FIT3158 Business Decision Modelling
Tutorial 11
Queuing Theory and Simulation

Topics covered:

- **Generating random numbers – Monte Carlo Simulation**
- **Application of Simulation in Newsboy Problem and Queuing**
- **Queuing Models – M/M/1 and M/M/s Model**

Download FIT3158 Tutorial 11.xlsx from Moodle.
Exercise 1: Simulating Newsboy Problem

News vendor Phyllis Pauley sells newspapers, and each day she must determine how many newspapers to order. Phyllis pays the company 20c for each newspaper and sells the paper for 30c each. Newspapers that are unsold by the end of the day are worthless.

Phyllis knows that each day she can sell between 6 and 10 papers, with varying probabilities. These are listed in the table below.

Deman	Probabilit
6	0.15
7	0.20
8	0.30
9	0.25
10	0.10

a) Use Monte Carlo simulation (with 10 simulations) to manually generate demand for the next 10 days. Use the random numbers given in the spreadsheet (B15:B24) to perform your simulation. (Hint, derive the cumulative probability table first.)

   [Once you've finished the exercise and sure that everything is working good, you can replace the random number with the RNG function: = RAND()]

b) Next, calculate the total profit for 10 days assuming a stock level of 9.

Exercise 2: Simulating Queuing Problem

Simulate the arrival of customers at Ezy Bright Car Wash using the exponential distribution, taking triplets of the random digits below as uniform random number. Calculate the inter-arrival time with mean of four minutes, using the following formula: $t_n = -b\log_e(r_n)$ to generate exponentially distributed random variables. Assume service time is a constant three minutes.

Random Numbers: 234793023942374619245628912629480927282384718234862349726189

Calculate arrival time, service time and waiting time for 20 customers.

a)

Customer	Random #	Inter-arrival	Arrival Time	Service Starts	Service Finish	Waiting Time
1	234 (0.234)	5.81	5.81	5.81	8.81	0.00
2	793 (0.793)	0.93	6.74	8.81	11.81	2.07
3	23 (0.023)	15.09	21.83	21.83	24.83	0.00
:						
20						

Exercise 3: (L & W [Lapin & Whisler] Problem 17-1)

For each of the following single-server queuing systems, determine the values of L, W, L_q, W_q, and ρ (=P_W):

	λ	μ
(a)	20	25
(b)	8	12
(c)	2	5
(d)	0.4	0.7

Exercise 4: (L & W Problem 17-5)

Sammy Lee is the sole operator of a barbershop. Between noon and 6 P.M. on Saturday afternoons, 1 customer arrives every 15 minutes on the average. Sammy takes an average of 10 minutes to trim each customer. His little shop has chairs for only 2 waiting customers in addition to the customer getting a haircut.

a) What is the probability that any particular customer will have to spend part of his waiting time standing up?

b) What percentage of an average Saturday afternoon is Sammy busy? How many hours is he idle on an average Saturday afternoon?

c) What is the average time that a customer will have to wait before getting a haircut?

d) What is the probability that any particular haircut takes more than 15 minutes?

Exercise 5: (L & W Problem 17-8)

Ace Airlines has one reservations clerk on duty at a time to handle information about flight times and make reservations. All calls to Ace Airlines are answered by an operator. If a caller requests information or reservations, the operator transfers the call to the reservations clerk. If the clerk is busy, the operator asks the caller to wait. When the clerk becomes free, the operator transfers the call of the person who has been waiting the longest. Assume that arrivals and services can be approximated by a Poisson process. Calls arrive at a rate of 10 per hour, and the reservations clerk can service a call in an average of 4 minutes.

a) What is the average number of calls waiting to be connected to the reservations clerk?

b) What is the average time that a caller must wait before reaching the reservations clerk?

c) What is the average time it takes for a caller to complete a call?

d) What is the probability that a caller takes more than 10 minutes to complete a reservation?

e) What is the probability that there will be no callers in a 15-minute period?

f) What is the probability that there will be 3 callers in a 15-minute period?

Exercise 6 (Ragsdale Problem 13-6)

Tri-Cities Bank has a single drive-in teller window. On Friday mornings, customers arrive at the drive-in window randomly, following a Poisson distribution at an average rate of 30 per hour.

a) How many customers arrive per minute, on average?

b) How many customers would you expect to arrive in a 10-minute interval?

c) Determine the probability of exactly 0, 1, 2, and 3 arrivals in a 10-minute interval.

d) What is the probability of more than three arrivals occurring in a 10-minute interval?

Solution:

Exercise 1 & 2: Refer to FIT3158 Tutorial 11 Solution.xlsx

Exercise 3: L&W17-1

(a) $L = 20/(25 - 20) = 4$
$W = 4/20 = .20$
$$L_q = \frac{(20)^2}{25(25-20)} = 3.2$$
$W_q = 3.2/20 = .16$
$\rho = \lambda/\mu = 20/25 = .8$

(b) $L = 8/(12 - 8) = 2$
$W = 2/8 = .25$
$$L_q = \frac{(20)^2}{25(25-20)} = 3.2$$
$W_q = 1.33/8 = .17$
$\rho = 8/12 = .67$

(c) $L = 2/(5 - 2) = .67$
$W = .67/2 = .34$
$$L_q = \frac{(2)^2}{5(5-2)} = .27$$
$W_q = .27/2 = .13$
$\rho = 2/5 = .4$

(d) $L = .4/(.7 - .4) = 1.33$
$W = 1.33/.4 = 3.33$
$$L_q = \frac{(.4)^2}{.7(.7-.4)} = .76$$
$W_q = .76/.4 = 1.90$
$\rho = .4/.7 = .57$

Exercise 4: L & W 17-5

$1/\lambda = 15$ minutes or 1/4 hour $1/\mu = 10$ minutes or 1/6 hour
$\lambda = 4$ customers/hour $\mu = 6$ customers/hour

(a) $P_0 = 1 - (\lambda/\mu) = 1/3$
$P_1 = (4/6)(1/3) = 4/18 = 2/9$
$P_2 = (4/6)^2(1/3) = 4/27$
$P_3 = (4/6)^3(1/3) = 8/81$
Pr[at most 3 customers] = 1/3 + 2/9 + 4/27 + 8/81 = 65/81 = 0.80

The probability that a customer must stand is the same as that for there being 3 or more customers ahead of the customer on his or her arrival.
Pr[stand] = 1 – 0.80 = 0.20

(b) $\rho = 4/6 = 2/3$; busy 66.67% of the time
Sammy is idle (1 – 2/3)6 = 2 hours.

(c) Wq = 4/3 * ¼ = 1/3 = 20 mins

(d) Pr[T > 15 min] = $e^{-15/10}$ (or Pr[T > ¼ hr] = $e^{-6(1/4)}$) = 0.2231

Exercise 5: L & W17-8

$\lambda = 10$/hour $\mu = 15$/hour

(a) $L_q = 4/3$ calls
(b) $W_q = 2/15$ hour or 8 minutes
(c) W = 1/5 hour or 12 minutes
(d) P[T > 10 min] = $e^{-10/4} = e^{-2.5} = .0821$ [→ service time = 1/15 = 4 mins]
(e) P[T > 15 min] = $e^{-15/6} = e^{-2.5} = .0821$ [→ arrival time = 1/10 = 6 mins]

(f) Using $\lambda t = 10(1/4) = 2.5$,

$$\Pr[x=3] = \frac{(2.5)^3}{3!}e^{-2.5} = \frac{15.625}{6}(0.082085) = 0.2138$$ ⍰ .2138

<u>Exercise 6:</u> Ragsdale 13-6

(a) There are 30/60 = 0.5 arrivals per minute, on average.

(b) In a ten minute interval we would expect 10*0.5 = 5 arrivals to occur.

(c) $P(x=0) = (5^0 e^{-5})/(0!) = 0.00674$ (NOTE: 0!=1)

$P(x=1) = (5^1 e^{-5})/(1!) = 0.03369$

$P(x=2) = (5^2 e^{-5})/(2!) = 0.08422$

$P(x=3) = (5^3 e^{-5})/(3!) = 0.14037$

(d) $P(x>3)=1-P(x\le 3) = 1 - 0.00674 - 0.03369 - 0.08422 - 0.14037 = 0.73497$