

FIT1047 - Week 2

hour 1

Introduction to computer systems, networks and security



MONASH University

Reference: <https://www.alexandriarepository.org/syllabus/introduction-to-computer-systems-networks-and-security/>

Reference: Linda Null, Julia Lobur. The essentials of computer organization and architecture. Fourth edition, 2015. Jones & Bartlett

Topics for week 2

- Error detection
- Boolean algebra
- Logical circuits

Boolean logic and Boolean algebra

Boolean logic is a rather simple possible (but useful) logic.

Basic concepts:

- TRUE, FALSE
- AND, OR
- NOT

Usually, TRUE is represented by 1 and FALSE is represented by 0.

TRUE and FALSE are values for statements.

Note that not all statements qualify:

- Today, the temperature is over 15 degrees Celsius.
- Today, the weather is good.
- Haggis tastes great.

Do not confuse binary and Boolean:

	Binary	Boolean
0	Zero	FALSE
1	One	TRUE

Boolean Algebra

- Boolean algebra is a mathematical system for the manipulation of variables that can have **one** of **two** values
 - In **formal logic**, these values are **True/False**
 - In **digital systems**, these values are **on/off, 1/0 or high/low**
- Boolean expressions are created by performing operations on Boolean variables
 - Common Boolean operators include **AND, OR, NOT**

Truth Tables: **AND** & **OR**

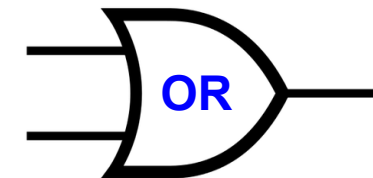
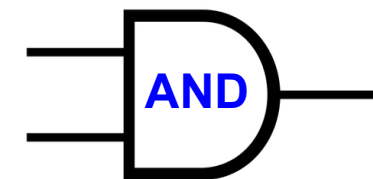
A Boolean operator can be completely described using a *Truth Table*

- Truth tables for the Boolean operators **AND** and **OR**

X AND Y		
X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X OR Y		
X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

Examples for gates

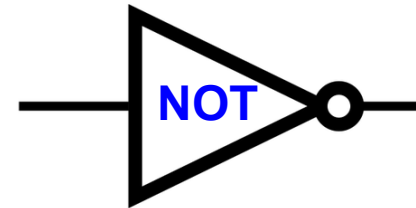


- The **AND** operator can also be represented as a period (a full stop) or space, so **X.Y** or **XY**
- The **OR** operator can also be represented as a + represented as **X+Y**

Truth Tables: NOT

- Truth table for the Boolean NOT operator

NOT x	
x	\bar{x}
0	1
1	0



- The NOT operation is often designated by an **overbar**, a **prime mark**, an “**elbow**” or a “**squiggle**”, e.g.,

$$\bar{A} \quad A' \quad \neg A \quad \sim A$$

Boolean Functions

A Boolean function has:

- At least **one** Boolean variable (x, y, \dots)
 - At least **one** Boolean operator (and, or, not, ..)
 - At least **one** input from the set $\{0, 1\}$ or $\{T, F\}$
- It produces an **output** that is also a member of the set $\{0, 1\}$ or $\{T, F\}$

Boolean Functions and Truth Tables

- Truth table for the Boolean function

$$F(x, y, z) = x\bar{z} + y$$

x	y	z	\bar{z}	$x\bar{z}$	$x\bar{z} + y$
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	0	1

$$F(x, y, z) = x\bar{z} + y$$

To make evaluation of the Boolean function easier, the truth table contains extra (shaded) columns to hold evaluations of subparts of the function

Rules of Precedence

- The **NOT** operator has **highest priority**, followed by **AND** & then **OR**
 - This is how we chose the (shaded) function subparts in our table

$$F(x, y, z) = x\bar{z} + y$$

x	y	z	\bar{z}	$x\bar{z}$	$x\bar{z} + y$
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	0	1

Boolean Algebra

- Digital computers contain circuits that implement Boolean functions
 - ✓ *The simpler we can make a Boolean function, the smaller the circuit that will result*
 - ✓ *Simpler circuits are cheaper to build, consume less power, and run faster than complex circuits*
- With this in mind
 - We always want to reduce our Boolean functions to their simplest form
 - There are several Boolean identities that help us to do this

Laws of Boolean Algebra

- Identity Law
- Null Law (or Dominance Law)
- Idempotent Law
- Complement Law
- Commutative Law
- Associative Law
- Distributive Law
- Absorption Law
- DeMorgans Law
- Double Complement Law

Identity Law

AND Form	OR Form
$1A = A$	$0+A = A$

Null Law (Dominance Law)

AND Form	OR Form
$0A = 0$	$1+A = 1$

Idempotent Law

AND Form	OR Form
$AA = A$	$A+A = A$

Complement Law

AND Form	OR Form
$A\bar{A} = 0$	$A+\bar{A} = 1$

Commutative Law

AND Form	OR Form
$AB = BA$	$A+B = B+A$

Laws of Boolean Algebra

- Identity Law
- Null Law (or Dominance Law)
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Associative Law

AND Form	OR Form
$(AB)C = A(BC)$	$A+(B+C) = (A+B)+C$

Distributive Law

AND Form	OR Form
$A+(BC) = (A+B)(A+C)$	$A(B+C) = AB+AC$

Absorption Law

AND Form	OR Form
$A(A+B) = A$	$A+AB = A$

DeMorgans Law

AND Form	OR Form
$\overline{AB} = \overline{A}+\overline{B}$	$\overline{(A+B)} = \overline{A} \times \overline{B}$

Double Complement Law

$$\overline{\overline{A}} = A$$

Laws of Boolean Algebra

- Identity Law
- Null Law (or Dominance Law)
- Idempotent Law
- Complement Law
- Commutative Law
- Associative Law
- Distributive Law
- Absorption Law
- DeMorgans Law
- Double Complement Law

Identity Name	AND Form	OR Form
Identity Law	$1x = x$	$0 + x = x$
Null Law	$0x = 0$	$1 + x = 1$
Idempotent Law	$xx = x$	$x + x = x$
Inverse Law	$x\bar{x} = 0$	$x + \bar{x} = 1$

(Inverse or Complement Law)

Identity Name	AND Form	OR Form
Commutative Law	$xy = yx$	$x+y = y+x$
Associative Law	$(xy)z = x(yz)$	$(x+y)+z = x + (y+z)$
Distributive Law	$x+yz = (x+y)(x+z)$	$x(y+z) = xy+xz$

Identity Name	AND Form	OR Form
Absorption Law	$x(x+y) = x$	$x + xy = x$
DeMorgan's Law	$\overline{(xy)} = \bar{x} + \bar{y}$	$\overline{(x+y)} = \bar{x}\bar{y}$
Double Complement Law	$\overline{(\bar{x})} = x$	

Simplifying a Function

Apply Boolean algebra identities to simplify the following Boolean function:

$(\overline{AB})A + A\overline{B}$

SOLUTION:

de Morgan

$(\overline{A} + \overline{B})A + A\overline{B}$

Commutative

$A(\overline{A} + \overline{B}) + A\overline{B}$

Distributive

$A\overline{A} + A\overline{B} + A\overline{B}$

Inverse

$0 + A\overline{B} + A\overline{B}$

Identity

$A\overline{B} + A\overline{B}$

Idempotent

$A\overline{B}$

Identity Name	AND Form	OR Form
Identity Law	$1x = x$	$0 + x = x$
Null Law	$0x = 0$	$1 + x = 1$
Idempotent Law	$xx = x$	$x + x = x$
Inverse Law	$x\overline{x} = 0$	$x + \overline{x} = 1$

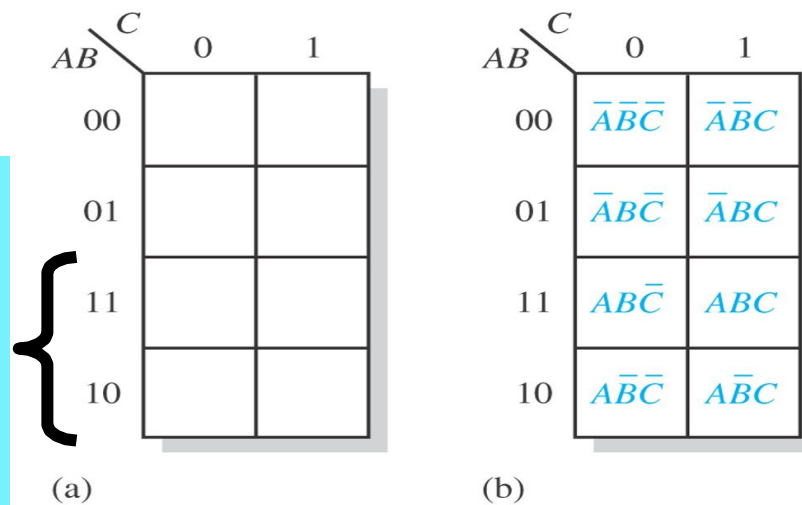
Identity Name	AND Form	OR Form
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Identity Name	AND Form	OR Form
Absorption Law	$x(x+y) = x$	$x + xy = x$
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Double Complement Law	$\overline{(\overline{x})} = x$	

Simplifying using Karnaugh-Maps (K-map)

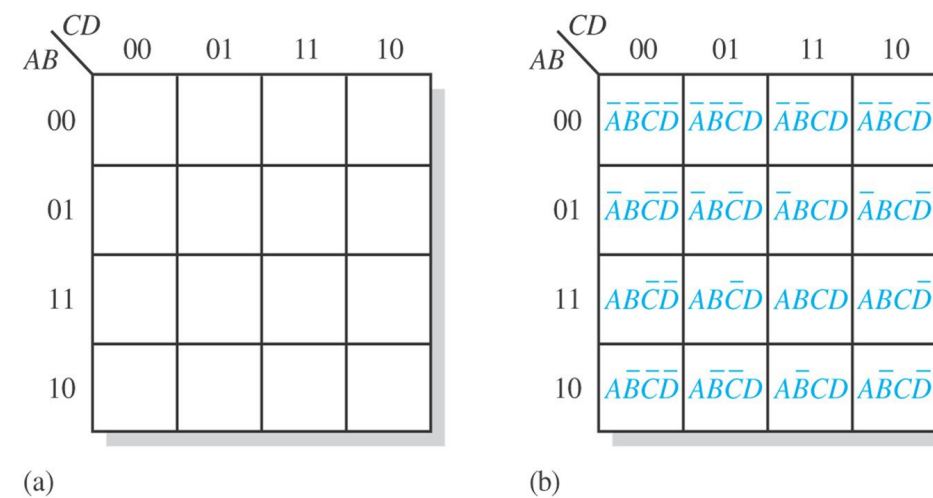
- ✓ Provides a method for simplifying Boolean expressions.
- ✓ It will produce the simplest Sums-Of-Products (SOP) and Product-Of-Sums (POS) expressions.
- ✓ Works best for less than 6 variables.
- ✓ Similar to a truth table → it maps all possibilities.
- ✓ A Karnaugh map is an array of cells arranged in a special manner.
- ✓ The number of cells is 2^n where n = number of variables

A 3-Variable Karnaugh Map:



- ✓ **Note:** The order of these values they are reverse of the usual order.
- ✓ They are arranged this way so that only one variable changes at a time.

A 4-Variable Karnaugh Map:



Simplifying using Karnaugh-Maps (K-map)

One of the most common forms of simplification in Boolean algebra is the following:

$$A\bar{B} + AB = A(\bar{B} + B) = A$$

The same with three variables:

$$A\bar{B}C + ABC = AC$$

Both functions are independent from the value of B.

Karnaugh-maps provide an easy graphical way to find minimal AND terms that are then combined with OR to get the complete function.

Truth table for A AND B

A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

Karnaugh-map for A AND B

		B	
		0	1
A	0	0	0
	1	0	1 = A.B

Simplifying using Karnaugh-Maps (K-map)

Let's look at a Karnaugh-map with 3 variables:


$$\overline{B}A\overline{C} + AB\overline{C} + ABC + \overline{A}BC + \overline{A}B\overline{C}$$

		BC			
		00	01	11	10
A	0	0	0	1	1
	1	1	0	1	1

- Find groups of 1s.
- The group of **four** 1s represents **B=1**. (Note: A=0/1, B=1, C=0/1)
- The wrapped group represents **A=1 AND C=0**. (Note: A=1, While B = 0/1, C=0)

Simplified version: $B + A\overline{C}$

7 rules for working with Karnaugh-Maps (K-map)


- ☞ Rule 1: No group can contain a zero. 

		B	
		0	1
A	0	0	1
A	1	0	1

Correct

		B	
		0	1
A	0	0	1
A	1	0	1

Wrong


- ☞ Rule 2: Groups may be horizontal/vertical/square, but never diagonal. 

		B	
		0	1
A	0	0	1
A	1	1	1

Correct

		B	
		0	1
A	0	0	1
A	1	1	1

Wrong


- ☞ Rule 3: Groups must contain 1, 2, 4, 8, 16, 32(powers of 2) cells. 
- ☞ Rule 4: **Each group must be as large as possible.**

				BC			
				00	01	11	10
A	0	0	0	1	0		
A	1	1	1	1	0		

Correct

				BC			
				00	01	11	10
A	0	0	0	1	0		
A	1	1	1	1	0		

Wrong

- ☞ Rule 5: **Groups can overlap.** 


				BC			
				00	01	11	10
A	0	0	0	1	1		
A	1	1	1	1	1		

Correct

				BC			
				00	01	11	10
A	0	0	0	1	1		
A	1	1	1	1	1		

Wrong

- ☞ Rule 6: Each 1 must be part of at least one group

- ☞ Rule 7: Groups may wrap around the map. 

				BC			
				00	01	11	10
A	0	1	0	0	1		
A	1	1	0	0	1		

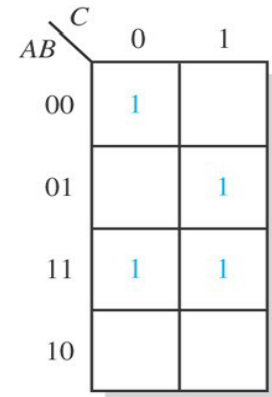
Wrap around

Karnaugh-maps are useful for smaller Boolean functions.

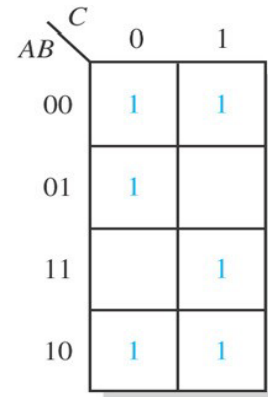
Automated algorithms for optimization are used for bigger functions.

Simplifying using Karnaugh-Maps (K-map)

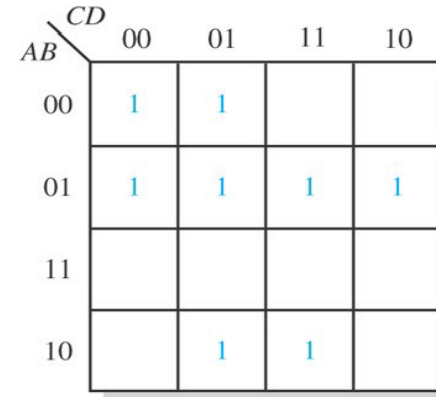
Simplification example



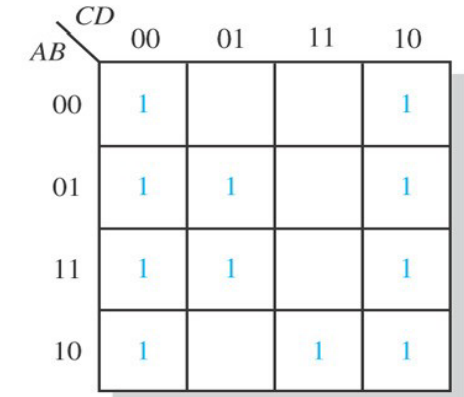
(a)



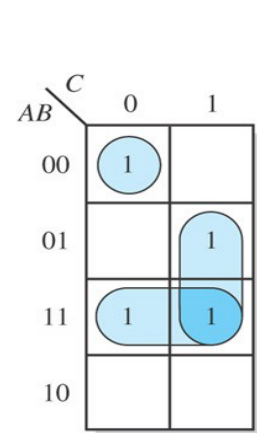
(b)



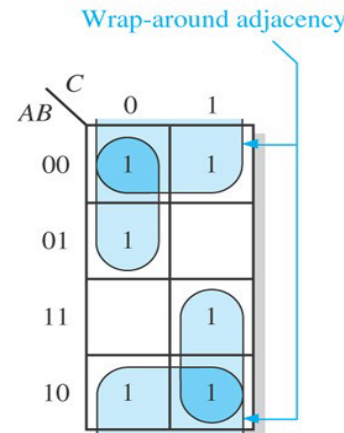
(c)



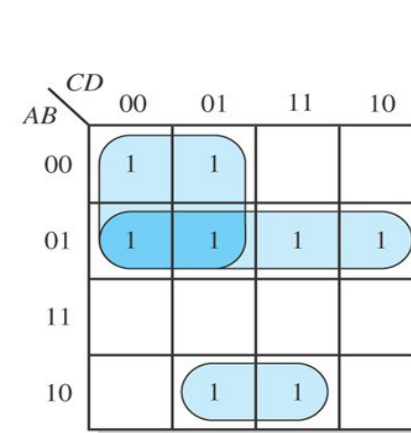
(d)



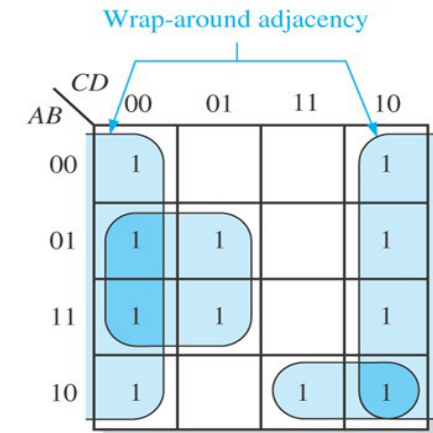
(a)



(b)



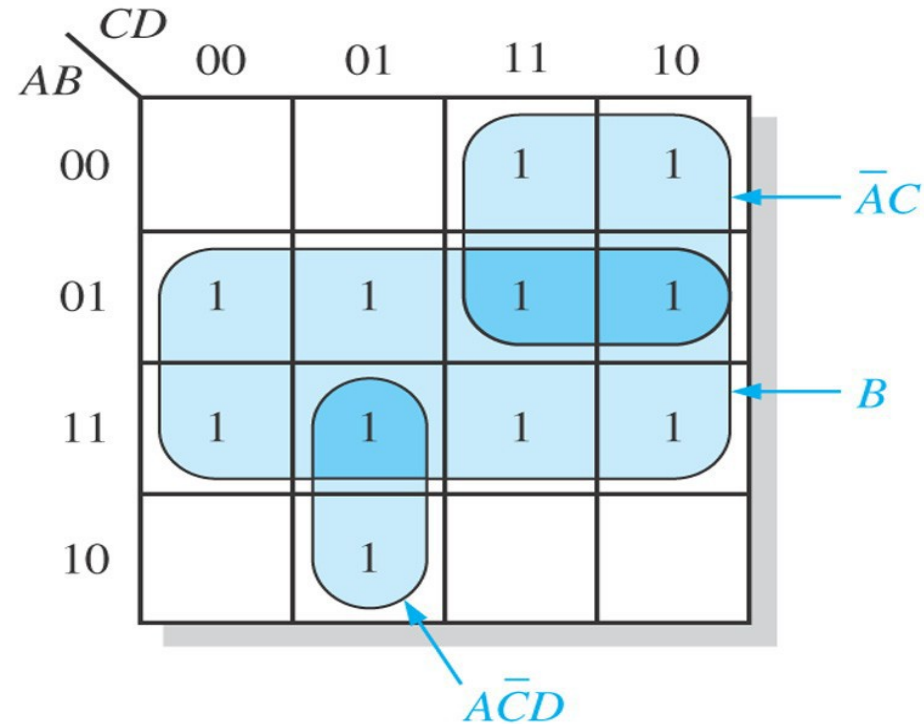
(c)



(d)

Simplifying using Karnaugh-Maps (K-map)

Simplification example:



$$F(A, B, C, D) = \bar{A}\bar{C} + B + A\bar{C}D$$

Logic Gates and Truth Tables

Logic Gates

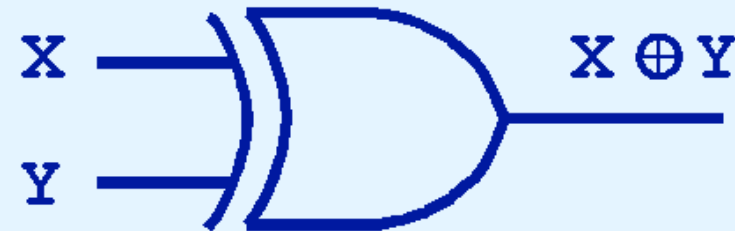
- The logic gate is the **building brick of digital logic**
- A logic gate is an electronic device that produces a result based on two or more input values
 - In reality, gates consist of one to six transistors, but digital designers think of them as a single unit
- **Integrated circuits** contain **collections of gates** suited to a particular purpose

XOR

- The output of the XOR operation is True (1) only when the values of the inputs differ

X XOR Y

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0



NAND and NOR (I)

NAND and NOR gates have special properties:

1. NAND can be realised very efficiently.
2. All other gates can be build only using NAND gates.

NAND is $= \overline{AB}$ or \overline{XY}

NOR is $= \overline{A + B}$ or $\overline{X + Y}$

X NAND Y

X	Y	X NAND Y
0	0	1
0	1	1
1	0	1
1	1	0



X NOR Y

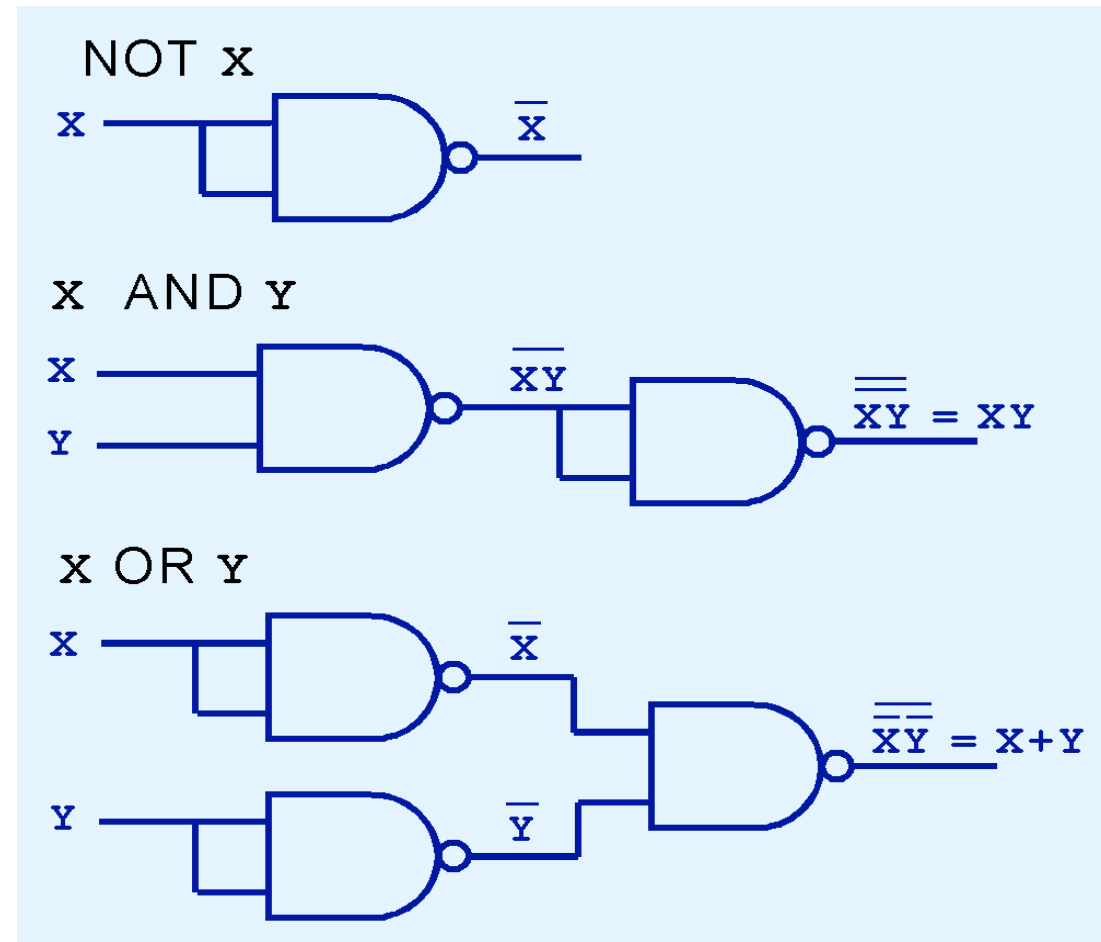
X	Y	X NOR Y
0	0	1
0	1	0
1	0	0
1	1	0



NAND and NOR (II)

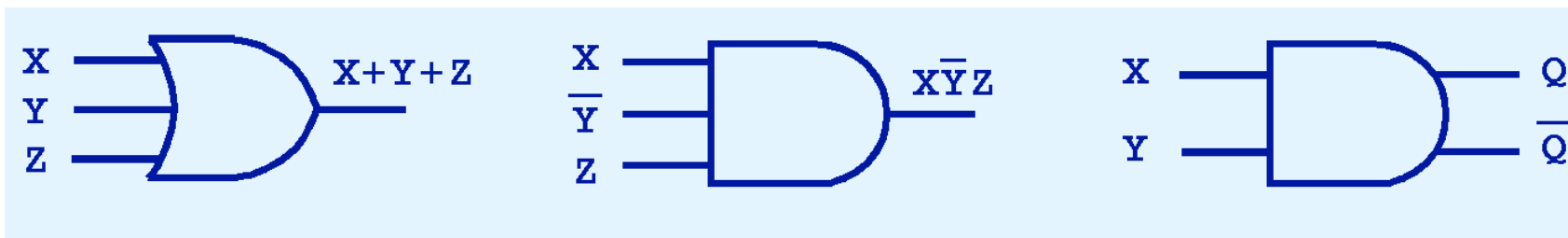
- Known as universal gates because they can be used to construct any gate

- Building **NOT** with NAND gates
- Building **AND** with NAND gates
- Building **OR** with NAND gates



Logic Gates: Inputs and Outputs

- Gates can have multiple inputs and more than one output
 - A second output can be provided for the complement of the operation



Summarize topics of first 2 weeks

- A little bit of history
- Vacuum tubes and transistors
- bit, byte, word (8bit/16bit/32bit/64bit)
- Numbering systems (base 2, base 10, base 16)
- Conversion between numbering systems
- Signed integer representations (sign/magnitude, 1's complement, 2's complement)

- Properties of 2's complement, adding 2's complement numbers
- Floating point representation
- Properties of floating point, precision, rounding
- Characters: ASCII and Unicode
- Error detection: Parity bits, Checksum, CRC

- Boolean Logic AND, OR, NOT, XOR, NAND, NOR
- Logic gates
- Simple logic circuits
- Boolean Algebra, Laws
- Karnaugh Maps/K-maps

End of Lecture Week-2 Hour 1

FIT1047 - Week 2

hour 2

Introduction to computer systems, networks and security



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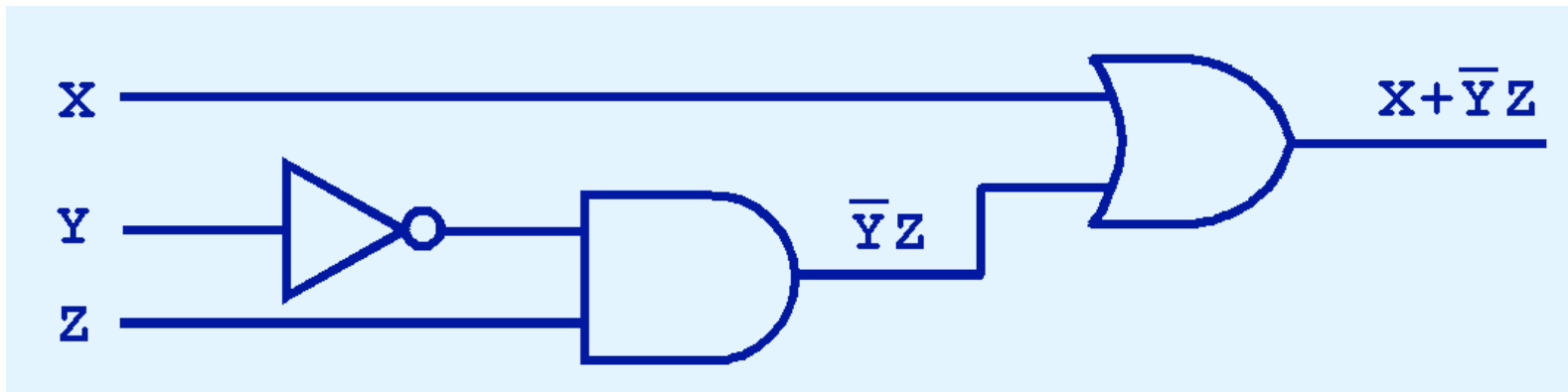
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Logic Gates and Boolean Functions

- Combinations of gates implement Boolean functions

Example $F(X, Y, Z) = X + \bar{Y}Z$



Simplifying a Function

- Sometimes it is more economical to build a circuit using the complement of a function (and complementing its result) than it is to implement the function directly
 - ✓ DeMorgan's law provides an easy way of finding the complement of a Boolean function
 - ✓ Recall DeMorgan's law states

$$\overline{(xy)} = \bar{x} + \bar{y} \quad \text{and} \quad \overline{(x+y)} = \bar{x}\bar{y}$$

- DeMorgan's law can be extended to any number of variables

Calculating the Complement of a Function

- Replace each variable by its complement and change all ANDs to ORs and all ORs to ANDs
- Example
 - Find the complement of:

$$F(X, Y, Z) = (XY) + (\bar{X}Z) + (Y\bar{Z})$$

$$\begin{aligned}\bar{F}(X, Y, Z) &= \overline{(XY) + (\bar{X}Z) + (Y\bar{Z})} \\ &= \overline{(XY)} \overline{(\bar{X}Z)} \overline{(Y\bar{Z})} \\ &= (\bar{X} + \bar{Y})(X + \bar{Z})(\bar{Y} + Z)\end{aligned}$$

Canonical Form (I)

- There are numerous ways of stating the same Boolean expression
 - These “synonymous” forms are *logically equivalent*
 - Logically equivalent expressions have identical truth tables
 - In order to eliminate as much confusion as possible, designers express Boolean functions in *standardized* or *canonical* form

Canonical Form (II)

There are two canonical forms for Boolean expressions

- **Sum-of-products** and **product-of-sums**
 - Recall the Boolean product is the **AND** operation and the Boolean sum is the **OR** operation
- **Sum-of-products**: **ANDed** variables are **ORed** together
 - For example: $F(X,Y,Z) = XY + XZ + YZ$
- **Product-of-sums**: **ORed** variables are **ANDed** together
 - For example: $F(X,Y,Z) = (X+Y)(X+Z)(Y+Z)$

Converting to **Sum of Products**

We are interested in the values of the variables that make the function **True (=1)**

1. Using the Truth Table, make groups of **ANDed** variables (or negated variables), such that each group results in a True function value
2. OR together each group of variables

Converting to **Sum of Products**: Example

$$F(x, y, z) = x\bar{z} + y$$

x	y	z	$x\bar{z} + y$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$$F(x, y, z) = \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + xy\bar{z} + xyz$$

Converting to **Product of Sums**

IDEA: use De Morgan Law to calculate the complement of the complement of the function

1. We are interested in the values of the variables that make the function **False (=0)**
2. **Complement** all the function values (function output)
3. Using the Truth Table, make groups of ANDed variables (or negated variables), such that each group results in a True value for the complement of the function (False value for the function)
4. OR together each group of variables
→ This yields the complement of the function
5. Take the complement of this complement
→ This yields the function itself

Converting to Product of Sums: Example

$$F(x, y, z) = x\bar{z} + y$$

x	y	z	$x\bar{z} + y$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$$\begin{aligned}
 F(x, y, z) &= \overline{\bar{x} \bar{y} \bar{z}} + \overline{\bar{x} \bar{y} z} + \overline{x \bar{y} z} \\
 &= (\overline{\bar{x} \bar{y} \bar{z}})(\overline{\bar{x} \bar{y} z})(\overline{x \bar{y} z}) \\
 &= (x + y + z)(x + y + \bar{z})(\bar{x} + y + \bar{z})
 \end{aligned}$$


Calculating the Complement of a Function

- Replace each variable by its complement and change all ANDs to ORs and all ORs to ANDs
- Example
 - Find the complement of:

$$F(X, Y, Z) = (XY) + (\bar{X}Z) + (Y\bar{Z})$$

$$\begin{aligned}\bar{F}(X, Y, Z) &= \overline{(XY) + (\bar{X}Z) + (Y\bar{Z})} \\ &= \overline{(XY)} \overline{(\bar{X}Z)} \overline{(Y\bar{Z})} \\ &= (\bar{X} + \bar{Y})(X + \bar{Z})(\bar{Y} + Z)\end{aligned}$$

7 rules for working with Karnaugh-Maps (K-map)


- ☞ Rule 1: No group can contain a zero. 

		B	
		0	1
A	0	0	1
A	1	0	1

Correct

		B	
		0	1
A	0	0	1
A	1	0	1

Wrong


- ☞ Rule 2: Groups may be horizontal/vertical/square, but never diagonal. 


		B	
		0	1
A	0	0	1
A	1	1	1

Correct

		B	
		0	1
A	0	0	1
A	1	1	1

Wrong

- ☞ Rule 3: Groups must contain 1,2,4,8,16,32(powers of 2) cells. 
- ☞ Rule 4: Each group must be as large as possible.

- ☞ Rule 5: Groups can overlap. 


		BC			
		00	01	11	10
A	0	0	0	1	1
A	1	1	1	1	1

Correct

		BC			
		00	01	11	10
A	0	0	0	1	1
A	1	1	1	1	1

Wrong

- ☞ Rule 6: Each 1 must be part of at least one group

- ☞ Rule 7: Groups may wrap around the map. 

		BC			
		00	01	11	10
A	0	1	0	0	1
A	1	1	0	0	1

Wrap around

Karnaugh-maps are useful for smaller Boolean functions.

Automated algorithms for optimization are used for bigger functions.

FIT1047 TUTORIAL 2 (week-2)

2d (Additional advanced task:) Use logisim and only AND, OR and NOT to build a circuit for the following function:

$$F(A, B, C) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

How many gates do you need? Simplify the function first, e.g. by using k-maps.

METHOD – II : Using K-maps

$$= A\bar{B} + C$$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}		1	1	
A	1	1	1	

	AB	$\bar{A}\bar{B}$	$\bar{A}B$	$A\bar{B}$
\bar{C}				1
C	1	1	1	1

$\Rightarrow C$

$A\bar{B}$

$$F(A, B, C) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$= C + A\bar{B}$$

FIT1047 TUTORIAL 2 (week-2)

2d (Additional advanced task:) Use logisim and only AND, OR and NOT to build a circuit for the following function:

$$F(A, B, C) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

How many gates do you need? Simplify the function first, e.g. by using k-maps.

METHOD – I : Using Boolean Identities

$$\begin{aligned} F(A, B, C) &= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC \\ &= \bar{A}C(\bar{B}+B) + A\bar{B}(\bar{C}+C) + ABC \\ &= \bar{A}C + A\bar{B} + ABC \\ &= \bar{A}C + A(\bar{B}+BC) \\ &= \bar{A}C + A(\bar{B}+B)(\bar{B}+C) \\ &= \bar{A}C + A(\bar{B}+C) \\ &= \bar{A}C + A\bar{B} + AC \\ &= C(\bar{A}+A) + A\bar{B} \end{aligned}$$

$$F(A, B, C) = C + A\bar{B}$$

Identity Name	AND Form	OR Form
Identity Law	$1x = x$	$0 + x = x$
Null Law	$0x = 0$	$1 + x = 1$
Idempotent Law	$xx = x$	$x + x = x$
Inverse Law	$x\bar{x} = 0$	$x + \bar{x} = 1$

Identity Name	AND Form	OR Form
Commutative Law	$xy = yx$	$x+y = y+x$
Associative Law	$(xy)z = x(yz)$	$(x+y)+z = x+(y+z)$
Distributive Law	$x+yz = (x+y)(x+z)$	$x(y+z) = xy+xz$

Identity Name	AND Form	OR Form
Absorption Law	$x(x+y) = x$	$x + xy = x$
DeMorgan's Law	$\overline{(xy)} = \bar{x} + \bar{y}$	$\overline{(x+y)} = \bar{x}\bar{y}$
Double Complement Law	$\overline{(\bar{x})} = x$	

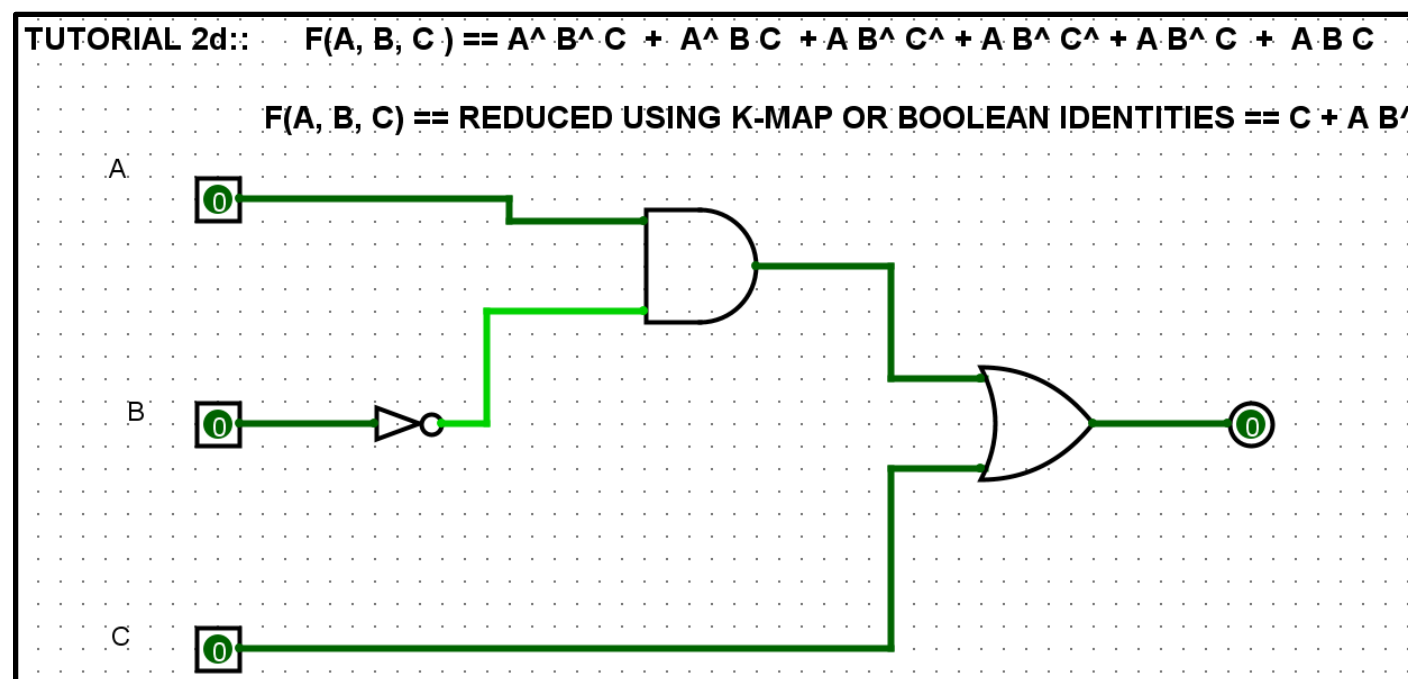
FIT1047 TUTORIAL 2 (week-2)

2d (Additional advanced task:) Use logisim and only AND, OR and NOT to build a circuit for the following function:

$$F(A, B, C) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC = A\bar{B} + C$$

How many gates do you need? Simplify the function first, e.g. by using k-maps.

Simulate using logisim



Summarize topics of first 2 weeks

- A little bit of history
- Vacuum tubes and transistors
- bit, byte, word (8bit/16bit/32bit/64bit)
- Numbering systems (base 2, base 10, base 16)
- Conversion between numbering systems
- Signed integer representations (sign/magnitude, 1's complement, 2's complement)
- Properties of 2's complement, adding 2's complement numbers
- Floating point representation
- Properties of floating point, precision, rounding
- Characters: ASCII and Unicode
- Error detection: Parity bits, Checksum, CRC
- Boolean Logic AND, OR, NOT, XOR, NAND, NOR
- Logic gates
- Simple logic circuits
- Boolean Algebra, Laws
- Karnaugh Maps/K-maps

END of Lecture 2