FIT1045 Algorithmic Problem Solving – Tutorial 5.

Objectives

The **objectives of this tutorial** are:

- To become familiar with some of the puzzles that will be used in this unit.
- To investigate the use of the greedy algorithm to solve the Knapsack Problem.
- To investigate the use of the brute force approach to solve the Farmer, Wolf, Goat and Cabbage Problem.
- To investigate the use of the brute force approach to solve the 4–Queens Problem.
- To build on strategies used in the above problems to solve the Water Jug Problem.

Task 1

The Knapsack Problem: Suppose you have n items and a knapsack that can carry at most a certain weight. Each item has a weight and a value. *The Knapsack Problem* is the problem of deciding which subset of the items to put in your knapsack in order to maximise the value of the contents of the knapsack.

Consider the following items:

Type	**1**	**2**	**3**	**4**	**5**
Value	$1.35	$1	$1	$1.85	$0.55
Weight	3kg	2kg	2kg	4kg	1kg

One way to solve the Knapsack problem is to exhaustively search all possible subsets of items and select the one of most value. However, there are 2^5 possible subsets of these items. Another approach is to use a greedy strategy and in this tute you will investigate three types of "greed".

Suppose you have a knapsack of capacity 4kg. Determine what items you would take if you followed each of the following strategies:

1. At each step choose the item with the **least weight** that can fit.
2. At each step choose the item with the **greatest value** that can fit.
3. At each step find the item with the **greatest value per unit weight** that can fit.

Discuss what is being minimised/maximised in each of these strategies. Which strategy gives you the largest value?

Task 2

The Farmer, Wolf, Goat and Cabbage Problem: A farmer has to transport himself and a wolf, a goat and a cabbage across the river in a small boat. The boat can only take the farmer and at most one of the wolf, goat or cabbage in a crossing. The wolf will eat the goat, if the farmer leaves them together while he crosses the river. Similarly, the goat will eat the cabbage if the farmer is not there to intervene. However, the wolf does not like to eat cabbage and the cabbage has promised not to harm the wolf. How can the farmer transport himself and the wolf, goat and cabbage safely to the other side of the river?

Part A

This problem has four objects: the Farmer, the Wolf, the Goat and the Cabbage. Each object has 2 states: it is on the left side of the river or it is on the right side of the river. Discuss how a bit list can be used to describe the current state, given we denote the left side of the river by 0 and the right side of the river by 1.

Create a list of 4–bit lists that correspond to valid states.

Part B

What bits list represents the start state? and what bit list represents the final state?

What happens to the current state, when the Farmer takes the boat across the river?

Create a graph where the vertices are the bit list corresponding to valid states and two vertices are connected by an edge if there is a *transition* between the two states. (A transition is where we can get to one state from the other by the Farmer (possibly with one of the Wolf, Goat or Cabbage) taking the boat to the other side of the river.)

Part C

Describe an algorithm that uses a brute force approach to solving the Farmer, Wolf, Goat and Cabbage problem.

Task 3

The N–Queens Problem The N–Queens Problem is the problem of placing N queens on an $N \times N$ board so that no two queens are in the same row, column or diagonal.

Part A

In this tutorial, we will investigate the case where $N = 4$, that is, the 4–Queens Problem.

Discuss how you could represent solutions to this problem.

Part B

Write an algorithm that generates all possible ways that 4–Queens can occupy a 4×4 board so that no 2 Queens occupy the same row or the same column. (**Note:** For this task, you need not check if any two Queens occupy the same diagonal. However, a proper solution for the N-Queens problem would require you to do so.)

Task 4

The Water Jug Problem You have two water jugs, a 4-Litre jug and a 3-Litre Jug, with no measurement markings. How can you use these jugs to measure 2 Litres of water?

Hint: You can represent the current contents of the two jugs as a pair (x, y). For example, $(1, 3)$ represents the 4-Litre jug containing 1 litre and the 3-Litre jug containing 3 litres.

Puzzle of the week

The Smart Pirates Puzzle: 100 pirates of different ages have a treasure of 1000 gold coins. On their ship, they decide to split the coins using the following scheme: The oldest pirate proposes how to share the coins, and ALL pirates (including the oldest) vote for or against it. If 50 percent or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain. As pirates tend to be a bloodthirsty bunch, if a pirate would get the same number of coins if he voted for or against a proposal, he will vote against the scheme so that the pirate who proposed the plan will be thrown overboard. Assuming that all 100 pirates are intelligent, rational, greedy, and do not wish to die, (and are rather good at maths for pirates) how will the oldest pirate here save his life and still get maximum coins?