FIT3158 Business Decision Modelling

Lecture 6

Network Modelling (Part 2)

Topics Covered:





Solving transportation problems with MODI (closed loop) Method

Modelling Assignment Problems



Introduction

Please recall ...

- A <u>network model</u> is one which can be represented by a set of nodes, a set of arcs, and functions (e.g., costs, supplies, demands, etc.) associated with the arcs (also called edges) and/or nodes (also called vertices).
- Each of these models can be formulated as a linear programming problem and solved by general purpose linear programming (LP) codes.

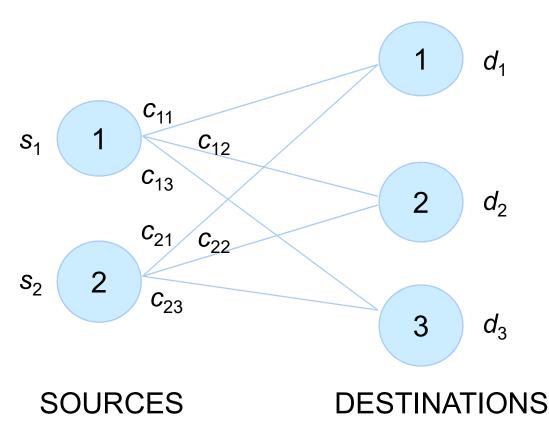
Introduction

- One of the most important applications of quantitative analysis in solving business problems is the physical distribution of products.
- Great cost savings can be achieved by more efficient routing, distribution and scheduling of goods and services from one node (<u>source</u>, where the supply is) to the required destination (<u>sink</u>, where the demand is).
- The <u>transportation problem</u> seeks to minimize the total shipping costs of transporting goods from *m* origins (each with a supply s_i) to *n* destinations (each with a demand d_j), when the unit shipping cost from an origin, *i*, to a destination, *j*, is c_{ii}.



Transportation Problem

 The following is a <u>network representation</u> of a transportation problem with two sources and three destinations



Transportation Problem - LP Formulation

The linear programming formulation in terms of the amounts shipped from the origins to the destinations, x_{ij} , can be written as:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \le s_i \quad for \ i = 1, 2, ..., m \quad \text{Supply}$$

$$\sum_{i=1}^{m} x_{ij} = d_j \quad for \ i = 1, 2, ..., n \quad \text{Demand}$$

$$x_{ij} \ge 0 \quad \text{for all } i \text{ and } j$$

Transportation Problem

LP Formulation Special Cases

Total supply exceeds total demand:

No modification of LP formulation is necessary.

Total demand exceeds total supply:

Add a dummy origin with supply equal to the shortage amount. Assign a zero shipping cost per unit. The amount "shipped" from the dummy origin (in the solution) will not actually be shipped.

LP Formulation Special Cases

The following special-case modifications to the linear programming formulation can be made:

❖ Minimum shipping guarantee from *i* to *j*:

$$x_{ij} \geq L_{ij}$$

❖ Maximum route capacity from *i* to *j*:

$$X_{ij} \leq L_{ij}$$

Unacceptable route:

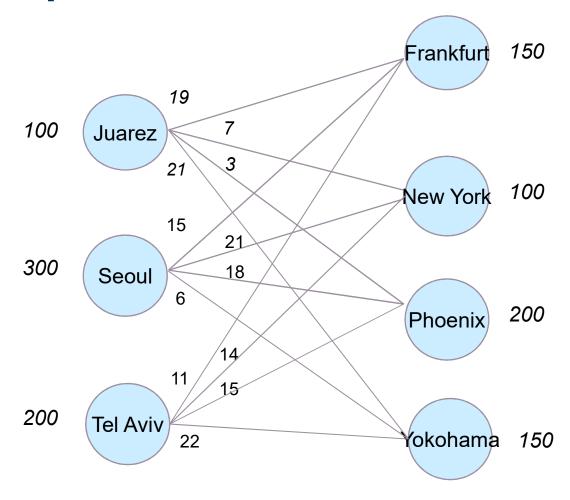
Remove the corresponding decision variable.

Example: Ski Shipment Scheduling

From		То	Warehou	se	
Plant	Frankfurt	New York	Phoenix	Yokohama	Capacity
Juarez	19	7	3	21	100
Seoul	15	21	18	6	300
Tel Aviv	11	14	15	22	200
Demand	150	100	200	150	



Ski Shipment – Network Model



Plant (Source)

Warehouse (Sink)



Implementing the Model & Solution

See file <u>Transportation.xlsm</u> (Method 1)

I amount						\			
	4	А	В	С	D	E	H		
	1		Ski Ship	oment-Sc	hedulin	g Illustratio	on		
2	2					=SUMPROD	UCT(B5:E7,B	12:E14)	
3	3	From	Marie Marie	To War	rehouse L		position in the		
4	4	Plant	Frankfurt	New York	Phoenix	Yokohama	Capacity		
E	5	Juarez	19	7	3	21	100		
(6	Seoul	15	21	18	6	300		
7	7	Tel Aviv	11	14	15	22	200		
8	8	Demand	150	100	200	150	C(min)		
SUM(B12	2:B	314)		Solution			\$6,250		
	0	From		To War	rehouse	=SUM(B			
1	1	Plant	Frankfurt	New York	Phoenix	Yokohama	Tota	I(DIZ.E	
1	2	Juarez	0	0	100	0	100		
1	3	Seoul	50	0	100	150	300		
1	4	Tel Aviv	100	100	0	0	200	V	
		Total	150	100	200	150	- 501	ution	
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Implementing the Model & the Solution

See file <u>Transportation.xlsm</u> (Method 2)

Ship	From	То	Unit Cost
0	1 Juarez	4 Frankfurt	\$19
0	1 Juarez	5 New York	\$7
100	1 Juarez	6 Phoenix	\$3
0	1 Juarez	7 Yokohama	\$21
50	2 Seoul	4 Frankfurt	\$15
0	2 Seoul	5 New York	\$21
100	2 Seoul	6 Phoenix	\$18
150	2 Seoul	7 Yokohama	\$6
100	3 Tel Aviv	4 Frankfurt	\$11
100	3 Tel Aviv	5 New York	\$14
0	3 Tel Aviv	6 Phoenix	\$15
0	3 Tel Aviv	7 Yokohama	\$22





Transportation Models

We will now look at some of the techniques used to solve transportation problems.

Northwest Corner Method

- Vogel's Approximation Method (VAM)
- MODI (The Closed-Loop Method)

Northwest Corner Method

Algorithm:

- Start in the top left hand (or Northwest) corner.
- Allocate the maximum supply possible to demand.
- Adjust the row and column entries.
- If demand is met, move to next column.
- If supply is exhausted, move to next row.
- Move from top left → bottom right



Northwest Corner Method – Example

Source		Destination									
Ocurce	D1	D2	D3	D4	Capacity						
S1	19 100	7	3	21	100						
S2	15 50	21 100	18 150	6	250 150 300						
S 3	11	14	15 50	22 (150)	150 200						
Demand	50	100	50 200	150	600						



Northwest Corner Method – Solution

To supply:	Quantity:	Unit Cost:	Total Cost:
S1 → D1	100	19	1900
S2 → D1	50	15	750
S2 → D2	100	21	2100
S2 → D3	150	18	2700
S3 → D3	50	15	750
S3 → D4	150	22	3300
			\$11,500

This is quite a 'simplistic' technique. It does not take the costs into consideration.

The solution generated is far from optimal – compare this with Slide 11 or 12 (where solution is far cheaper).



Vogel's Approximation Method (VAM)

This method was originally used for ammunition distribution.

The Basic Principle:

In choosing a route,

- Try to avoid high cost routes.
- Will be implicitly making decisions about alternative routes.
- Does not only consider direct costs but also the next best alternative.



Vogel's Approximation Method (VAM)

Algorithm:

- 1. Calculate the potential opportunity loss for rows. The opportunity loss is conservatively estimated as the difference between the lowest cost cell and the next lowest cost cell.
- 2. Do the same thing for columns.
- 3. Locate the highest potential opportunity loss. Break ties arbitrarily.
- 4. Allocate the maximum supply possible to the minimum cost cell in the row or column located in (3).
- 5. Adjust rows and columns.
- 6. Iterate



VAM – Example

0		Destination									
Source	D1 D2		D3	D4	Capacity						
S1	19	7	3 100	21	100	7-3=4 0					
S2	15 50	21	18 100	6 150	150 -300	15-6=9 18-15=3					
S 3	11 100	14 100	15	22	_100 _ 20 0	14-11=3 15-11=4					
Demand	50 150	100	100 200	150	600						
	15-11=4	14-7=7 21-14=7 0	15-3=12 18-15=3	21-6=15 0							



Vogel's Approximation Method – Solution

To supply:	Quantity:	Unit Cost:	Total Cost:
S1 → D3	100	3	300
S2 → D1	50	15	750
S2 → D3	100	18	1800
S2 → D4	150	6	900
S3 → D1	100	11	1100
S3 → D2	100	14	1400
			\$6,250

Clearly, this gives a better solution than the Northwest Corner Method. In fact, this is the optimal solution (approx. Slide 11).

But, VAM does not always guarantee an optimal solution. MODI does!

MODI (Closed-Loop Method)

Also known as:

- Modified Distribution Method; or
- Modified Dantzig Iteration Algorithm

What we have noticed so far:

If there are 3 sources (N) and 4 destinations (M), the total number of allocations is 3 + 4 - 1 = 6

So, for non-degenerate solutions, there will always be: N + M - 1 allocations.



MODI (Closed-Loop Method)

Algorithm:

(Recall that the C_{ii} are the edge costs.)

- 1. Generate a basic feasible solution (e.g., using Northwest Corner or Vogel's Approximation Method [VAM]).
- Derive R_i + K_j = C_{ij} for any cell with a shipment
 (where R_i = Row Indicators, K_j = Column Indicators).
 By convention, we always set R₁ = 0
- 3. Calculate the C_{ij} $(R_i + K_j)$ values for cells with no shipment.

MODI (Closed-Loop Method)

- 4. Put a "+" sign in the most negative cell.
 - a) If there is more than 1 negative, choose the biggest reduction. Break ties arbitrarily.
 - b) If there are no cells with a negative $C_{ij} (R_i + K_j)$ values $\rightarrow STOP$ the solution is optimal.
- 5. Form a closed loop.
- 6. Determine maximum adjustment and modify solution.
- 7. Iterate

1. Start with a basic feasible solution – let's use the one from Northwest Corner Method (see above, approx. slide 15).

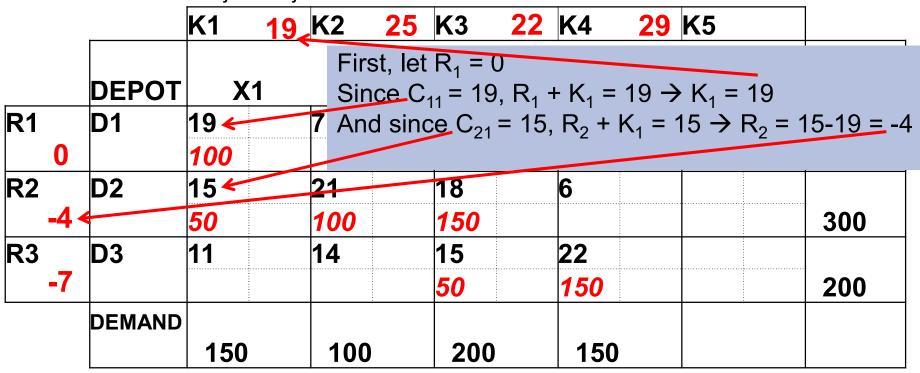
Source	Destination									
Source	D1	D2	D3	D4	Capacity					
S 1	19 100	7	3	21	100					
S 2	15 50	21 100	18 150	6	300					
S 3	11	14	15 50	22 150	200					
Demand MONASH Un	150	100	200	150	600					

2. Derive $R_i + K_j = C_{ij}$ for any cell with a shipment

		K 1	19	K2 2	5 K3	22 K4	29 K5	
				First,	let R₁ = C)		
	DEPOT		X1			$R_1 + K_1 =$	•	
R1	D1	19	4	7 And s	since C ₂₁	$= 15, R_2 +$	$K_1 = 15 - \frac{1}{2}$	$R_2 = 15-19 =$
0		100)					100
₹2	D2	15	<u> </u>	21	18	6		
-4	-	50		100	150			300
R 3	D3	11		14	15	22		
-7					50	150		200
	DEMAND							
		15	50	100	200	150		

MONASH University So,
$$K_2 = 21 - (-4) = 25$$
, $K_3 = 18 - (-4) = 22$

2. Derive $R_i + K_j = C_{ij}$ for any cell with a shipment



There are (M+N-1) allocations. And, setting $R_1 = 0$, there are (M+N)-1 values of R_i and K_j to find.

So, $K_1 = 19 - 0 = 19$, $R_2 = 15 - 19 = -4$, $K_2 = 21 - (-4) = 25$, $K_3 = 18 - (-4) = 22$, $R_3 = 15 - 22 = -7$, $K_4 = 22 - (-7) = 29$ (and this completes step 2). We now go to step 3 and calculate $C_{ij} - (R_i + K_j)$ values for cells with no shipment.

3. Calculate the $C_{ij} - (R_i + K_j)$ values for cells with no shipment.

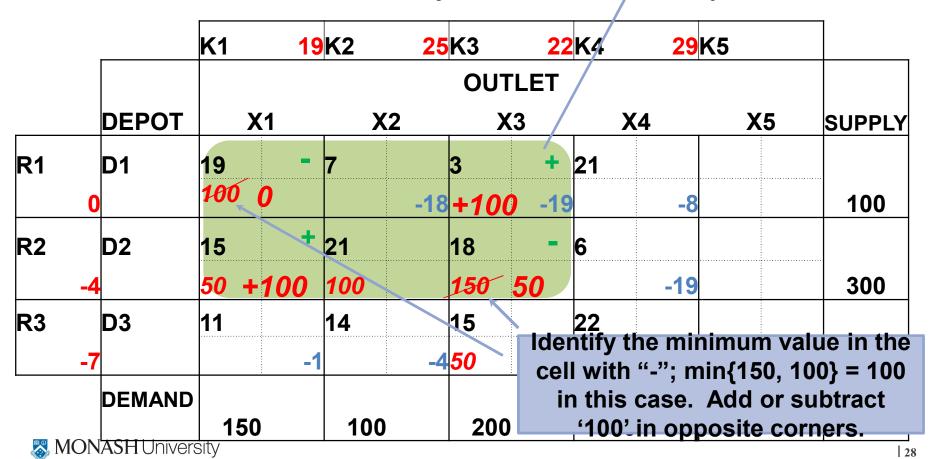
			K1	19	K2	25	K 3	22K4	29 K	(5	
			OUTLET								
		DEPOT	X1		X	2	X 3		X 4	X5	SUPPLY
R1		D1	19		7		3	21			
	0		100			-18	-	19	-8		100
R2		D2	15		21		E.a.: C	ell C ₁₂ &	$C_{42} - n$	o shipm	ent
	-4		50		100			$-(R_1 +$			
R3		D3	11		14		• -	•	·	•	22) = -19
	-7	•		-1		-4	<i>50</i>	150			200
		DEMAND									
			150		100		200	150	0		



- 4. Put a "+" sign in the most negative cell.
- 5. Form a closed loop.

Entering cell
Put a "+" sign here

6. Determine maximum adjustment and modify solution



After 2nd Iteration:

		K1	<mark>0</mark> K2	6K3	3K4	10K	5				
			OUTLET								
	DEPOT	X1	X2	X 3	X	(4	X5	SUPPLY			
R1	D1	19	7	3	21						
	0		19	1 100		11		100			
R2	D2	15	21	18	- 6	+					
•	15	150	100	<i>50</i> 0	+50	-19		300			
R3	D3	11	14	15	+ 22	-					
	12		-1	-4 <mark>50</mark> +	·50 15 0	100		200			
	DEMAND										
		150	100	200	150)					



After 3rd Iteration:

		K1	19	K2	25	K 3	3	K4	10K	5	
		OUTLET									
	DEPOT	X1		X	2	X3	3	>	(4	X5	SUPPLY
R1	D1	19		7		3		21			
	0		0		-18	100			11		100
R2	D2	15	4	21	-	18		6	+		
-4	4	150		100	0		19	50	+100		300
R3	D 3	11	,	14	+	15		22	_		
12	2		-20	+100	-23	100		100	0		200
	DEMAND										
	 ASH Univers	150		100		200		150			

After 4th Iteration:

		K1	-4	K2	2	K 3	3	K4	-13K	.5	
		OUTLET									
	DEPOT	×	(1	X2	2	X	3	X	4	X5	SUPPLY
R1	D1	19		7		3		21			
(23	3	5	100			34		100
R2	D2	15	-	21		18	+	6			
19		150	50	0		+100	-4	150			300
R3	D3	11	+	14		15	-	22			
12		+100	3	100		100	0		23		200
	DEMAND										
MON	ASH Universi	$_{\mathrm{by}}$ 150		100		200		150			31

After 5th Iteration:

There are no cells with a negative $C_{ij} - (R_i + K_j)$ values \rightarrow STOP – the solution is optimal.

		K1	<mark>0</mark> K2	3K3	<mark>3</mark> K4	-9K5	
		OUTLET					
	DEPOT	X1	X2	Х3	X4	X5	SUPPLY
R1 0	D1	19	7	3	21		
			19	4100		30	100
R2 15	D2	15	21	18	6		
		50		3100	150		300
R3 11	D3	11	14	15	22		
		100	100		1	20	200
	DEMAND						
MON	 ASH Univers	150	100	200	150		

MODI (Closed-Loop Method) – Solution

To supply:	Quantity:	Unit Cost:	Total Cost:
S1 → D3	100	3	300
S2 → D1	50	15	750
S2 → D3	100	18	1800
S2 → D4	150	6	900
S3 → D1	100	11	1100
S3 → D2	100	14	1400
			\$6,250

MODI always gives the optimal solution.



Some issues to take note of:

- 1. When supply does not equal demand
 - When demand < supply, add a dummy column for demand to make up for the difference.

	X1	X2	Dummy	Supply
D1	8	9	0	200
D2	12	7	0	200
Demand	200	100	100	

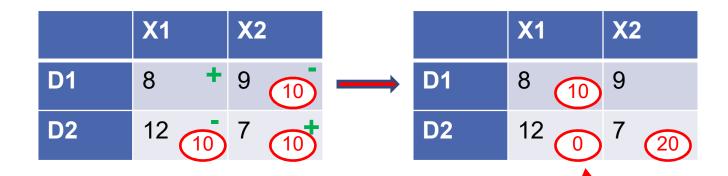
 Similarly, when demand > supply, add a dummy row for supply

In fact, the first step in doing any allocations is to check whether the demand and supply are equal.



Some issues to take note:

2. When there's a case of degeneracy E.g.:



We now ended up having 2 allocations, thus breaking the (M + N - 1) rule.

To handle such a situation, we put a zero (0) in one of the cells.

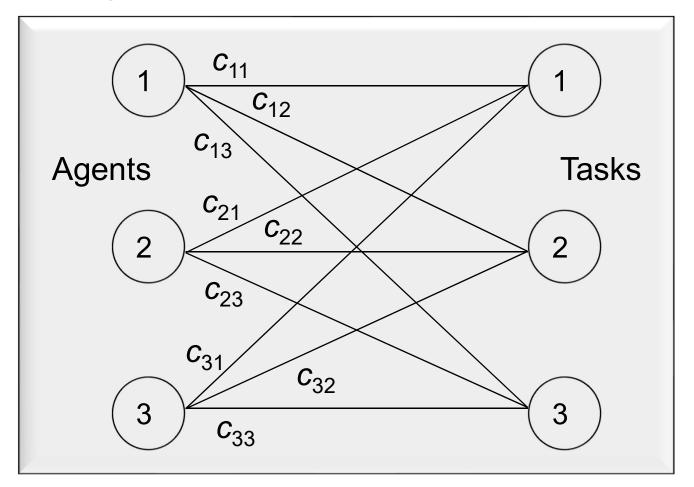


Assignment Problem

- An <u>assignment problem</u> seeks to minimize the total cost assignment of *m* workers to *m* jobs, given that the cost of worker *i* performing job *j* is *c*_{ij}.
- It assumes all workers are assigned and each job is performed.
- An assignment problem is a special case of a <u>transportation</u> <u>problem</u> in which all supplies and all demands are equal to 1; hence assignment problems may be solved as linear programs.
- The <u>network representation</u> of an assignment problem with three workers and three jobs is shown on the next slide.



Network Representation





Linear Programming Formulation

Using the notation:

$$x_{ij} = \begin{cases} 1 & \text{if agent } i \text{ is assigned to task } j \\ 0 & \text{otherwise} \end{cases}$$

 c_{ij} = cost of assigning agent i to task j

continued ---

Linear Programming Formulation (continued)

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad for \ j = 1, 2, ..., n \text{ Tasks}$$

$$\sum_{j=1}^{n} x_{ij} \le 1 \quad for \ i = 1, 2, ..., m \text{ Agents}$$

$$x_{ij} \ge 0 \text{ for all } i \text{ and } j$$

- LP Formulation Special Cases
 - Number of agents exceeds the number of tasks:

Extra agents simply remain unassigned.

• Number of tasks exceeds the number of agents:

Add enough dummy agents to equalize the number of agents and the number of tasks. The objective function coefficients for these new variables would be zero.

- LP Formulation Special Cases (continued)
 - The assignment alternatives are evaluated in terms of revenue or profit:

Solve as a maximization problem.

• An assignment is unacceptable:

Remove the corresponding decision variable.

• An agent is permitted to work t tasks:

$$\sum_{j=1}^{n} x_{ij} \le t \quad \text{for } i = 1, 2, ..., m \text{ Agents}$$

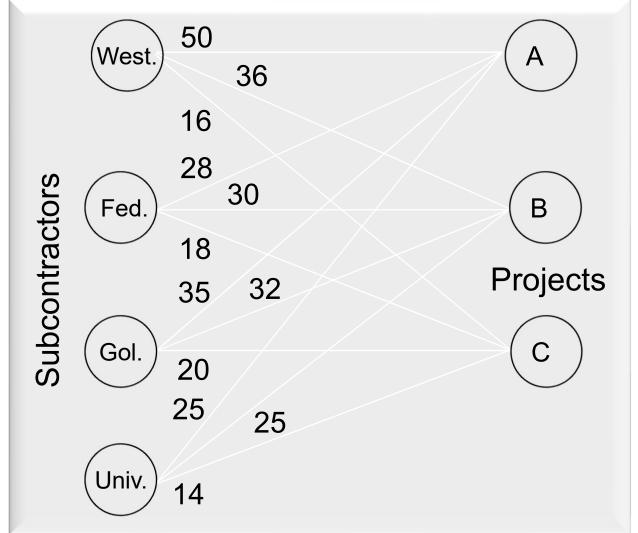
An electrical contractor pays her subcontractors a fixed fee plus mileage for work performed. On a given day the contractor is faced with three electrical jobs associated with various projects. Given below are the distances between the subcontractors and the projects.

	<u>Projects</u>		
Subcontractor	<u>A</u>	<u>B</u>	<u>C</u>
Westside	50	36	16
Federated	28	30	18
Goliath	35	32	20
Universal	25	25	14

How should the contractors be assigned so that total mileage is minimized?



NetworkRepresentation



Linear Programming Formulation

Min
$$50x_{11}+36x_{12}+16x_{13}+28x_{21}+30x_{22}+18x_{23}$$

 $+35x_{31}+32x_{32}+20x_{33}+25x_{41}+25x_{42}+14x_{43}$
s.t. $x_{11}+x_{12}+x_{13} \le 1$
 $x_{21}+x_{22}+x_{23} \le 1$
 $x_{31}+x_{32}+x_{33} \le 1$
 $x_{41}+x_{42}+x_{43} \le 1$
 $x_{11}+x_{21}+x_{31}+x_{41} = 1$
 $x_{12}+x_{22}+x_{32}+x_{42} = 1$
 $x_{13}+x_{23}+x_{33}+x_{43} = 1$
Tasks
 $x_{ij} = 0$ or 1 for all i and j

Implementation of Model & Solution:

See file <u>Assignment Problems.xlsx</u> (Contractor Assignment)

			Ma	trix of Indic	cators	
	Projects		A	В	C	(Sum <= 1)
		Westside	0	0	1	1
Matrix of	Subcontractors	Federated	1	0	0	1
Indicators		Goliath	0	0	0	0
		Universal	0	1	0	1
		(Sum = 1)	1	1	1	
	Projects		A	В	C	
Matrix of Weights	Subcontractors	Westside	50	36	16	
		Federated	28	30	18	
		Goliath	35	32	20	
		Universal	25	25	14	
	Projects		A	В	C	
Matrix of		Westside	0	0	16	
Matrix of Products	Subcontractors	Federated	28	0	0	
		Goliath	0	0	0	
		Universal	0	25	0	
~		Obje				
University						



The optimal assignment is:

Subcontractor	<u>Project</u>	<u>Distance</u>
Westside	С	16
Federated	Α	28
Goliath	(unas	ssigned)
Universal	В	<u>25</u>
Tot	al Distand	ce = 69 miles

Job Assignment based on maximising preferences

- We have 10 people and we wish to assign each person one job to do. Each person makes a list of 3 preferences and we assign the jobs accordingly.
- This problem is probably getting quite close to the size that you could do with the solver in practice and it provides a good example of how the solver works.
- By observing partial solutions, we can see that the solver initially relaxes the constraint that indicators be integers and gradually enforces this condition as a solution is approached.



Setup: Input Data

Input data, shown coded as a table of weights.

Pro	efere	ences		I	Prefere	ences (coded	10 = f	irst, 5	= seco	nd, 1	= thire
Person	P1	P2	P3	1	2	3	4	5	6	7	8	9
a	2	7	8	0	10	0	0	0	0	5	1	0
b	2	4	5	0	10	0	5	1	0	0	0	0
c	9	8	6	0	0	0	0	0	1	0	5	10
d	6	9	1	1	0	0	0	0	10	0	0	5
e	6	4	7	0	0	0	5	0	10	1	0	0
f	6	7	9	0	0	0	0	0	10	5	0	1
g	6	8	7	0	0	0	0	0	10	1	5	0
h	1	7	5	10	0	0	0	1	0	5	0	0
I	3	7	4	0	0	10	1	0	0	5	0	0
j	1	9	8	10	0	0	0	0	0	0	1	9

See file <u>Assignment Problems.xlsx</u> (Job Choices)



Indicator Matrix

Final settings showing indicator values and constraint values.

	1	2	3	4	5	6	7	8	9	10	Check
	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
Check	1	1	1	1	1	1	1	1	1	1	



The Travelling Salesperson Problem A salesperson wants to find the least costly route for visiting

A salesperson wants to find the least costly route for visiting clients in n different cities, visiting each city exactly once before returning home.

n	(n-1)!
3	2
5	24
9	40,320
13	479,001,600
17	20,922,789,888,000
20	121,645,100,408,832,000

Ragsdale, C.. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e), Cengage Learning Ragsdale (2021, 9e), chap. 8, sec. 8.14



Example: The Traveling Salesperson Problem

 Wolverine Manufacturing needs to determine the shortest distance for a drill bit to drill 9 holes in a fiberglass panel.

See file TSP.xlsm

Note: This is a Non-linear Programming (NLP) problem.

Ragsdale, C.. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) , Cengage Learning. Ragsdale (9e, 2021), chap. 8



End of Lecture 6

References:

Ragsdale, C. (2021). 9th edition, chapter 5,

Ragsdale, C. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) Cengage Learning: Chapter 8

Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 12

Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning: Chapter 10



Homework

- Go through today's lecture examples :
 - ✓ Familiarise yourself with the different algorithms used:
 - Northwest Corner Method
 - Vogel's Approximation Method
 - MODI (Closed-loop) Method
 - ✓ Understand how the spreadsheets are being modeled for Assignment problems and Transportation problems
- Readings for next Lecture:
- C. T. Ragsdale (9th edn), chapter 8, secs. 8.4 8.5, Economic Order Quantity (EOQ)
- Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 15 - Inventory Decisions under Certainty



Tutorial 5 this week:

Network Modelling:

- The Shortest Route Problem
- Maximal Flow Problem
- Minimal Spanning Tree Problem

