FIT1045 Algorithmic Problem Solving – Tutorial 4.

Objectives

The **objectives of this tutorial** are:

- To solve problems by identifying invariants
- To be able to identify invariants in algorithms
- To understand the greedy approach and its limitations
- To be able to find a Minimum Spanning Tree

Task 1

A *loop invariant* is a condition that holds for every iteration of the loop. Identify the loop invariants in the following pseudocode fragment.

```
max ← L[0]
i ← 1
while(i < length(L))
        if(L[i] >max)
                max ← L[i]
        i ← i + 1
```

Discuss how can you write a loop invariant so that it remains true at each iteration.

Hint: Can I write my invariant in terms of a variable or variables?

Task 2

Several coins are placed in a line on a table. Some of the coins are heads up, and some are heads down. You are required to turn all the coins heads up. However, you must turn two coins at a time. From which initial states is it possible to turn all the coins heads up?

Hint: Look for invariants - what does not change when you turn two coins over?

Task 3

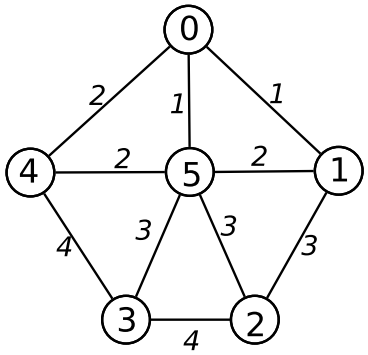
Consider an urn that is filled with black and white balls. At each step take two balls out until there is only one ball. If both balls have the same colour, throw them away and put another black ball into the urn. If they have different colours, throw the black ball away and put the white ball back into the urn. What can be said about the colour of the final ball in relation to the original number of black and white balls?

Task 4

Find three different denominations for coins such that:

- Any amount of money can be expressed in with combinations of those coins
- There is an amount of money for which the Greedy approach, using given denominations, does not find the fewest number of coins that can make up that amount.

Task 5



Break into groups and use Prim's algorithm find a minimum spanning tree for the graph above.

What is the weight of the spanning tree you found?

Compare the spanning tree you found with that found by other groups. Were the spanning trees all the same? And did they have the same weight? Discuss.

Puzzle of the week

Tricky Coin Puzzle: There is a table on which an even number of coins is placed. You also know that there are as many coins with Heads up as there are coins with Tails up. Now you have to divide the coins into two equal piles such that number of coins with Heads up and Tails up in each piles is the same. The catch is you are blind folded and so cannot determine which side of each coin is facing up.