Solutions:

Lecture 7: Deterministic Inventory

1. $Q^* = \sqrt{\dfrac{2(400)20}{4(.10)}} = 200$ door knobs

2. (a) $Q^* = \sqrt{\dfrac{2(100)2{,}500}{.4(5)}} = 500$ medallions

   (b) $T^* = 500/2{,}500 = .2$ year

3. (a) $A = 1{,}000$ $\quad c = \$1.00 + .10 = \$1.10$
   $k = \$50$ $\quad h = \$.18 + .10 = \$.28$

   (b) $Q^* = \sqrt{\dfrac{2(1{,}000)50}{.28(1.10)}} = 569.8$ or 570

   (b) Q* = 2(1,000)(50) = 569.8 or 570

4. (a) $Q^* = \sqrt{\dfrac{2(1{,}200)20}{.3(1)}}\sqrt{\dfrac{.10+.30}{.10}} = 400(2) = 800$ pounds

   (b) $S^* = \sqrt{\dfrac{2(1{,}200)20}{.3(1)}}\sqrt{\dfrac{.10}{.10+.30}} = 400(1/2) = 200$ pounds

   (c) $T^* = 800/1{,}200 = 2/3$ year

5. (a) $Q^* = \sqrt{\dfrac{2(100)4}{.16(2)}} = 50$

   (b) (1) $Q* = \sqrt{\frac{2(100)4}{0.16(2)}}\sqrt{\frac{0.04+0.16(2)}{0.04}} = 50 \times 3 = 150$

   (2) $S* = \sqrt{\frac{2(100)4}{0.16(2)}}\sqrt{\frac{0.04}{0.04+0.16(2)}} = \frac{50}{3} = 16\frac{2}{3}$

   (3) $Q^* - S^* = 150 - 16\ 2/3 = 133\ 1/3$

6. (a) $Q^* = \sqrt{\dfrac{2(10{,}000)2{,}000}{10(.25)}}\sqrt{\dfrac{50{,}000}{50{,}000-10{,}000}} = 4{,}000(1.118) = 4{,}472$

   (b) $T_1^* = 4{,}472/50{,}000 = .089$ year
   (c) $T^* = 4{,}472/10{,}000 = .4472$ year

   (d) $TC(4{,}472) = \left(\dfrac{10{,}000}{4{,}472}\right)2{,}000 + .25(10)\left(\dfrac{4{,}472}{2}\right)\left(\dfrac{50{,}000-10{,}000}{}50{,}000\right)$
   $= \$4{,}472 + \$4{,}472 = \$8{,}944$

7. $Q^* = \sqrt{\dfrac{2(45{,}000)20}{.10(200)}} = 300$

$$TC(300) = \left(\frac{45{,}000}{300}\right)20 + .10(200)\left(\frac{300}{2}\right) = \$3{,}000 + \$3{,}000 = \$6{,}000$$

8. (a) $Q^* = \sqrt{\dfrac{2(1{,}000)25}{.25(500)}}\sqrt{\dfrac{5+125}{5}} = 101.98$

(b) $S^* = \sqrt{\dfrac{2(1{,}000)25}{.25(500)}}\sqrt{\dfrac{5}{5+125}} = 3.92$

(c) Q* – S* = 101.98 – 3.92 = 98.06

(d) $TC(101.98, 3.92) = \left(\dfrac{1{,}000}{101.98}\right)25 + \dfrac{.25(500)(3.92)^2}{2(101.98)} + \dfrac{5(101.98-3.92)^2}{2(101.98)}$

=\$245.15 + \$9.43 + \$235.72 = \$490.29

9. (a) $Q^* = \sqrt{\dfrac{2(100{,}000)10{,}000}{.10(100)}}\sqrt{\dfrac{500{,}000}{500{,}000-100{,}000}} = 44{,}721.36(1.118) = 50{,}000$

(b) T_1^* = 50,000/500,000 = .1 year

(c) T^* = 50,000/100,000 = .5 year

(d) $TC(50{,}000) = \left(\dfrac{100{,}000}{50{,}000}\right)10{,}000 + .10(10)\left(\dfrac{50{,}000}{2}\right)\left(\dfrac{500{,}000-100{,}000}{500{,}000}\right)$

= \$20,000 + \$20,000 = \$40,000

10. (a) $Q^* = \sqrt{\dfrac{2(4{,}800)10}{.20(12)}} = 200$

$$TC(200) = \left(\frac{4{,}800}{200}\right)10 + .20(12)\left(\frac{200}{2}\right) = \$240 + \$240 = \$480$$

(b) $Q^* = \sqrt{\dfrac{2(4{,}800)100}{.20(10)}}\sqrt{\dfrac{7{,}200}{7{,}200-4{,}800}} = 692.82(1.732) = 1{,}200$

$TC(1{,}200) = \left(\dfrac{4{,}800}{1{,}200}\right)100 + .20(10)\left(\dfrac{1{,}200}{2}\right)\left(\dfrac{7{,}200-4{,}800}{7{,}200}\right)$

= \$400 + \$400 = \$800

(a) Purchase ~~in-house~~ because the total annual relevant cost is less.

Lecture 8: Stochastic Inventory:

16-3 (a)

Demand d	Probability Pr[D=d]	Cumulative Probability Pr[D ≤ d]
100	.15	.15
150	.20	.35
200	.30	.65
250	.20	.85
300	.15	1.00

(b) $c = \$2$ $h_E = \$.10$ $P_S = 0$ $P_R = \$3$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0+3-2}{(0+3-2)+(.1+2)} = .32$$

The smallest level for Q having a cumulative probability at least as large is Q* = 150.

16-4 (a)

Demand d	Pr[D = d]	Pr[D ≤ d]	d x Pr[D = d]
2,000	.05	.05	100
3,000	.20	.25	600
4,000	.25	.50	1,000
5,000	.30	.80	1,500
6,000	.20	1.00	1,200
			μ = 4,400

$c_v = h_E + c = 0.40$
$c_u = P_S + P_S - c = 0.55$

(b) $c = \$.50$ $p_S = .05$ $p_R = \$1.00$ $h_E = -\$.10$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{.05+1.00-.50}{(.05+1.00-.50)+(-.10+.50)} = .58$$

The smallest cumulative probability greater than or equal to the above occurs for a demand of 5,000 calendars. Thus, Q* = 5,000 calendars.

(c)

		(1) Shortage		(2) Surplus	
d	Pr[D = d]	d – Q*	(d – Q*)Pr[D = d]	Q* – d	(Q* – d)Pr[D = d)
2,000	.05	0	0	3,000	150
3,000	.20	0	0	2,000	400
4,000	.25	0	0	1,000	250
5,000	.30	0	0	0	0
6,000	.20	1,000	200	0	0
			B(Q) = 200 expected shortage		800 expected surplus

Not examinable

(3) $TEC(Q) = c\mu + (h_E + c)[Q - \mu + B(Q)] + (p_S + p_R - c)B(Q)$

$$TEC(5{,}000) = \$.50(4{,}400) + (-.10 + .50)(800) + (.05 + 1.00 - .50)(200)$$
$$= \$2{,}200 + 320 + 110 = \$2{,}630$$

Or simply:
$TEC(Q) = c\mu + [c_v \text{ x Expected Surplus}] + [c_u \text{ x Expected Shortage}]$
$= \$0.50(4{,}400) + (0.4 \text{ x } 800) + (0.55 \text{ x } 200) = \$2{,}630$

(d) Pr[shortage] = Pr[D > 5,000] = .20

16-5 (a) $c = \$.50$ $p_S = 0$ $p_R = \$.90$ $h_E = \$.10$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + .90 - .50}{(0 + .90 - .50) + (\ .10 + .50)} = .40$$

$\Pr[D \leq Q^*] = .40$ $z = -.25$
$Q^* = \mu + z\sigma = 1{,}000 - .25(200) = 950$ pumpkins

(b) (1) $B(Q) = \mu - Q + \sigma L[(\mu - Q)/\sigma]$

$$B(950) = 1{,}000 - 950 + 200L([1{,}000 - 950]/200)$$
$$= 50 + 200L(.25)$$
$$= 50 + 200(.2863)$$
$$= 107.26$$

Not examinable

(2) Expected surplus $= Q - \mu + B(Q)$
$= 950 - 1{,}000 + 107.26$
$= 57.26$

(3) $TEC(950) = \$.50(1{,}000) + (.10 + .50)(950 - 1{,}000 + 107.26) + (0 + .90 - .50)(107.26)$
$= \$500 + 34.36 + 42.90 = \$577.26$

(c) Pr[shortage] = Pr[D > 950]
=1 – .40 = .60

16-6 (a) $c = \$.25$ $p_S = 0$ $p_R = \$.50$ $h_E = -\$.05$ (negative, since revenue is received)

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + .50 - .25}{(0 + .50 - .25) + (-.05 + .25)} = .56$$

$Q^* = 62$ papers, since this quantity involves a cumulative demand probability of .60, the smallest one exceeding the above ratio.

(b)

Demand d	Probability	Demand × Prob. dPr[D = d]	Cumulative Probability	Holding Cost .20(62 – d)	Cost × Prob.	Shortage Cost .25(d – 62)	Cost × Prob.
51	.05	2.55	.05	2.2	.11	-	-
52	.05	2.60	.10	2.0	.10	-	-
53	.05	2.65	.15	1.8	.09	-	-
54	.05	2.70	.20	1.6	.08	-	-
55	.05	2.75	.25	1.4	.07	-	-
56	.05	2.80	.30	1.2	.06	-	-
57	.05	2.85	.35	1.0	.05	-	-
58	.05	2.90	.40	.8	.04	-	-
59	.05	2.95	.45	.6	.03	-	-
60	.05	3.00	.50	.4	.02	-	-
61	.05	3.05	.55	.2	.01	-	-
62	.05	3.10	.60	0.0	.00	-	-
63	.05	3.15	.65	-	-	.25	.0125
64	.05	3.20	.70	-	-	.50	.0250
65	.05	3.25	.75	-	-	.75	.0375
66	.05	3.30	.80	-	-	1.00	.0500
67	.05	3.35	.85	-	-	1.25	.0625
68	.05	3.40	.90	-	-	1.50	.0750
69	.05	3.45	.95	-	-	1.75	.0875
70	.05	3.50	1.00	-	-	2.00	.1000
		μ = 60.50		66			\$.4500

$c\mu$ = \$.25(60.5) = \$15.125

TEC(62) = \$15.125 + .66 + .4500 = \$16.235

Not examinable

(c) This newsvendor's maximum expected daily profit is achieved by stocking Q* = 62 Berkeley Barbs This may be obtained in the same manner as with the Fortune problem. Or, more directly, we may first multiply expected demand of 60.5 copies (the median value may be used to quickly find this, since the demand distribution is symmetrical) by the revenue per copy:

\$.50(60.5) = \$30.25

Then subtracting TEC(62) from the above, we have

Maximum expected profit = \$30.25 – 16.235

= \$14.015

16-7 (a)

Demand d	Probability Pr[D = d]	Cumulative Probability Pr[D ≤ d]	d × Pr[D = d]
5	.10	.10	.50
10	.15	.25	1.50
15	.30	.55	4.50
20	.20	.75	4.00
25	.15	.90	3.75
30	.10	1.00	3.00
			μ = 17.25

Not examinable

(b) $c = .50 \qquad p_S = 0 \qquad p_R = 1.00 \qquad h_E = 0$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + 1.00 - .50}{(0 + 1.00 - .50) + (0 + .50)} = .50$$

The smallest level for demand having a cumulative probability at least as large is for 15 dozen donuts, so Q* = 15.

(c)

		(1) Shortage		(2) Surplus	
d	Pr[D = d]	d – Q*	(d – Q*)Pr[D = d]	Q* – d	(Q* – d)Pr(D = d]
5	.10	0	0	10	1.00
10	.15	0	0	5	.75
15	.30	0	0	0	0
20	.20	5	1.00	0	0
25	.15	10	1.50	0	0
30	.10	15	1.50	0	0
			B(Q*) = 4.00 Expected Shortage		1.75 Expected Surplus

Not examinable

(3) TEC(15) = \$.50(17.25) + (0 + .50)[1.75] + (0 + 1.00 -.50)(4.00)
=\$8.625 + .875 + 2.00 = \$11.50

(d) Pr[shortage] = Pr[D > 15] = .20 + .15 + .10 = .45

16-8 (a) $c = \$2 \qquad p_S = 0 \qquad p_R = \$4 \quad h_E = -\$1.50$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + 4 - 2}{(0 + 4 - 2) + (-1.50 + 2)} = .80$$

Q* = 500 kg

(b)

			(1) Shortage		(2) Surplus	
d	Pr[D = d]	d × Pr[D = d]	d – Q*	(d – Q*)Pr[D = d]	Q* – d	(Q* – d)Pr(D = d]
100	.05	5.00	0	0	400	20.00
200	.12	24.00	0	0	300	36.00
300	.18	54.00	0	0	200	36.00
400	.25	100.00	0	0	100	25.00
500	.22	110.00	0	0	0	0
600	.09	54.00	100	9.00	0	0
700	.09	63.00	200	18.00	0	0
		410.00		B(Q*) = 27.00 Expected Shortage		117.00 Expected Surplus

Not examinable

(3) TEC(500) = \$2(410) + (–1.50 + 2)(117) + (0 + 4 – 2)(7)

= \$820 + 58.50 + 54 = \$932.50

16-9 (a) $c = \$5$ $\mu = 5{,}000$
$p_S = \$20$ $\sigma = 1{,}000$
$p_R = \$15$
$h_E = -\$.50$ (negative, since revenue is received)

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{20 + 15 - 5}{(20 + 15 - 5) + (-.50 + 5)} = .8696$$

Area = .8696 – .5000 = .3696
$z = 1.12$
$Q^* = \mu + z\sigma = 5{,}000 + 1.12(1{,}000)$
$= 6{,}120$ trees

Not examinable

Professor Dull should order 6,120 trees.

(b) (1)

$$B(6{,}120) = 1{,}000L\left(\frac{6{,}120 - 5{,}000}{1{,}000}\right) = 1{,}000L(1.12)$$
$$= 1{,}000(.06595) = 65.95 \text{ or } 66$$

(2) Expected surplus = 6,120 – 5,000 + 65.95 = 1,185.95 or 1,186

(3) TEC(6,120) = \$5(5,000) + (–.50 + 5)(6,120 – 5,000 + 65.95) + (20 + 15 – 5)(65.95)
= \$25,000 + 5,336.78 + 1,978.50 = \$32,315.28