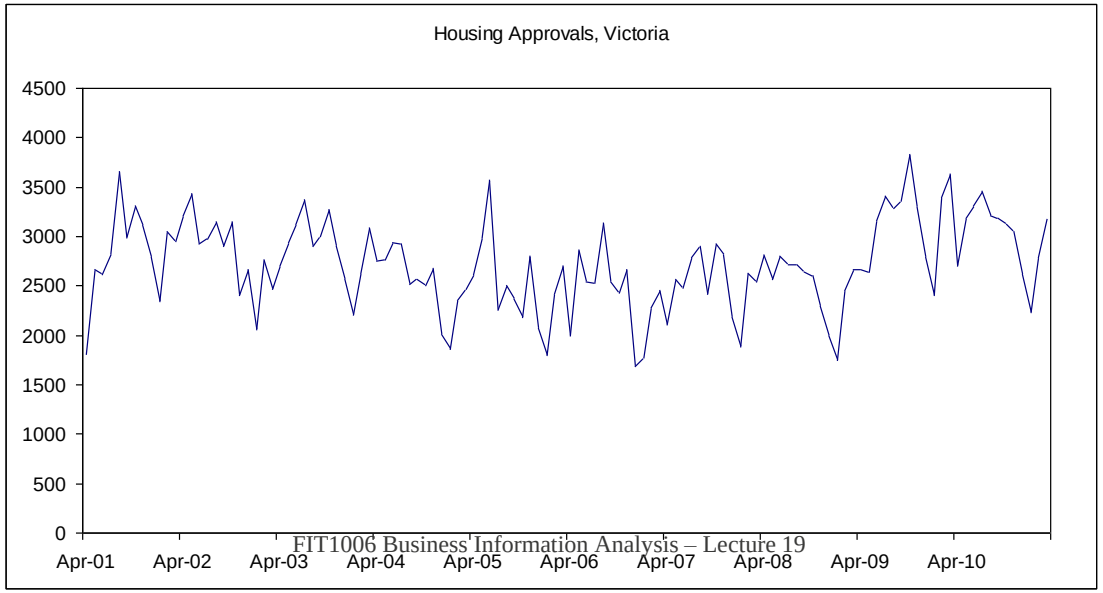
The main features of this time series are:

A. trend & random

✓ B. seasonal & random

C. trend & seasonal

D. trend & random & seasonal



https://flux.qa (Feed code: SJ6KGV)

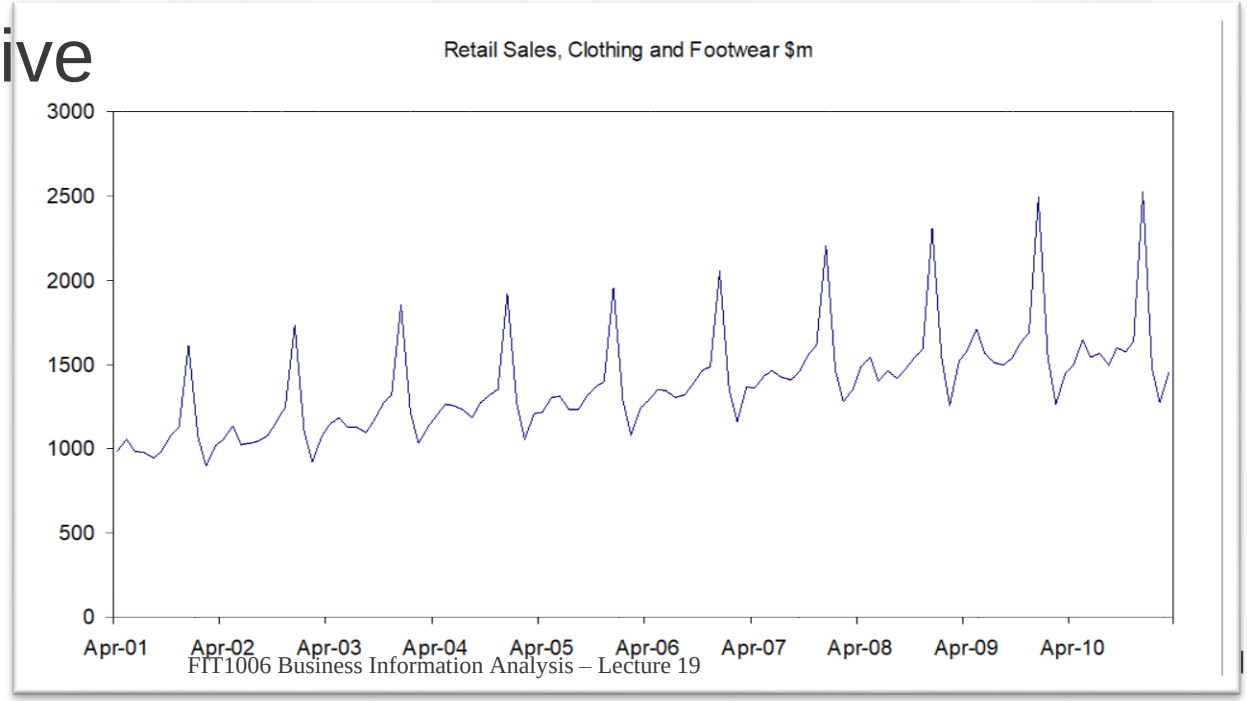
Question 5

The main features of this time series are:

A. trend & random & seasonal additive

✓ B. trend & random & seasonal multiplicative

C. Inconclusive

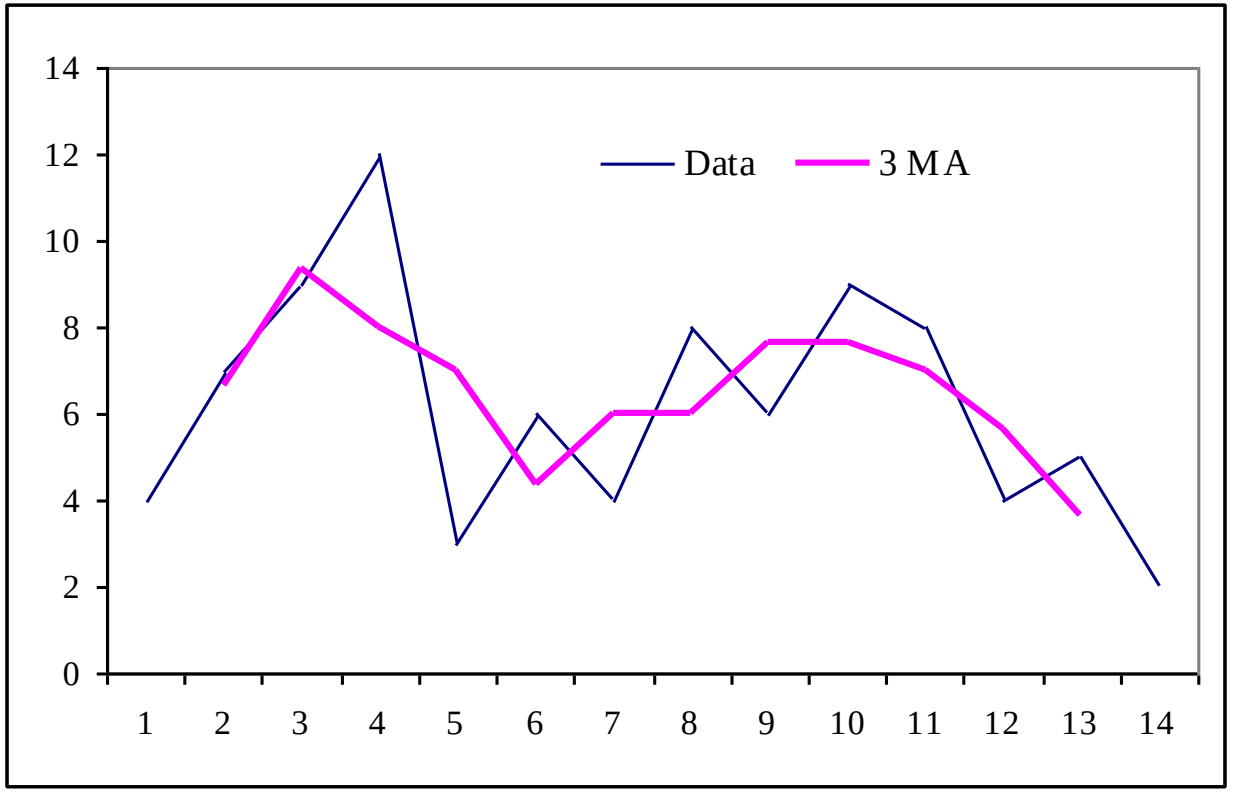


Moving Averages

- One of the first tasks in the analysis of **additive** time series is to smooth the data using a moving average.
- As the name suggests, a moving average works by successively taking observations over a number of periods and averaging. The average of the time indexes locates the moving average in time.
- Odd numbers of data are preferred for MA's because the data remains centred (time index is an integer), 3, 5, 7 being usual lengths. For quarterly data, a centred 4 period average is used. Medians are also used for robust smoothing.

3 Period Moving Average

Data	3 MA	3 MA
4		
7	=(4+7+9)/3	6.67
9	=(7+9+12)/3	9.33
12	=(9+12+3)/3	8.00
3	=(12+3+6)/3	7.00
6	…	4.33
4	…	6.00
8	…	6.00
6	…	7.67
9	…	7.67
8	…	7.00
4	…	5.67
5	…	3.67
2		



5 Period Moving Average

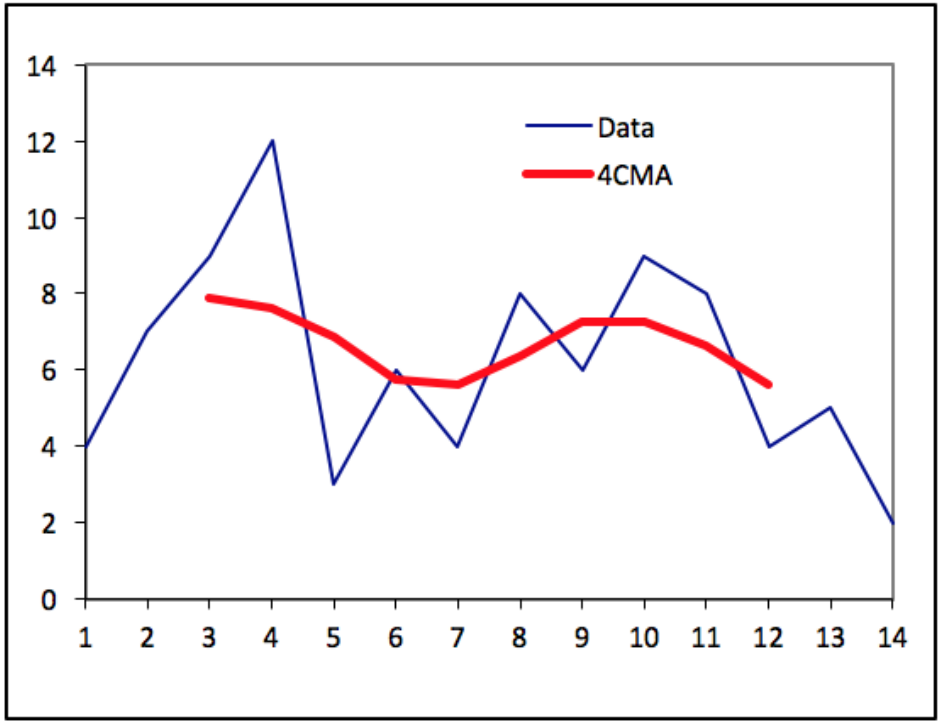
Data	5 MA	5 MA
4		
7		
9	=(4+7+9+12+3)/5	7.00
12	=(7+9+12+3+6)/5	7.40
3	=(9+12+3+6+4)/5	6.80
6	…	6.60
4	…	5.40
8	…	6.60
6	…	7.00
9	…	7.00
8	…	6.40
4	…	5.60
5		
2		

Centred 4 Period Moving Average

- For quarterly data, or other data with cycles of 4 periods, a centred 4 period moving average is often used.
- The reasoning for this method is as follows:
  - The moving average contains 4 observations, which comprise a single cycle (Summer Autumn Winter Spring).
  - For observations in periods 1, 2, 3 and 4, the time index of the average is at period 2.5, i.e., between observations 2 and 3.
  - We thus take the average of pairs of off-centred observations to re-centre them.
  - This method can be adapted for other even numbered cycles.

Centered 4 period moving average

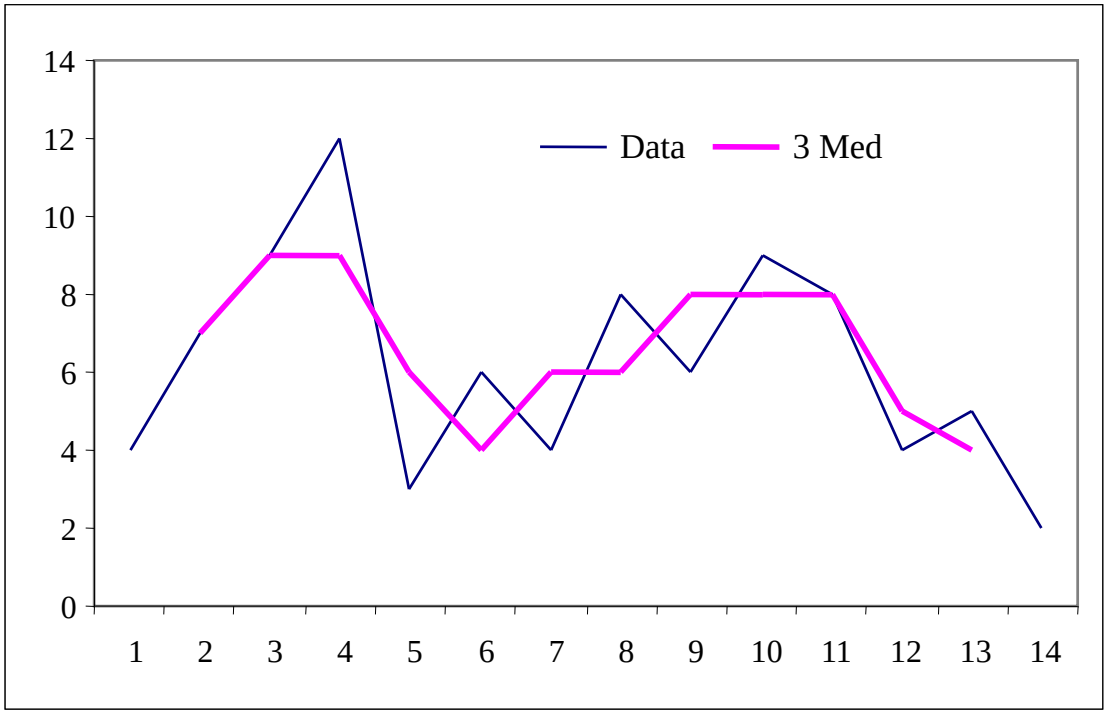
Period	Data	4 MA		4CMA	4CMA
1	4				
2	7	=(4+7+9+12)/4	8.00		
3	9	=(7+9+12+3)/4	7.75	=(8.00+7.75)/2	7.88
4	12	=(9+12+3+6)/4	7.50	=(7.75+7.50)/2	7.63
5	3	=(12+3+6+4)/4	6.25	=(7.50+6.25)/2	6.88
6	6	...	5.25	...	5.75
7	4	...	6.00	...	5.63
8	8	...	6.75	...	6.38
9	6	...	7.75	...	7.25
10	9	...	6.75	...	7.25
11	8	...	6.50	...	6.63
12	4	...	4.75	...	5.63
13	5			...	
14	2				



Smoothing With Medians

- Medians can also be used to smooth data. They are robust to outliers, although not as ‘smooth’ as means.

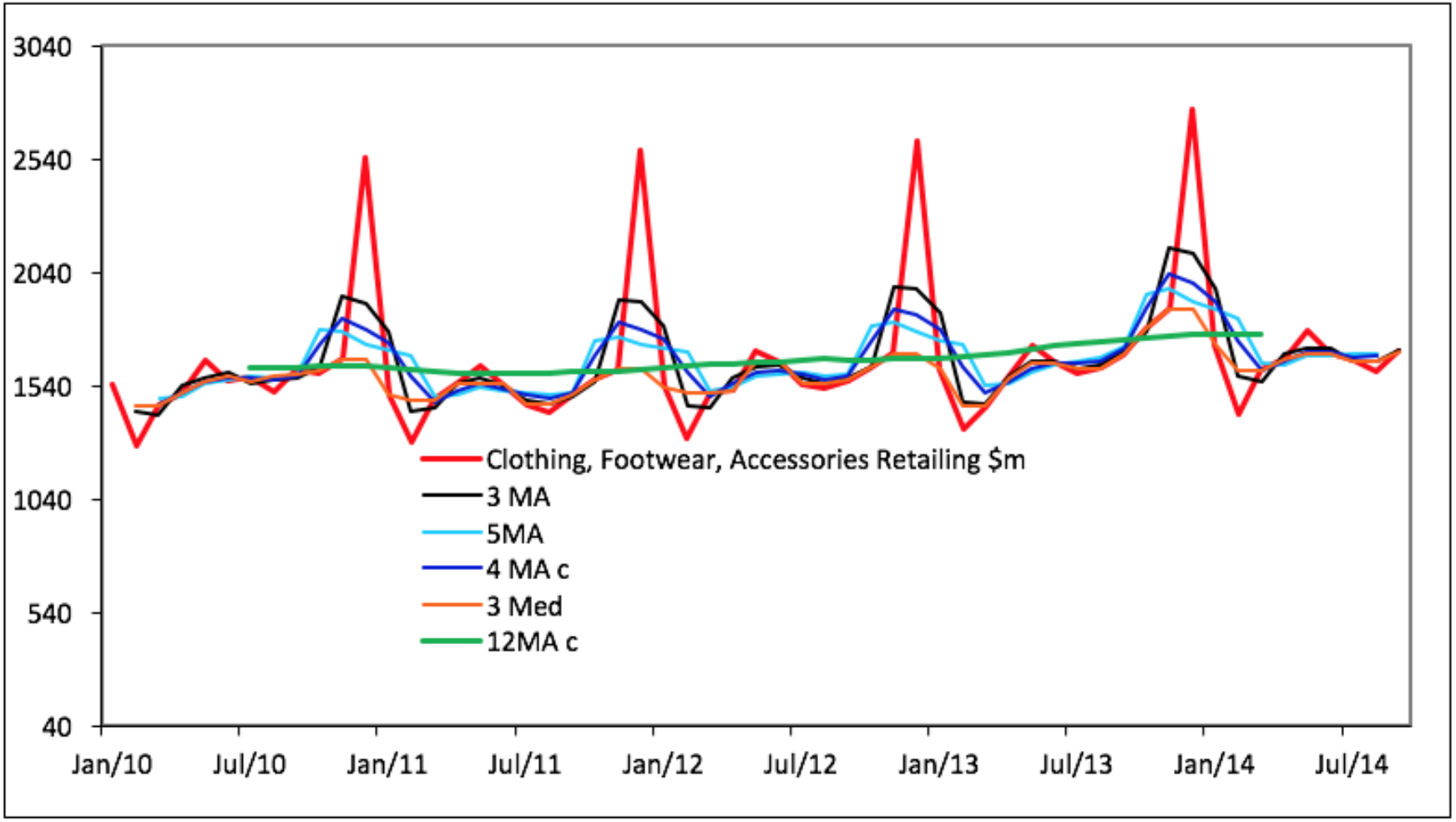
Data	3 Median	3 Med
4		
7	=Median(4, 7, 9)	7.00
9	=Median(7, 9, 12)	9.00
12	=Median(9, 12, 3)	9.00
3	…	6.00
6	…	4.00
4	…	6.00
8	…	6.00
6	…	8.00
9	…	8.00
8	…	8.00
4	…	5.00
5	…	4.00
2		



Methods Compared (FIT1006 Lecture 19 and 20.xlsx)

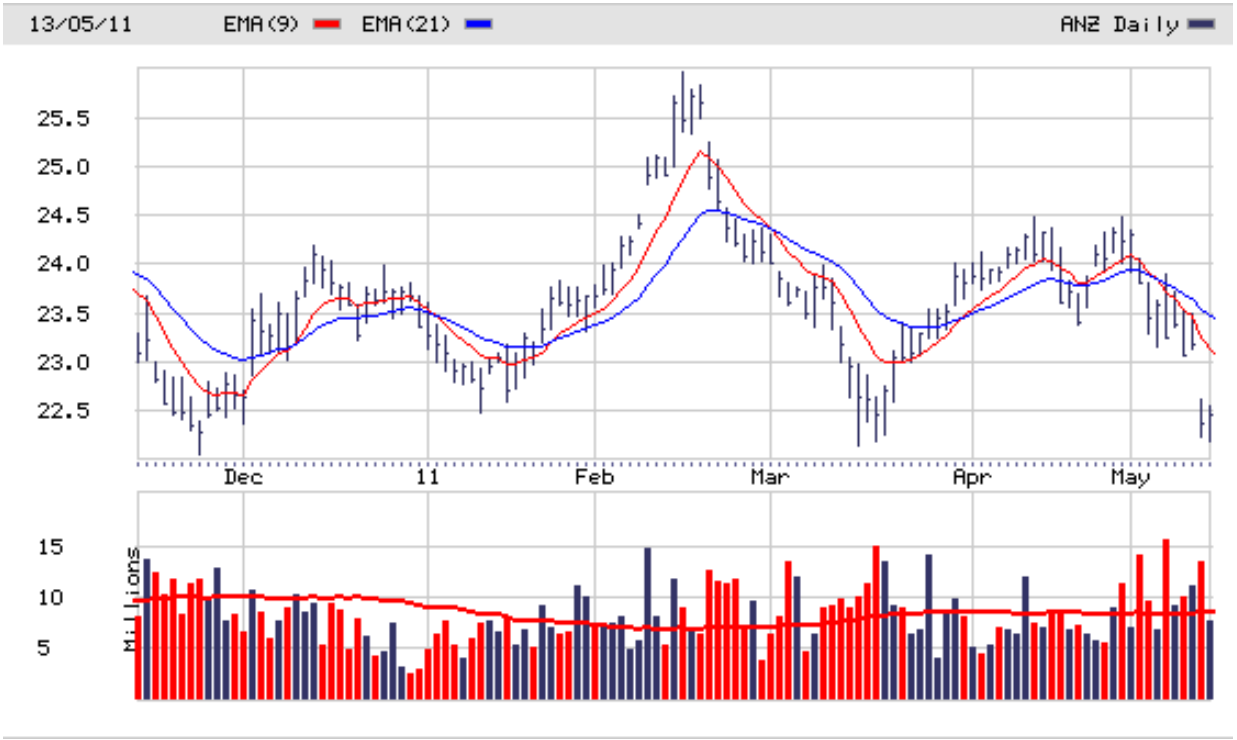
Month Year	Clothing, Footwear, Accessories Retailing $m	3 MA	5MA	4 MA c	3 Med	12MA c
1/01/2010	1545.9					
1/02/2010	1273.6	1423.9			1452.3	
1/03/2010	1452.3	1411.1	1486.6	1458.3	1452.3	
1/04/2010	1507.5	1537.8	1489.1	1507.4	1507.5	
1/05/2010	1653.6	1573.2	1550.2	1558.8	1558.6	
1/06/2010	1558.6	1597.1	1561.9	1575.1	1579.0	
1/07/2010	1579	1549.5	1582.5	1570.1	1558.6	1622.2
1/08/2010	1511	1566.7	1570.7	1569.2	1579.0	1621.0
1/09/2010	1610.1	1571.9	1590.2	1583.3	1594.6	1622.8
1/10/2010	1594.6	1620.3	1783.8	1722.5	1610.1	1625.6
1/11/2010	1656.3	1932.6	1781.4	1838.1	1656.3	1626.3
1/12/2010	2546.8	1900.7	1717.7	1786.3	1656.3	1624.6
1/01/2011	1499	1779.2	1694.4	1726.2	1499.0	1618.9
1/02/2011	1291.8	1423.0	1673.1	1579.3	1478.2	1610.1
1/03/2011	1478.2	1440.0	1489.0	1470.6	1478.2	1601.7
1/04/2011	1549.9	1551.4	1498.4	1518.3	1549.9	1596.0
1/05/2011	1626.1	1574.0	1531.3	1547.3	1549.9	1592.9

Methods Compared: Graph



Analysis of share prices

- Moving average forecasts over different periods are used by some share analysts to determine when a share is trending up or down. Figure below is from E*trade.

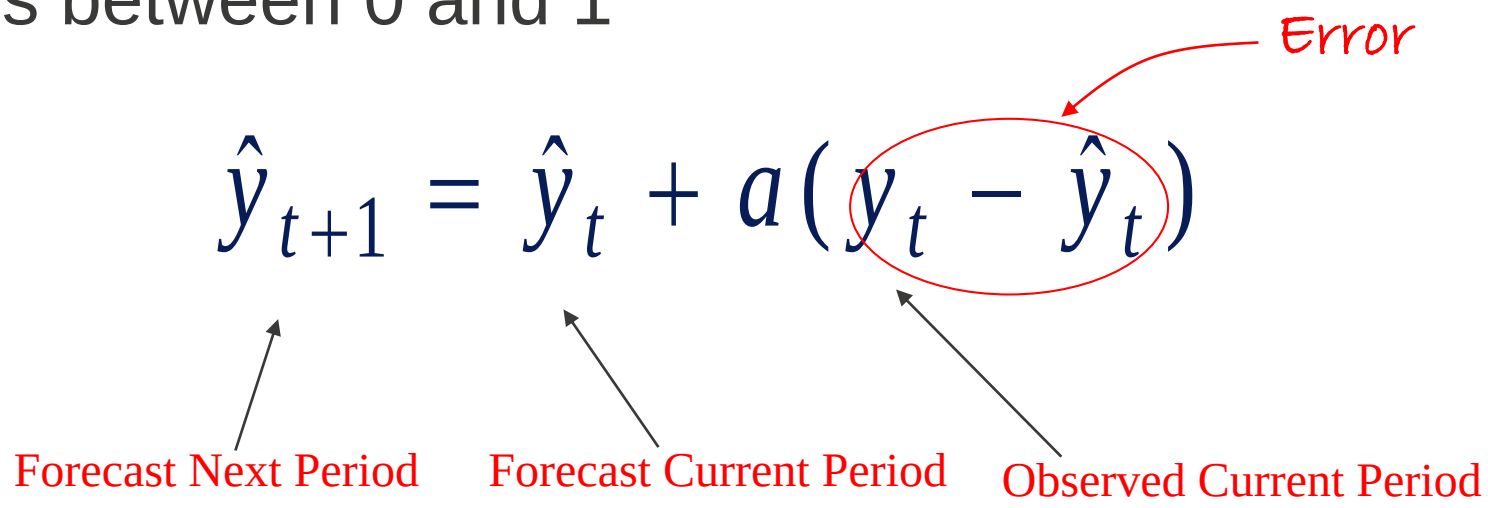


Exponential Smoothing

- Exponential smoothing is a way of forecasting one, two, three … periods ahead, using historical data.
- Exponential smoothing uses an observation at time t as well as the forecast value at time *t*. The forecast for the following period is based on the current observation less a proportion of the error observed in the current period.
- This method is called an adaptive technique as it makes use of the most recent information to correct (update) the forecast.

Exponential Smoothing cont.

- New forecast = previous forecast + α(previous actual - previous forecast)
- New forecast = previous forecast - α(error)
- α is between 0 and 1



Example (Class Activity)

- The process of exponential smoothing when $\alpha = 0.6$

Observed	Forecast	Error
55	55.0	0.00
59	55.0	4.00
53	57.4	-4.40
48	54.8	-6.76
44	50.7	-6.70
50	46.7	3.32
	48.6	*and so on ...*

For first value, Forecast = Observed

Error = 55 - 55

Error = 59 - 55

Forecast = 55 + 0.6(0)

Forecast = 55 + 0.6(4.0)

$$\hat{y}_{t+1} = \hat{y}_t + a(y_t - \hat{y}_t)$$

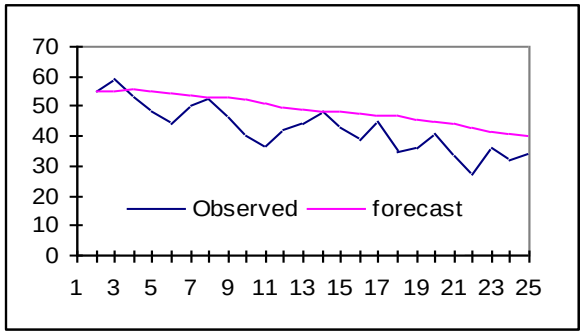
Example

Exponential Smoothing		
0.6		
Observed	forecast	error
55	55.00	0.00
59	55.00	4.00
53	57.40	-4.40
48	54.76	-6.76
44	50.70	-6.70
50	46.68	3.32
52	48.67	3.33
46	50.67	-4.67
40	47.87	-7.87
37	43.15	-6.15
42	39.46	2.54

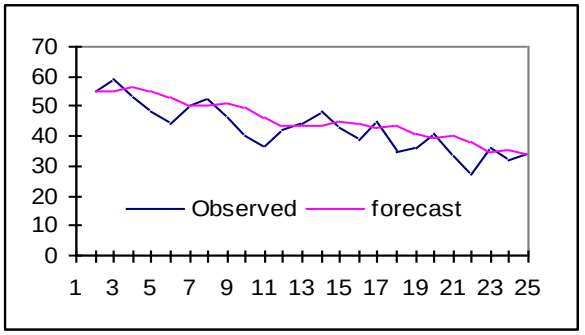
Simple Exponential Smoothing			
alpha =	0.6		
Period	Observ	forecast	error
JAN	55	=55	=0
FEB	59	=55+0.6*0	=59-55
MAR	53	=55+0.6*4	=53-57.40
APR	48	=57.4+0.6*-4.4	…
MAY	44	…	…
JUN	50	…	…
JUL	52	…	…
AUG	46	…	…
SEP	40	…	…
OCT	37	…	…
NOV	42	…	…

The Value of *a*

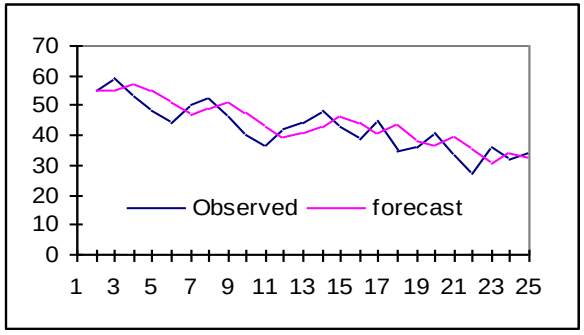
$a = 0.1$



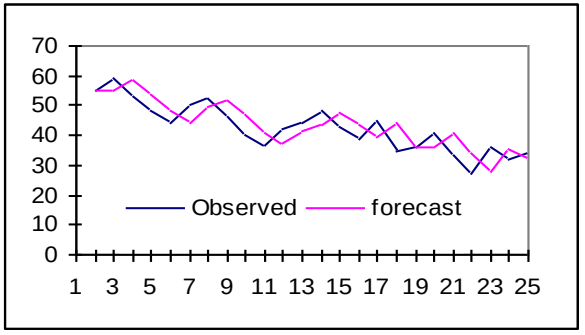
$a = 0.3$



$a = 0.6$



$a = 0.9$



Forecast Accuracy

- One approach to measuring the accuracy of a forecast is to use Mean Absolute Percent Error (MAPE). This is the average error of a series of forecasts.

$$MAPE = \frac{\sum_{i=1}^{n} \frac{|\hat{Y}_i - y_i|}{y_i}}{n}$$

$\hat{y}_i = forecast\ at\ period\ i$

$y_i = actual\ value\ period\ i$

$n = number\ of\ terms\ evaluated$

Example

	Alpha =	0.3	
Data	Forecast	Error	APE
4	4.00	0.00	
7	4.00	3.00	0.43
9	4.90	4.10	0.46
12	6.13	5.87	0.49
3	7.89	-4.89	1.63
6	6.42	-0.42	0.07
4	6.30	-2.30	0.57
8	5.61	2.39	0.30
6	6.33	-0.33	0.05
9	6.23	2.77	0.31
8	7.06	0.94	0.12
4	7.34	-3.34	0.84
5	6.34	-1.34	0.27
2	5.94	-3.94	1.97
		MAPE	58%

We don't include first value as it is not a forecast.

$$APE = \frac{|\hat{Y}_i - y_i|}{y_i}$$

APE
=Error/Actual
= 3/7 =0.43

A common technique for exponential smoothing is to choose an α that minimises MAPE using the Excel Solver.

$$MAPE = \frac{1}{n} \times \sum_{i=1}^{n} \frac{|\hat{Y}_i - y_i|}{y_i}$$

This is the average value of the APE

Summary

You should be able to:

- Plot a time series graph.
- Recognise the 3 components of a time series:
  - Trend;
  - Seasonal or cyclic component;
  - Random fluctuations (or noise).
- Construct a moving average.
- Make a one period forecast using exponential smoothing.
- Know the effect of different values of α.
- Calculate the accuracy of a forecast using MAPE.

Reading/Questions (Selvanathan)

- Reading: Time Series
    - 7th Ed. Sections 17.1, 17.2, 17.7.
- Questions: Time Series
    - 7th Ed. Questions 17.1, 17.3, 17.5, 17.6, 17.8, 17.38, 17.40.
    - Tutorial 11 Questions.
    - Create moving averages and exponential smoothed forecasts of some historical time series:
    - Ref: FIT1006 Lecture 19 and 20.xlsx

www.abs.gov.au