Part A: Multiple Choice *(10 marks in total)*

Select the best answer for each question. Correct response scores 1 mark. There is no penalty for incorrect answers. Selecting more than one answer for a particular question will result in zero marks for that question.

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1. A company uses 4 pounds of resource 1 to make each unit of X1 and 3 pounds of resource 1 to make each unit of X2. There are only 150 pounds of resource 1 available. Which of the following constraints reflects the relationship between X1, X2 and resource 1?

   A. $4\ X_1 + 3\ X_2 \geq 150$
   B. $4\ X_1 + 3\ X_2 \leq 150$
   C. $4\ X_1 + 3\ X_2 = 150$
   D. $4\ X_1 \leq 150$

   ANSWER: A $\boxed{B}$ C D (circle the correct answer)

2. What is the goal in optimization?

   A. Find the best decision variable values that satisfy all constraints.
   B. Find the values of the decision variables that use all available resources.
   C. Find the values of the decision variables that satisfy all constraints.
   D. None of the above.

   ANSWER: $\boxed{A}$ B C D (circle the correct answer)

3. How many constraints are there in a transportation problem which has 5 supply points and 4 demand points? (ignore the non-negativity constraints)

   A. 4
   B. 5
   C. 9
   D. 20

   ANSWER: A B $\boxed{C}$ D (circle the correct answer)

4. A production company wants to ensure that if Product 1 is produced, production of Product 1 not exceed production of Product 2. Which of the following constraints enforce this condition?

   A. $X_1 \geq M_2Y_2$
   B. $X_1 \leq M_2X_2$
   C. $X_1 \leq M_1Y_1$, $X_1 \leq Y_1X_2$
   D. $X_1 \leq X_2$

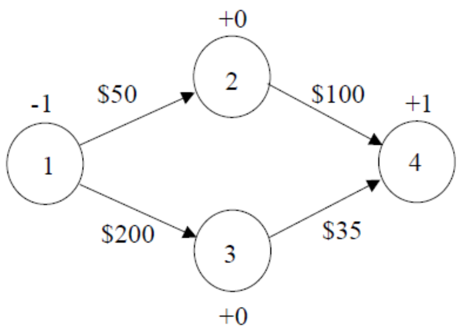
   ANSWER: A B C $\boxed{D}$ (circle the correct answer)

5. The decision rule which determines the maximum payoff for each alternative and then selects the alternative associated with the largest payoff is the

   A. maximax decision rule.
   B. maximin decision rule.
   C. minimax regret decision rule.
   D. minimin decision rule.

   ANSWER: $\boxed{A}$ B C D (circle the correct answer)

6. What is the constraint for node 2 in the following shortest path problem?



A. $-X_{12} - X_{13} = 0$
B. $-X_{12} - X_{24} = 1$
C. $X_{12} + X_{13} = 0$
D. $-X_{12} + X_{24} = 0$

ANSWER: A B C D (circle the correct answer)

Questions 7 to 10 use the following information.

You are considering 4 investments, A, B, C and D. The payoff from each investment is a dependant on the economic condition over the next 2 years. The economy can expand or decline. The following payoff matrix has been developed for the decision problem.

	A	B	C	D
1		Payoff Matrix		
2	Probability	0.7	0.3	
3		Economy		
4	Investment	Decline	Expand	
5	A	-10	90	
6	B	20	50	
7	C	40	45	
8	D	15	20	

7. What decision should be made according to the expected regret decision rule?
A. A
B. B
C. C
D. D

ANSWER: A B C D (circle the correct answer)

8. What decision should be made according to the expected monetary value decision rule?
A. A
B. B
C. C
D. D

ANSWER: A B C D (circle the correct answer)

9. What is the expected monetary value of Investment A?
   A. 15
   B. 20
   C. 30
   D. 34

   ANSWER: A $\boxed{B}$ C D (circle the correct answer)

10. A time-series which has no significant upward or downward trend is referred to as:

   A. static
   B. stationary
   C. non-moving
   D. non-stationary

   ANSWER: A $\boxed{B}$ C D (circle the correct answer)

Part B: Short Answer Questions *(60 marks in total)*

Question 11 &12: **(10 marks)**

Q11) A farmer is planning his spring planting. He has 20 acres on which he can plant a combination of Corn, Pumpkins and Beans. He wants to maximize his profit but there is a limited demand for each crop. Each crop also requires fertilizer and irrigation water which are in short supply. There are only 50 acre ft of irrigation available and only 8,000 pounds/acre of fertilizer available. The following table summarizes the data for the problem.

Crop	Profit per Acre ($)	Yield per Acre (lb)	Maximum Demand (lb)	Irrigation (acre ft)	Fertilizer (pounds/acre)
Corn	2,100	21,000	200,000	2	500
Pumpkin	900	10,000	180,000	3	400
Beans	1,050	3,500	80,000	1	300

a) Formulate the LP model for this linear programming problem.

ANSWER: **4 marks**

Let

X_1 = acres of corn
X_2 = acres of pumpkin
X_3 = acres of beans

MAX: $2100X_1 + 900X_2 + 1050X_3$
Subject to: $21X_1 <= 200$
$10X_2 <= 180$
$3.5X_3 <= 80$
$X_1 + X_2 + X_3 <= 20$
$2X_1 + 3X_2 + 1X_3 <= 50$
$5X_1 + 4X_2 + 3X_3 <= 80$
$X_1, X_2, X_3 \geq 0$

Question 12:

The problem in question 11 is modelled in an Excel Spreadsheet and the Sensitivity Report is generated as shown below:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Acres of Corn	9.52	0	2100	1E+30	350
C4	Acres of Pumpkin	0	-500.01	899.99	500.01	1E+30
D4	Acres of Beans	10.79	0	1050	210	375.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E8	Corn demand Used	200000	0.017	200000	136000	152000
E9	Pumpkin demand Used	0	0	180000	1E+30	180000
E10	Bean demand Used	37777.78	0	80000	1E+30	42222.22
E11	Water Used	29.84	0	50	1E+30	20.15
E12	Fertilizer Used	8000	3.5	8000	3619.04	3238.09

Answer the following question based on the above report.

a). What is the optimal solution to this problem?

ANSWER: **1 mark**

9.52 acres of corn and 10.79 acres of beans

Total profit: 9.52 x $2100 + 10.79 x $1050

= 19,992 + 11,329.50 = $31,321.50

b). Suppose the farmer can purchase more fertilizer for $2.50 per pound, should he purchase it and how much can he buy and still be sure of the value of the additional fertilizer?

ANSWER: **2 marks**

Yes, because the cost of $2.50 is less than the shadow price of $3.50. The allowable increase is 3619.04 pounds and so he can buy up to this amount and still be sure that the shadow price will still hold.

c). What does the reduced cost for pumpkin indicate?

ANSWER: **1 mark**

It indicates that for every acre that the farmer dedicates to pumpkin, he will incur a loss of $500.01

d). Identify ~~the~~ a binding resource and state how much would you pay to acquire an additional unit of that resource?

ANSWER: **1 mark**

Fertiliser is a binding resource and the farmer can pay up to $3.50 on top of what he is currently paying to acquire an additional unit of the resource. Note: Corn is also a binding resource

e). What can you infer from the shadow price of corn in this report?

ANSWER: **1 mark**

If we are able to increase the demand for corn, we can potentially increase the profit by $0.017 for every extra pound (up to a limit of 136,000 pounds)

-------------------------------- END OF QUESTION 12 --------------------------------

Question 13: (10 marks)

Consider the following distribution problem for Ace Widgets:

	Shipping Costs to Warehouses				Capacity
Depot	W1	W2	W3	W4	
P1	2	6	4	12	100
P2	7	3	10	11	250
P3	5	8	9	13	300
Demand	50	150	200	250	

a) Formulate an LP formulation for Ace Widgets including an objective function and constraints.

ANSWER: **3 marks**

Let X_{ij} be the trip from Depot (i) to Warehouse (j); where i = P1, P2, P3 and j = W1, W2, W3, W4

C_{ij} be the cost from Depot (i) to Warehouse (j)

MIN: $\sum C_{ij}X_{ij}$

ST: $X_{11} + X_{12} + X_{13} + X_{14} = 100$ (or you can write $\sum X_{1j} = 100$)

$\sum X_{1j} = 100$

$\sum X_{2j} = 250$

$\sum X_{3j} = 300$

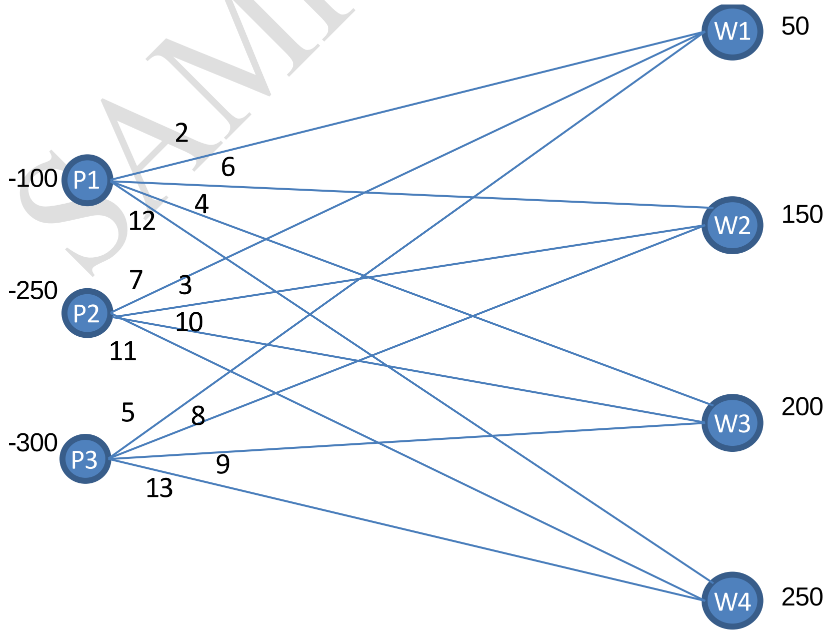
$\sum X_{i1} = 50$

$\sum X_{i2} = 150$

$\sum X_{i3} = 200$

$\sum X_{i4} = 250$

Network model to depict this problem. (Not required, given for clarity)



b) Solve the problem using the North-West Corner method. **2 marks**

		K1=	K2=	K3=	K4=	K5=	
	Source	Destination W1	W2	W3	W4		SUPPLY
R1=	P1	2 / *50*	6 / *50*	4	12		~~100~~ 50
R2=	P2	7	3 / *100*	10 / *150*	11		~~250~~ 150
R3=	P3	5	8	9 / *50*	13 / *250*		~~300~~
R4=							
	DEMAND	50	~~150~~ 100	~~200~~ 50	~~250~~		

c) Using the solution generated in b) apply the ***MODI method (closed-loop path)*** to determine the optimized allocation for one iteration only (i.e stop after forming the first closed loop to find the updated allocations). **3marks**

MODI: 1st Iteration

		K1= 2	K2= 6	K3= 13	K4= 17	K5=	
	Source	Destination W1	W2	W3	W4		SUPPLY
R1= 0	P1	2 / *50*	6 / *50* -	4 + / -9	12 / -5		100
R2= -3	P2	7 / 8	3 + / *150*	10 - / *100*	11 / -3		250
R3= -4	P3	5 / 7	8 / 6	9 / *50*	13 / *250*		300
R4=							
	DEMAND	50	150	200	250		

MODI: 2nd Iteration

		K1= 2	K2= -3	K3= 4	K4= 8	K5=	
	Source	Destination W1	W2	W3	W4	D5	SUPPLY
R1= 0	P1	2 / *50*	6	4 / *50*	12 / 4		100
R2= 6	P2	7 / -1	3 / *150*	10 -	11 + / *100* / -3		250
R3= 5	P3	5 / -2	8 / 6	9 + / *150*	13 - / *150*		300
R4=							
	DEMAND	50	150	200	250		

d) Is the solution from c) degenerate? How much has the solution from b) improved? (2 marks)

No

From North-West corner method:

Total cost = 2 x 50 + 6 x 50 + 3 x 100 + 10 x 150 + 9 x 50 + 13 x 250 = $5,900

From MODI (after [1st] iteration):

Total cost = 2 x 50 + 4 x 50 + 3 x 150 + 10 x 100 + 9 x 50 + 13 x 150 = $4,150

Improvement = 5900 -4150 = $1750

Note: The iterations beyond first are not required for this question but if you want to work on this question, this is final solution:

MODI: 3rd Iteration

		K1= 2	K2= 0	K3= 4	K4= 8	K5=	
	Source	Destination W1	W2	W3	W4		SUPPLY
R1= 0	P1	2 - ~~50~~	6 6	4 + 50 +50	12 4		100
R2= 3	P2	7 2	3 150	10 3	11 100		250
R3= 5	P3	5 + +50 -2	8 3	9 - ~~150~~ 100	13 150		300
R4=							
	DEMAND	50	150	200	250		

MODI: Final

		K1= 0	K2= 0	K3= 4	K4= 8	K5=	
	Source	Destination W1	W2	W3	W4		SUPPLY
R1= 0	P1	2 2	6 6	4 100	12 4		100
R2= 3	P2	7 4	3 150	10 3	11 100		250
R3= 5	P3	5 50	8 3	9 100	13 150		300
R4=							
	DEMAND	50	150	200	250		

Total cost = 4 x 100 + 3 x 150 + 11 x 100 + 5 x 50 + 9 x 100 + 13 x 150 = $5,050

------------------------------- END OF QUESTION 13 -------------------------------

Question 14 & 15: **(10 marks)**

Q14) A baseball card dealer must determine how many 1955 reproduced Willie Mays cards to stock. He experiences an annual demand of 100 cards. Each card is acquired from a big dealer for $2. Each shipment must be sent by registered mail at a cost of $4 regardless of quantity. Inventory is financed through a 16% bank loan.
Suppose a shortage penalty applies in the amount of $0.04 per card short (on an annual basis).

a). What is the economic order quantity?

ANSWER: **3 marks**

$$Q^* = \sqrt{\frac{2*100*4}{2*0.16}}\sqrt{\frac{0.04+(2*0.16)}{0.04}} = 150$$

b). What is the optimal order level?

ANSWER: **2 mark**

$$S^* = \sqrt{\frac{2*100*4}{2*0.16}}\sqrt{\frac{0.04}{0.04+(2*0.16)}} = 16\frac{2}{3}$$

c). If the optimal policy is used, determine the number of cards on backorder when a shipment arrives.

ANSWER: **1 mark**

Q* – S* = 150 – 16 2/3 = 133 1/3

b) Q15) The demand for Halloween pumpkins at the Black Cat's Patch is normally distributed with a mean of 1,000 and a standard deviation of 200. Each pumpkin costs $0.50 and sells for $0.90. Unsold pumpkins are disposed of at a cost of $0.10 each.

a) How many pumpkins should be ordered?

ANSWER: **3 marks**

$c = \$.50$ $p_S = 0$ $p_R = \$.90$ $h_E = \$.10$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + .90 - .50}{(0 + 0.90 - 0.50) + (0.10 + 0.50)} = .40$$

$\Pr[D \le Q^*] = .40$ $z = -.25$

$Q^* = \mu + z\sigma = 1{,}000 - .25(200) = 950$ pumpkins

(ii) For the quantity in (a), determine the probability that there will be a shortage?

ANSWER: **1 mark**

$\Pr[D > Q^*] = 1 - \Pr[D \le Q^*] = 1 - 0.40 = 0.60$

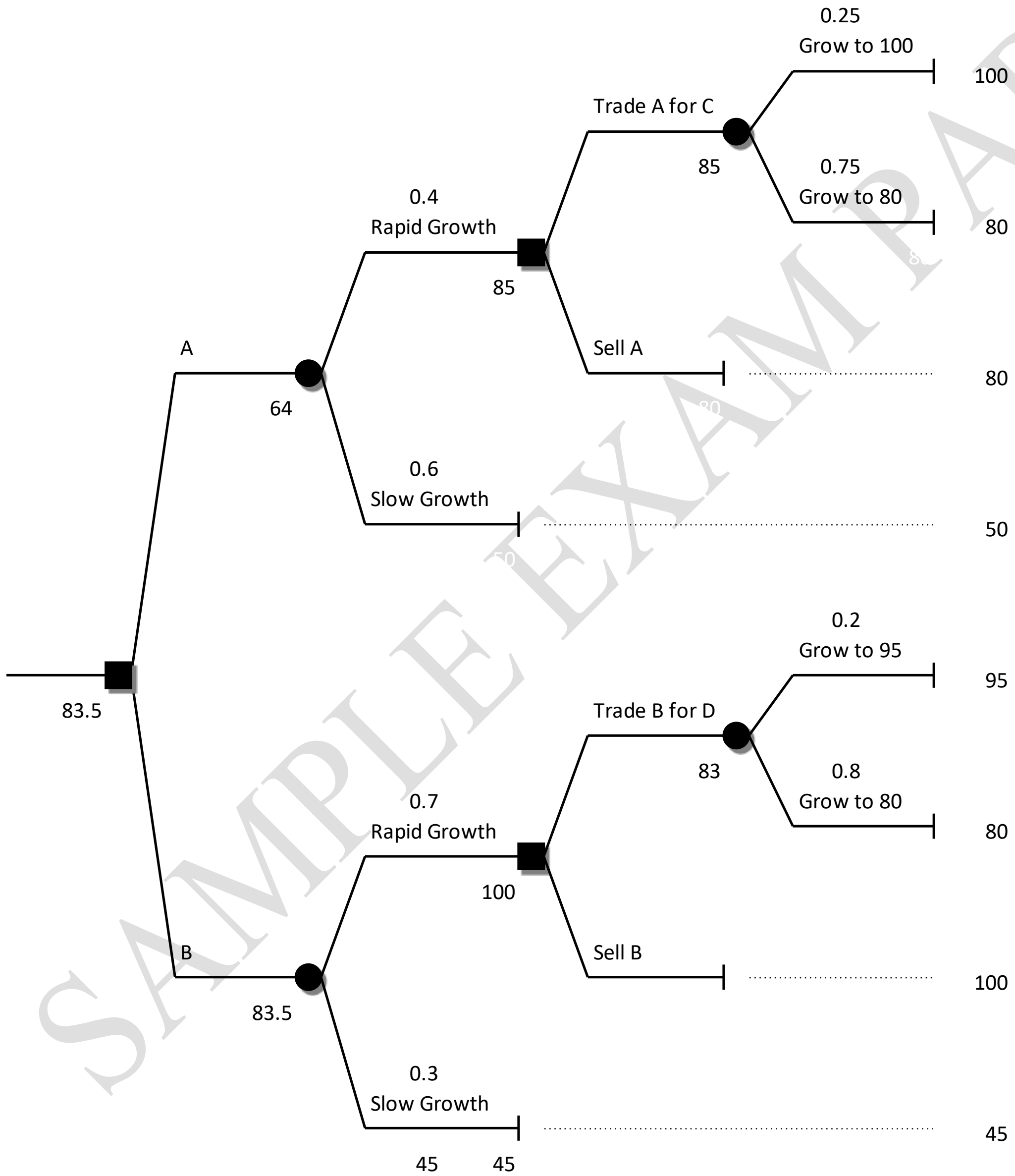
-------------------------------- END OF QUESTION 14 & 15 --------------------------------

Question 16 & 17: **(10** marks)

Q16) An investor is considering 2 investments, A, B, which can be purchased now for $10. There is a 40% chance that investment A will grow rapidly in value and a 60% chance that it will grow slowly. If A grows rapidly, the investor can cash it in for $80 or trade it for investment C, which has a 25% chance of growing to $100 and a 75% chance of reaching $80. If A grows slowly, it is sold for $50. There is a 70% chance that investment B will grow rapidly in value and a 30% chance that it will grow slowly. If B grows rapidly, the investor can cash it in for $100 or trade it for investment D, which has a 20% chance of growing to $95 and an 80% chance of reaching $80. If B grows slowly, it is sold for $45. A decision tree for the problem can be constructed as below.

a) . Using backward induction and the expected monetary value (EMV) approach, what is the EMV at decision nodes 1 and 2 (indicated in red in the given diagram).

ANSWER:



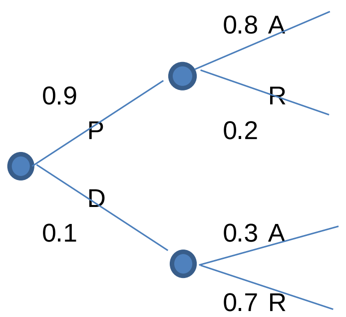
b). Evaluate the complete decision tree using the Expected Monetary Value (EMV) criteria and advice on the course of action. What is the EMV at decision node 3 as indicated in red in the diagram?

ANSWER: **3 marks**

Choose B and sell B regardless whether it's growing slowly or rapidly. EMV = $83.5

Q17) Eagle Credit Union (ECU) has experienced a 10% default rate with its commercial loan customers (i.e. 90% of commercial loan customers pay back their loans). ECU has developed a statistical test to assist in predicting which commercial loan customers will default. The test assigns either a rating of 'Approve' or 'Reject' to each loan applicant. When applied to recent loan commercial customers who paid their loans, the test gave an 'Approve' rating in 80% of the cases examined. When applied to recent loan commercial customers who defaulted, it gave a 'Reject' rating in 70% of the cases examined.

Note: To answer this question it may be easier to draw the decision tree or the joint probability table:



P(P ∩ A) = 0.9 x 0.8 = 0.72

P(P ∩ R) = 0.9 x 0.2 = 0.18

P(D ∩ A) = 0.1 x 0.3 = 0.03

P(D ∩ R) = 0.1 x 0.7 = 0.07

a)Fill in the joint probability table below

1 mark

	Joint Probabilities		
	Pay	**Default**	**Total**
Approve	0.720	0.030	0.750
Reject	0.180	0.070	0.250
Total	0.900	0.100	

b). What is the conditional probability of a 'Reject' rating given that the customer defaulted?

ANSWER: **1 marks**

P(Reject|Default) = 0.070/0.100 = 0.7

c). What is the conditional probability of an 'Approve' rating given that the customer defaulted?

ANSWER: **1 marks**

P(Approve|Default) = 0.030/0.100 = 0.3

d). Suppose a new customer receives a 'Reject' rating. If that customer gets the loan anyway, what is the probability of default?

ANSWER: **2 marks**

P(Default|Reject) = 0.070/0.250 = 0.28

-------------------------------- END OF QUESTIONS 16 & 17--------------------------------

Question 18 & 19: (10 marks)

Q18) The customer service desk at Joe's Discount Electronics store receives 5 customers per hour on average. On average, each customer requires 10 minutes for service. The customer service desk is staffed by a single clerk.

a). Determine the Arrival rate and the Service rate.

ANSWER: **1 mark**

$\lambda = 5$ (per hour)

$\mu = 1/10 \times 60 = 6$ (per hour)

b). What is the average time a customer spends in the customer service area?

ANSWER: **1 mark**

$W = 1/(\mu - \lambda) = 1 / (6 - 5) = 1$ Note: you will get the same answer if you use the formula given in the formula sheet

c). What is the probability that the customer service clerk takes more than 10 minutes?

ANSWER: **1 mark**

$P(x > 10\text{mins}) = e^{-10/10} = 0.368$

d). What is the average number of customers in the queue?

ANSWER: **1 mark**

$Lq = \frac{\lambda^2}{\mu(\mu - \lambda)} = 4.17$

e). What is the probability that there are less than 5 customers arriving in an hour?

ANSWER: **2 mark**

$f(x) = \frac{\theta^x e^{-\theta}}{x!}$ for a distribution having mean $\theta, (e = 2.71828...)$

$P(x=0) = (5^0 e^{-5})/(0!) = 0.00674$ (NOTE: 0!=1)

$P(x=1) = (5^1 e^{-5})/(1!) = 0.03369$

$P(x=2) = (5^2 e^{-5})/(2!) = 0.08422$

$P(x=3) = (5^3 e^{-5})/(3!) = 0.14037$

$P(x=4) = (5^4 e^{-5})/(4!) = 0.17547$

P(less than 5 customers) = 0.441

Q19) Simulate the arrival of patients at a clinic using the uniform random number given in the table. The mean inter-arrival time is two minutes. Using the formula $t_n = -b \log_e(r_n)$, calculate the arrival time for 5 customers.

a). If service time is a constant at five minutes, complete the table below:

Customer	Random Number	Interarrival Time	Arrival Time	Service Starts	Service Ends	Number of patient in the clinic
1	0.42	-2 x -0.87 = 1.74	1.74	1.74	6.74	4
2	0.96	-2 x -0.04 = 0.08	1.82	6.74	11.74	4
3	0.37	-2 x -0.99 = 1.99	3.81	11.74	16.74	3
4	0.52	-2 x -0.65 = 1.31	5.11	16.74	21.74	2
5	0.23	-2 x -1.47 = 2.94	8.05	21.74	26.74	1

b). Calculate the average number of patients in the clinic.

Average number of patients in the clinic is 2.8 (but in the long run there will be a 'never-ending' queue!)

-------------------------------- **END OF QUESTIONS 18 & 19** --------------------------------

Question 20: (10 marks)

The following is a set of quarterly sales data recorded over a period of 3 years. The deseasonalised sales data has also been worked out for you:

Period	Actual Sales	Seasonally adjusted
1	5	6.16
2	6	6.53
3	8	6.36
4	7	6.92
5	6.2	7.63
6	6.5	7.07
7	11	8.75
8	9	8.90
9	7.4	9.11
10	10	10.88
11	12	9.54
12	10.3	10.19

a) Fit a least square regression line for the above data.

ANSWER: **4 marks**

	X	Y	XY	X^2
	1	6.16	6.2	1
	2	6.53	13.1	4
	3	6.36	19.1	9
	4	6.92	27.7	16
	5	7.63	38.2	25
	6	7.07	42.4	36
	7	8.75	61.2	49
	8	8.90	71.2	64
	9	9.11	82.0	81
	10	10.88	108.8	100
	11	9.54	105.0	121
	12	10.19	122.2	144
Sum:	78	98.04278	697.0	650
Mean:	6.5	8.170232		

Regression line: $y = mx + c$

$$m = \boxed{\frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}} = 0.417829$$

$$c = \bar{y} - m\bar{x} = 5.454345$$

y = 0.42x + 5.45

b) Using the least square regression line from part (a), forecast the sales for the next 4 quarters in the next year (Period 13, 14, 15 and 16) using the multiplicative model. Assume the following seasonal index,

Quarter	Index
1	81.22%
2	91.89%
3	125.76%
4	101.13%

ANSWER: **2 mark**

Y_{13} = (0.42 x 13 + 5.45) x 0.8122 = 8.8

Y_{14} = (0.42 x 14 + 5.45) x 0.9189 = 10.4

Y_{15} = (0.42 x 15 + 5.45) x 1.2576 = 14.7

Y_{16} = (0.42 x 16 + 5.45) x 1.0113 = 12.3

c) Considering the following coefficients generated from regression summary statistics. Forecast the sales for the next 4 quarters in the next year (Period 13, 14, 15 and 16) using the additive model.

Coefficients:

Intercept	Period	1	2	3	4
5.34	0.43	-1.28	-0.41	1.99	0

ANSWER: **2 mark**

Y_{13} = (0.43 x 13 + 5.34) – 1.28 = 9.625

Y_{14} = (0.43 x 14 + 5.34) – 0.41 = 10.925

Y_{15} = (0.43 x 15 + 5.34) + 1.99 = 13.758

Y_{16} = (0.43 x 16 + 5.34) + 0 = 12.192

d) If the sales in 2018the next year turn out to be Quarter 1: 8.5, Quarter 2: 10, Quarter 3: 15 and Quarter 4: 12, calculate the MAPE of the forecast in b) and c) and comment which model should be used.

ANSWER: **2 marks**

Multiplicative model MAPE = $\frac{0.04+0.039+0.017+0.023}{4}$ = 0.029

Additive model MAPE = $\frac{0.132+0.093+0.083+0.016}{4}$ = 0.081

The multiplicative seems to be a better forecast technique for this se of data.

------------------------------- END OF QUESTION 20-------------------------------

Formula Sheet

Expected Value of a Project

If a decision has a number of outcomes, i , each having a payoff x_i , with probability $p(x_i)$ then the expected value of the decision is given by $\sum_i x_i p(x_i)$

Bayes' Theorem

To find the posterior probability that event A_i will occur given that event B has occurred

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \ldots + P(A_n)P(B|A_n)}$$

Tabular form for calculations

States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
A1	$P(A_1)$	$P(B \mid A_1)$	$P(B \cap A_1)$	$P(A_1 \mid B)$
A2	$P(A_2)$	$P(B \mid A_2)$	$P(B \cap A_2)$	$P(A_2 \mid B)$
			$P(B)$	

Least Squares Regression

For bivariate data consisting of n pairs of observations (x, y), the Least Squares Line of Best Fit is $y = mx + c$,

where $m = \dfrac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$ and $c = \bar{y} - m\bar{x}$.

Simple exponential smoothing

$\hat{y}_{t+1} = \hat{y}_t + a(y_t - \hat{y}_t)$,

where : $\hat{y}_t$: forecast value, y_t : observed value, α : smoothing factor, t : period (time) index

Mean Squared Error

$$MSE = \frac{\sum_{i=1}^{n} (Y_i - F_i)^2}{n}$$

Mean Absolute Percent Error

$$MAPE = \frac{\sum_{i=1}^{n} \frac{|F_i - Y_i|}{Y_i}}{n}$$

Y_i *are the actual (observed) values*

F_i *are the fitted (forecast) values*

n *is the number of forecast values*

Queuing, Probability and Simulation

Service and waiting times for a single server queue, Poisson arrivals, Exponential service:

$\lambda =$ the average number of arrivals per time period (arrival rate)

$\frac{1}{\lambda} =$ the average time between arrivals

$\mu =$ the average number of services per time period (service rate)

$\frac{1}{\mu} =$ the average time taken for each service

$P_0 = 1 - \frac{\lambda}{\mu}$ the probability that no units are in the system

$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$ the average number of units in the waiting line

$L = L_q + \frac{\lambda}{\mu}$ the average number of units in the system

$W_q = \frac{L_q}{\lambda}$ the average time a unit spends in the waiting line

$W = W_q + \frac{1}{\mu}$ the average time a unit spends in the system

$P_w = \frac{\lambda}{\mu}$ the probability that an arriving unit has to wait for service

$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ the probability of n units in the system

Probability distributions:

The Poisson distribution

$f(x) = \frac{\theta^x e^{-\theta}}{x!}$ for a distribution having mean $\theta, (e = 2.71828...)$

The exponential distribution

$f(x) = \frac{1}{\theta} e^{-x/\theta}$ for a distribution having mean $\theta, (e = 2.71828...)$

$P(x \le x_0) = 1 - e^{-x_0/\theta}$

$P(x \ge x_0) = e^{-x_0/\theta}$ for a given value of x_0

Linear congruential generation of uniform random variables

> Let X_0 be an integer chosen at random (the random seed) then uniformly distributed integers are generated as $X_{n+1} = AX_n \bmod B$ where A and B are large co prime integers. Random numbers between 0 and 1 are calculated as $r_n = \frac{X_n - 1}{B - 2}$.

Generation of Exponentially distributed random variables

> Exponential variates with mean b are generated from uniform [0,1] random numbers, r_n, by the transformation $t_n = -b \log_e(r_n)$.

Inventory Models: Deterministic Demand

Holding cost $(per\ item) = h$ Order cost $= k$ Item cost $= c$ Back order cost $(per\ item) = p$
Annual demand $= A$ Production Rate $= B$

Economic Order Quantity

Optimal order quantity: $Q^* = \sqrt{\dfrac{2Ak}{ch}}$

Number of orders per year $= \dfrac{A}{Q^*}$

Time between orders (cycle time) $= \dfrac{Q^*}{A}$ years

Total annual cost = ordering cost + holding cost $= \dfrac{Ak}{Q} + \dfrac{Qch}{2}$

Economic Production Quantity

Optimal production lot size: $Q^* = \sqrt{\dfrac{2Ak}{ch}}\sqrt{\dfrac{B}{B-A}}$

Number of production runs per year $= \dfrac{A}{Q^*}$

Time between setups (cycle time) $= \dfrac{Q^*}{A}$ years

Total annual cost = setup cost + holding cost $= \dfrac{Ak}{Q} + \dfrac{chQ}{2}\left(\dfrac{B-A}{B}\right)$

EOQ with back orders

Optimal order quantity, $Q^* = \sqrt{\dfrac{2Ak}{ch}\left(\dfrac{p+ch}{p}\right)}$

Quantity at the beginning of each cycle, $S^* = \sqrt{\dfrac{2Ak}{ch}\left(\dfrac{p}{p+ch}\right)}$

Maximum number of backorders $= Q^* - S^*$

Number of orders per year $= \dfrac{A}{Q^*}$

Time between orders (cycle time) $= \dfrac{Q^*}{A}$ years

$$\text{Total annual cost} = \text{setup} + \text{holding} + \text{backorder}$$
$$= \frac{Ak}{Q} + \frac{chS^2}{2Q} + \frac{p(Q-S)^2}{2Q}$$

Formula for total cost using Quantity discounts.

$$\text{Total annual cost} = \text{purchase cost} + \text{holding cost} + \text{item cost}$$
$$= \frac{Ak}{Q} + \frac{chQ}{2} + Ac$$

Inventory models under random demand, assumed to be normally distributed with mean μ and standard deviation σ.

Single Period Order Quantity

$$P(demand < Q^*) = \frac{C_u}{C_u + C_v}, \quad Q^* = \mu + z\sigma$$

Cost of overestimating demand: $C_v = h_E + c$

Cost of underestimating demand: $C_u = P_S + P_R - c$

Where:

Unit Cost $= c$

Penalty for item held at end of inventory cycle $= h_E$

Penalty for each item short (goodwill etc.) $= P_S$

Selling Price $= P_R$

Reorder Point Model

Reorder Point: $r = \mu + z\sigma$

Average Inventory: $\frac{Q}{2} + z\sigma$

Total Annual Cost: $\left(\frac{Q}{2} + z\sigma\right)ch + \frac{Ak}{Q}$

Cumulative Probabilities for the Standard Normal Distribution

Table gives P(Z<z) for Z = N(0,1)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000