Lecture 4: Integer Linear Programming

1. A small town wants to build some new recreational facilities. The proposed facilities include a swimming pool, recreation center, basketball court and baseball field. The town council wants to provide the facilities which will be used by the most people, but faces budget and land limitations. The town has $400,000 and 14 acres of land. The pool requires locker facilities which would be in the recreation center, so if the swimming pool is built the recreation center must also be built. Also the council has only enough flat land to build the basketball court or the baseball field. The daily usage and cost of the facilities (in $1,000) are shown below.

Variable	Facility	Usage	Cost ($1,000)	Land
X_1	Swimming pool	400	100	2
X_2	Recreation center	500	200	3
X_3	Basketball court	300	150	4
X_4	Baseball field	200	100	5

   a. Formulate the ILP for this problem.

   b. Based on this ILP formulation of the problem and if the optimal solution is: (X1, X2, X3, X4) = (1, 1, 0, 1) what values should go in cells B5:G12 of the following Excel spreadsheet?

	A	B	C	D	E	F	G
1							
2							
3			Facilities				
4		Pool	Rec center	Basketball	Baseball	Total usage:	
5	Select (0=no, 1=yes)	1	1	0	1		
6	Usage	400	500	300	200		
7							
8	Resources					Used	Available
9	Cost						
10	Land						
11	Pool & Rec center						
12	Basket or Baseball						

   c. What formulas should go in cells F5:F12 of the above Excel spreadsheet?

2. A company has four projects, numbered 1 through 4. If any project is selected for implementation, each lowered numbered project must also be selected for implementation. Formulate the constraints to enforce these conditions.

3. An investor has $500,000 to invest and wants to maximize the money they will receive at the end of one year. They can invest in condos, apartments and houses. The profit after one year, the cost and the number of units available are shown below.

Variable	Investment	Profit ($1,000)	Cost ($1,000)	Number Available
X_1	Condos	6	50	10
X_2	Apartments	12	90	5
X_3	Houses	9	100	7

a. Formulate the ILP for this problem.

b. Based on this ILP formulation of the problem and if the optimal integer solution is: (X1, X2, X3) = (1, 5, 0), what values should go in cells B5:F12 of the following spreadsheet?

	A	B	C	D	E	F
1			Investment selection			
2						
3			Investment			
4		Condos	Apartments	Houses	Profit:	
5	Number Invested					
6	Profit					
7						
8	Resources				Used	Available
9	Cost					
10	Condos					
11	Apartments					
12	Houses					

c. Based on the ILP formulation of the problem what formulas should go in cells E5:E12 of the above spreadsheet?

4. A shipping company wants to build two new warehouses to cover 4 regions. The company wants to minimize the cost of building the two warehouses while strategically trying to locate the warehouse to cover most regions. The company has identified the regions that can be covered by each warehouse site and are indicated by a 1 in the following table:

	Warehouse Sites			
Region	1	2	3	4
A		1		1
B	1		1	1
C	1	1	1	
D	1			1
COST ($000s)	210	160	200	260

a. Formulate the ILP for this problem.

b. Based on this ILP formulation of the problem and the solution (X1 , X2 , X3 , X4)=(1, 1, 0, 0) what values should go in cells B6:G14 of the following Excel spreadsheet?

	A	B	C	D	E	F	G
1							
2							
3			Warehouse location				
4		1	2	3	4	Total	
5	Location selected					coverage	
6	Cost						

7							
8						Warehouse	
9	Requirements					Covering	Required
10	Region A						
11	Region B						
12	Region C						
13	Region D						
14	Warehouse Required						

c. Based on this ILP formulation of the problem what formulas should go in cells F6:F14 of the above Excel spreadsheet?