Lecture 1: Introduction to Management Science
Introduction to Optimization and Linear Programming

1. Solve the following LP problem graphically by enumerating the corner points.

   MAX: $2 X_1 + 7 X_2$

   Subject to: $5 X_1 + 9 X_2 \leq 90$

   $9 X_1 + 8 X_2 \leq 144$

   $X_2 \leq 8$

   $X_1, X_2 \geq 0$

ANSWER: Obj = 63.20
$X_1 = 3.6$
$X_2 = 8$

2. Solve the following LP problem graphically using level curves.

   MAX: $7 X_1 + 4 X_2$

   Subject to: $2 X_1 + X_2 \leq 16$

   $X_1 + X_2 \leq 10$

   $2 X_1 + 5 X_2 \leq 40$

   $X_1, X_2 \geq 0$

ANSWER: Obj = 58
$X_1 = 6$
$X_2 = 4$

3. Solve the following LP problem graphically by enumerating the corner points.

   MIN: $8 X_1 + 3 X_2$

   Subject to: $X_2 \geq 8$

   $8 X_1 + 5 X_2 \geq 80$

   $3 X_1 + 5 X_2 \geq 60$

   $X_1, X_2 \geq 0$

ANSWER: Obj = 48
$X_1 = 0$
$X_2 = 16$

4. Solve the following LP problem graphically using level curves.

   MAX: $5 X_1 + 3 X_2$

   Subject to: $2 X_1 - 1 X_2 \leq 2$

   $6 X_1 + 6 X_2 \geq 12$

   $1 X_1 + 3 X_2 \leq 5$

   $X_1, X_2 \geq 0$

ANSWER: Obj = 11.29
$X_1 = 1.57$
$X_2 = 1.14$

5. Solve the following LP problem graphically using level curves.

MIN: $5 X_1 + 7 X_2$

Subject to: $4 X_1 + 1 X_2 \geq 16$

$6 X_1 + 5 X_2 \geq 60$

$5 X_1 + 8 X_2 \geq 80$

$X_1, X_2 \geq 0$

ANSWER: Obj = 72.17
$X_1 = 3.48$
$X_2 = 7.83$

6. Jones Furniture Company produces beds and desks for college students. The production process requires carpentry and varnishing. Each bed requires 6 hours of carpentry and 4 hour of varnishing. Each desk requires 4 hours of carpentry and 8 hours of varnishing. There are 36 hours of carpentry time and 40 hours of varnishing time available. Beds generate $30 of profit and desks generate $40 of profit. Demand for desks is limited so at most 8 will be produced.

Formulate the LP model for this problem.

ANSWER: Let X_1 = Number of Beds to produce
X_2 = Number of Desks to produce

MAX: $30 X_1 + 40 X_2$
Subject to: $6 X_1 + 4 X_2 \leq 36$ (carpentry)
$4 X_1 + 8 X_2 \leq 40$ (varnishing)
$X_2 \leq 8$ (demand for X_2)
$X_1, X_2 \geq 0$

Solve the problem using the graphical method.

ANSWER: Obj = 240
$X_1 = 4$
$X_2 = 3$

7. The Big Bang explosives company produces customized blasting compounds for use in the mining industry. The two ingredients for these explosives are agent A and agent B. Big Bang just received an order for 1400 pounds of explosive. Agent A costs $5 per pound and agent B costs $6 per pound. The customer's mixture must contain at least 20% agent A and at least 50% agent B. The company wants to provide the least expensive mixture which will satisfy the customers requirements.

Formulate the LP model for this problem.

ANSWER: Let X_1 = Pounds of agent A used
X_2 = Pounds of agent B used

MIN: $5 X_1 + 6 X_2$
Subject to: $X_1 \geq 280$ (Agent A requirement)
$X_2 \geq 700$ (Agent B requirement)
$X_1 + X_2 = 1400$ (Total pounds)

$X_1, X_2 \geq 0$

Solve the problem using the graphical method.

ANSWER: Obj = 7700
$X_1 = 700$
$X_2 = 700$

8. Bob and Dora Sweet wish to start investing $1,000 each month. The Sweets are looking at five investment plans and wish to maximize their expected return each month. Assume interest rates remain fixed and once their investment plan is selected they do not change their mind. The investment plans offered are:

Fidelity	9.1% return per year
Optima	16.1% return per year
CaseWay	7.3% return per year
Safeway	5.6% return per year
National	12.3% return per year

Since Optima and National are riskier, the Sweets want a limit of 30% per month of their total investments placed in these two investments. Since Safeway and Fidelity are low risk, they want at least 40% of their investment total placed in these investments.

Formulate the LP model for this problem.

ANSWER:
MAX: $0.091X_1 + 0.161X_2 + 0.073X_3 + 0.056X_4 + 0.123X_5$
Subject to:

$X_1 + X_2 + X_3 + X_4 + X_5 = 1000$
$X_2 + X_5 \leq 300$
$X_1 + X_4 \geq 400$
$X_1, X_2, X_3, X_4, X_5 \geq 0$

Reference:

Ragsdale, C. T., Spreadsheet Modelling and Decision Analysis 5th Ed, Thomson, 2008: Chapter 1 & 2