FIT1045 Algorithmic Problem Solving – Tutorial 1.

Objectives

The **objectives of this tutorial** are:

- For students to get to know each other and your tutor.
- To start thinking about: what are algorithms, where algorithms are used, how do you develop algorithms, and how can you can communicate algorithms to others.
- To get familiar with trees and graphs.
- To get familiar with Eulerian circuits and Hamiltonian cycles.

Task 1

Get into a group of 4 or 5 students.

- In 10 minutes try to list as many examples of algorithms in the real world as you can. (You can use familiar examples, like how to get to university or how to fix a flat tire.)

Task 2

In your group, write an algorithm for making a cup of tea.

- List the **assumptions** that you have made.
- Using the definition of algorithm, justify why your answer is an algorithm. (Remember the properties of an algorithm: Input, Output, Finiteness, Definiteness, Effectiveness.)
- Identify the parts of your algorithm that are **selection**, **condition** and **iteration**.

Task 3

Graphs are used to represent objects (by vertices) and the relationships between pairs of objects (by edges). Some examples of graphs are given in Figure 1.

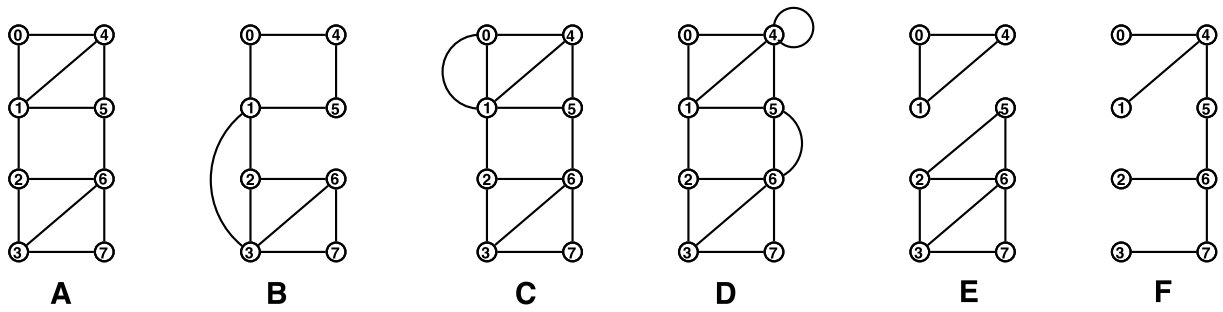

Figure 1: Some Examples of Graphs.

Part A

For each of the following properties, find a graph in Figure 1 with the property.

1. A graph is a *multigraph* if it has multiple edges joining the same pair of vertices.
2. A *loop* is an edge where both end vertices are the same.
3. A graph is *simple* if it has no loops and no multiple edges.

Select a graph in Figure 1. Can you find two vertices that are not adjacent? What are the degrees of these two vertices?

- Two vertices are *adjacent* if they are connected by an edge.
- The *degree* of a vertex in a simple graph is the number of edges incident to the vertex.

Part B

Imagine walking around a graph by moving between vertices along edges. We can write this *walk* as a sequence of vertices $v_0, v_1, \ldots, v_k$ where $v_i v_{i+1}$ is an edge in our graph for $i \in [0, k-1]$. For example, $0, 1, 2, 3, 1, 5$ is a walk in Graph B, but is not a walk in Graph C. Can you see why?

In this task, we look at restricted walks: the first restriction is that the walk can only visit each edge at most once and the second restriction is that the walk can only visit each vertex at most once. Discuss the following types of walk:

1. A *trail* is a walk with no repeated edges.
2. A *circuit* is a trail that begins and ends on the same vertex.
3. A *path* is a walk with no repeated vertices.
4. A *cycle* is a path that begins and ends on the same vertex. (Note: the only repeated vertex is the first and last vertex on this path.)

BTW: We say a graph is *connected* if every pair of vertices in the graph are connected by a path, and a connected graph with no cycles is called a *tree*. Can you find an example of a graph that is not connected? and another graph that contains no cycles?

Task 4

In Königsberg people used to wonder whether it was possible to walk across all seven bridges without crossing any bridge twice (see Figure 2). Try to solve this problem. Does it make a difference if you either add or remove a bridge? (**Hint:** Represent the problem as a graph.)

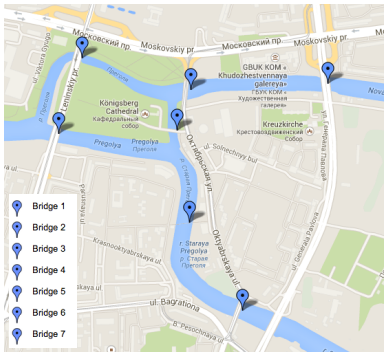

Figure 2: Königsberg, East Prussia; now known as Kaliningrad in Russia (`http://goo.gl/maps/b2mDM`).

A connected graph is called *Eulerian* if you can find a trail which starts and ends at the same vertex and contains each edge exactly once. Such a trail is called an *Eulerian circuit*.

Describe, in your own words, an algorithm that determines whether a simple connected graph is Eulerian.

Task 5

A *Hamiltonian cycle* is a cycle that visits every vertex in the graph exactly once. A graph is called *Hamiltonian* if it has a Hamiltonian cycle.

What graphs seen in this tute are Hamiltonian?

Puzzle of the week

Among n persons,a "celebrity" is defined as someone who is known by everyone but does not know anyone. The problem is to identify the celebrity, if one exists, by asking the question only of the form, "Excuse me, do you know the person over there?" (The assumption is that all the answers are correct, and even that celebrity will also answer.) The goal is to minimize the number of questions.