Lecture 2: Modeling and Formulating LP Problems

1. Jones Furniture Company produces beds and desks for college students. The production process requires carpentry and varnishing. Each bed requires 6 hours of carpentry and 4 hour of varnishing. Each desk requires 4 hours of carpentry and 8 hours of varnishing. There are 36 hours of carpentry time and 40 hours of varnishing time available. Beds generate $30 of profit and desks generate $40 of profit. Demand for desks is limited, so at most 8 will be produced.

   Formulate the LP for this problem.

   ANSWER:

   Let X_1 = Number of Beds to produce
   X_2 = Number of Desks to produce
   The LP model for the problem is
   MAX: $30 X_1 + 40 X_2$
   Subject to: $6 X_1 + 4 X_2 \leq 36$ (carpentry)
   $4 X_1 + 8 X_2 \leq 40$ (varnishing)
   $X_2 \leq 8$ (demand for X_2)
   $X_1, X_2 \geq 0$

2. A financial planner wants to design a portfolio of investments for a client. The client has $300,000 to invest and the planner has identified four investment options for the money. The following requirements have been placed on the planner. No more than 25% of the money in any one investment, at least one third should be invested in long-term bonds which mature in seven or more years, and no more than 25% of the total money should be invested in C or D since they are riskier investments. The relevant information regarding each investment option is given in the following table. The objective is to maximize the total return of the portfolio.

Investment	Return	Years to Maturity	Rating
A	6.45%	9	1-Excellent
B	7.10%	8	2-Very Good
C	8.20%	5	4-Fair
D	9.00%	8	3-Good

   Formulate the LP for this problem.

   ANSWER:

   Let X_1 = Dollars invested in A
   X_2 = Dollars invested in B
   X_3 = Dollars invested in C
   X_4 = Dollars invested in D

   MAX: $.0645 X_1 + .071 X_2 + .082 X_3 + .09 X_4$
   Subject to: $X_1 + X_2 + X_3 + X_4 \leq 300000$
   $X_1 \leq 75000$
   $X_2 \leq 75000$
   $X_3 \leq 75000$
   $X_4 \leq 75000$
   $X_1 + X_2 + X_4 \geq 100000$
   $X_3 + X_4 \leq 75000$
   $X_1, X_2, X_3, X_4 \geq 0$

3. A farmer is planning his spring planting. He has 20 acres on which he can plant a combination of Corn, Pumpkins and Beans. He wants to maximize his profit but there is a limited demand for each crop. Each crop also requires fertilizer and irrigation water which are in short supply. There are only 50 acre ft of irrigation available and only 8,000 pounds/acre of fertilizer available. The following table summarizes the data for the problem.

Crop	Profit per Acre ($)	Yield per Acre (lb)	Maximum Demand (lb)	Irrigation (acre ft)	Fertilizer (pounds/acre)
Corn	2,100	21,000	200,000	2	500
Pumpkin	900	10,000	180,000	3	400
Beans	1,050	3,500	80,000	1	300

Formulate the LP for this problem.

ANSWER:

Let

X_1 = acres of corn
X_2 = acres of pumpkin
X_3 = acres of beans

MAX: $2100X_1 + 900X_2 + 1050X_3$
Subject to: $21X_1 <= 200$
$10X_2 <= 180$
$3.5X_3 <= 80$
$X_1 + X_2 + X_3 <= 20$
$2X_1 + 3X_2 + 1X_3 <= 50$
$5X_1 + 4X_2 + 3X_3 <= 80$
$X_1, X_2, X_3 \geq 0$

4. A hospital needs to determine how many nurses to hire to cover a 24 hour period. The nurses must work 8 consecutive hours but can start work at the start of 6 different shifts. They are paid different wages depending on when they start their shifts. The number of nurses required per 4-hour time period and their wages are shown in the following table.

Time period	Required # of Nurses	Wage ($/hr)
12 am - 4 am	20	15
4 am - 8 am	30	16
8 am - 12 pm	40	13
12 pm - 4 pm	50	13
4 pm - 8 pm	40	14
8 pm - 12 am	30	15

Formulate the LP for this problem.

ANSWER:

Let X_i = number of nurses working in time period i; i = 1,6

MIN: $15X_1 + 16X_2 + 13X_3 + 13X_4 + 14X_5 + 15X_6$
Subject to: $1X_1 + 1X_2 \geq 20$
$1X_2 + 1X_3 \geq 30$
$1X_3 + 1X_4 \geq 40$
$1X_4 + 1X_5 \geq 50$
$1X_5 + 1X_6 \geq 40$

$1X_6 + 1X_1 \geq 30$
$X_i \geq 0$

5. A company needs to purchase several new machines to meet its future production needs. It can purchase three different types of machines A, B, and C. Each machine A costs $80,000 and requires 2,000 square feet of floor space. Each machine B costs $50,000 and requires 3,000 square feet of floor space. Each machine C costs $40,000 and requires 5,000 square feet of floor space. The machines can produce 200, 250 and 350 units per day respectively. The plant can only afford $500,000 for all the machines and has at most 20,000 square feet of room for the machines. The company wants to buy as many machines as possible to maximize daily production.

   Formulate the LP for this problem.

   ANSWER:

   Let X_i = number of machines of type i purchased

   MAX: $200X_1 + 250X_2 + 300X_3$
   Subject to: $2X_1 + 3X_2 + 5X_3 <= 20$
   $80X_1 + 50X_2 + 40X_3 <= 500$
   $X_1, X_2, X_3 \geq 0$