FIT3158 Business Decision Modelling

Tutorial 7 Solution

Deterministic Inventory

Topics covered:

- **Economic Order Quantity (EOQ) Model**
- **Quantity Discounts for the EOQ Model**
- **An Inventory Model with Planned Shortages**
- **Economic Production Lot Size Model**

Exercise 1: (Ragsdale 8E Chapter 8 Question 15/Ragsdale 9E Chapter 8 Question 19)

SuperCity is a large retailer of electronics and appliances. The store sells three different models of TVs that are ordered from different manufacturers. The demands, costs, and storage requirements for each model are summarized in the following table:

	Model 1	**Model 2**	**Model 3**
Annual Demand	800	500	1,500
Unit Cost	$300	$1,100	$600
Storage space required	9 sq m	25 sq m	16 sq m

It costs $60 to do the administrative work associated with preparing, processing, and receiving orders, and SuperCity assumes a 25% annual carrying cost for all items that it holds in inventory. There are 3,000 square metres of total warehouse space available for storing these items, and the store never wants to have more than $45,000 invested in inventory for these items. The manager of this store wants to determine the optimal order quantity for each model of TV.

a) Solve using the EOQ model formula for Q*.
b) What are the optimal order quantities?
c) How many orders of each type of TV will be placed each year?
d) Assuming that demand is constant throughout the year, how often should the orders be placed.
e) Formulate an NLP model for this problem.
f) Implement your model in a spreadsheet and solve it.

Exercise 2: (Ragsdale 8E Chapter 8 Question 16/Ragsdale 9E Chapter 8 Question 20)

The Radford hardware store expects to sell 1,500 electric garbage disposal units in the coming year. Demand for this product is fairly stable over the year. It costs $20 to place an order for these units and the company assumes a 15% annual holding cost on inventory. The following price structure applies to Radford's purchases of this product:

	Order Quantity		
	0 to 499	500 to 999	1000 and up
Price per Unit	\$35	\$33	\$31

So if Radford orders 135 units, they pay \$35 per unit; if they order 650, they pay \$33 per unit; and if they order 1,200, they pay \$31 per unit.

a) What is the most economical order quantity and total cost of this solution? *(Hint:* Solve a separate EOQ problem for each of the order quantity ranges given and select the solution that yields the lowest total cost.)

b) Suppose the discount policy changed so that Radford had to pay \$35 for the first 499 units ordered, \$33 for the next 500 units ordered, and \$31 for any additional units. What is the most economical order quantity and what is the total cost of this solution?

<u>Exercise 3:</u> (Lapin & Whisler Question 15-5)

ZIP Electric Bike Company buys special batteries to power its most popular model, called the Zippy. Since ZIP's successful IPO last year and because of high-profile endorsements by several well-known sports figures, demand has taken off, and the company is selling about 100,000 Zippies per year. The batteries cost \$50 each. ZIP figures that the impact of holding inventory is 15% per year and placing an order costs the company \$200.

a) Using the economic order quantity model, determine
   i. the optimal order quantity,
   ii. the number of orders placed per year, and
   iii. the total annual relevant cost.

b) Using a one variable data table, find the values in i-iii above for demand ranging from 10,000 – 200,000 Zippies.

c) Using Excel's Chart Wizard, create a chart of Q* vs demand.

<u>Exercise 4</u> (Lapin & Whisler Question 15-10)

In Exercise 3, suppose that when ZIP is out of batteries, the company cannot sell its Zippy electric bike. The company determines that this costs it \$20 per bike per year because of the unhappy customers.

a. Answer the following questions:
   i. What are the optimal order quantity and the maximum inventory level?
   ii. How many times per year should ZIP Electric Bike Company place orders for batteries?
   iii. What is the total annual relevant cost?
   iv. Calculate the maximum number of backorders.

b. Using a one variable data table, find each of the values in i-iv above for a penalty cost ranging from \$5 to \$100.

c. Using Excel's Chart Wizard, create a chart of Q*, S* and TC (total costs) on the y-axis vs the shortage cost, p, on the x-axis. Comment on how the solution varies as the penalty cost varies.

Exercise 5 (Lapin & Whisler Question 15-15)

The manufacturing vice-president of ZIP Electric Bike Company (refer to Ex 3) is thinking about manufacturing the special batteries used in the company's electric bikes, because of recent shortages by its supplier. He estimates that the company could do it cheaper, for about $40 per battery, and that it could make about 150,000 batteries per year with the current idle capacity in the manufacturing plant. The setup cost to manufacture the batteries would be $1000.

a. Find the optimal production quantity.
b. What is the maximum inventory of batteries that ZIP will have during the year?
c. How many times per year should ZIP make the batteries?
d. What is the duration of the optimal production run?
e. What is the total annual relevant cost?
f. Should ZIP continue to buy the batteries as in Ex 4 above, or start making them?
g. What happens to the solution when the demand is 150,000 per year? Why?

Solutions

Exercise 1

b. Model 1 = 35.7, Model 2 = 14.7, Model 3 = 34.6
c. Model 1 = 800/35.7 = 22.4 , Model 2 = 500/14.7 = 35.7 , Model 3 = 1500/34.6= 43.4
d. Model 1 = about every 16.3 days , Model 2 = about every 10.7 days , Model 3 = about every 8.4 days

e. MIN $60*(800/Q_1 + 500/Q_2 + 1500/Q_3) + 0.25*(300Q_1 + 1100Q_2 + 600Q_3)/2$

ST $300Q_1 + 1100Q_2 + 600Q_3 \leq 45{,}000$

$9Q_1 + 25Q_2 + 16Q_3 \leq 3{,}000$

$Q_1 , Q_2, Q_3 \geq 1$ and integer

See file: Tutorial 7 Solutions.xlsm
Solver solution would be slightly different due to considered constraints.

Exercise 2

a. Answer: Q=1000, Cost = $48,855
   Working:
   A = 1,500
   k = 20
   h=.15
   (1) When c=$35 (for orders less than 500)

$$Q* = \sqrt{\frac{2(1{,}500)(20)}{(35)(0.15)}} = 106.90$$

   (2) When c=$33 (for orders more than 500 but less than 1000)

$$Q* = \sqrt{\frac{2(1{,}500)(20)}{(33)(0.15)}} = 110.10$$

This quantity is not feasible as they need to order 500 or more to be eligible for $33

(3) When c=$31 (for orders more than 1000)

$$Q* = \sqrt{\frac{2(1{,}500)(20)}{(31)(0.15)}} = 113.60$$

This quantity is not feasible as they need to order 1000 or more to be eligible for $31

Working out the total cost:

TC = Purchasing cost + Ordering cost + Holding cost

(1) TC_1 = (1,500 * $35) + (1,500/106.9 * $20) + (107/2 * $35 * 015) = $53,061

(2) TC_2 = (1,500 * $33) + (1,500/500 * $20) + (500/2 * $33 * 015) = $50,797.5

(3) TC_3 = (1,500 * $31) + (1,500/1,000 * $20) + (1,000/2 * $31 * 015) = $48,855

Therefore, they should order 1,000 units @$31 – this will give the lowest total cost.

b. Q=107, Cost = $53,061 (This is a situation where the price is staggered. This question will be difficult to work without spreadsheet/solver and will not be examinable)

See file: Tutorial 7 Solutions.xlsm

Exercise 3

(a) (i) A = 100,000
k = 200
h=.15
c=50

$$Q^* = \sqrt{\frac{2(100{,}000)(200)}{(0.15)(50)}} = 2{,}309.4$$

(ii) The number of orders placed per year is

A/Q* = 100,000/2,309.4 = 43.3

(iii) $$TC(Q^*) = \left(\frac{100{,}000}{2{,}309.4}\right)(200) + (0.15)(50)\left(\frac{2{,}309.4}{2}\right)$$
$$= \$8{,}660.26 + \$8{,}660.25 = \$17{,}320.51$$

Exercise 4

Q* = $\sqrt{\frac{2Ak}{ch}\left(\frac{p+ch}{p}\right)}$=2708

S* = $\sqrt{\frac{2Ak}{ch}\left(\frac{p}{p+ch}\right)}$= approx. 1970

TC = $= \frac{Ak}{Q} + \frac{chS^2}{2Q} + \frac{p(Q-S)^2}{2Q}$ = $14771

Max backorders = Q*-S* = approx. 739

Exercise 5

(a) A = 100,000
B = 150,000
k = 1,000
h = 0.15
c = 40

$$Q^* = \sqrt{\frac{2(100{,}000)(1{,}000)}{(0.15)(40)}}\sqrt{\frac{150{,}000}{150{,}000 - 100{,}000}} = 10{,}000$$

(b) The maximum inventory is

$$= Q\left(\frac{B - A}{B}\right) = 10{,}000\left(\frac{150{,}000 - 100{,}000}{150{,}000}\right) = 3{,}333$$

(c) The number of times batteries are made per year is
A/Q* = 100,000/10,000 = 10

(d) The duration of the production run (in years) is

$$T_1 = \frac{Q}{B} = \frac{10{,}000}{150{,}000} = 0.0667$$

(e)

$$TC(Q^*, S^*) = \left(\frac{100{,}000}{10{,}000}\right)(1{,}000) + 0.15(40)\left(\frac{10{,}000}{2}\right)\left(\frac{150{,}000 - 100{,}000}{150{,}000}\right)$$
$$= \$10{,}000 + \$10{,}000 = \$20{,}000$$

(f) Buy the batteries because the total annual relevant cost is $17,320.51 (from Exercise 3) compared with $20,000 for making them from part (e) above.

(g)

$$Q^* = \sqrt{\frac{2(100{,}000)(1{,}000)}{(0.15)(40)}}\sqrt{\frac{150{,}000}{150{,}000 - 150{,}000}} = \text{undefined}$$

Since a division by zero occurs. This occurs because the production rate equals the demand rate and thus no solution exists.