FIT3158 Business Decision Modelling
Tutorial 2 Solution
Modelling and Solving LP Problems in a Spreadsheet

Topic: **Linear Programming – Spreadsheet modelling**

Note:

1. **Remember the goals for good spreadsheet design: (Lecture 2 Slide 14)**
   - Communication
   - Reliability
   - Auditability
   - Modifiability
2. **Also, recall the guidelines for (good) spreadsheet design (Lecture 2 Slide 15 and 16)**

Exercise 1 (Ragsdale, Q3.14 8E/Ragsdale, Q3.13 9E) - Do part c - e

A furniture manufacturer produces two types of tables (country and contemporary) using three types of machines. The time required to produce the tables on each machine is given in the following table:

Machine	Country	Contemporary	Total machine time available per week
Router	1.5	2.0	1,000
Sander	3	4.5	2,000
Polisher	2.5	1.5	1,500

Country tables sell for $350 and contemporary tables sell for $450. Management has determined that at least 20% of the tables should be country and at least 30% should be contemporary. How many of each type of table should the company produce if it wants to maximize its revenue?

a) Formulate the LP model for this problem (– done last week).
b) Determine optimal solution using graphical method (– done last week).
c) Create a spreadsheet model for this problem and solve it using Solver.
d) What is the optimal solution? [Compare this with your answer in part (b)]
e) How will your spreadsheet model differ if there are 25 types of tables and 15 machine processes involved in manufacturing them?

Exercise 2: Virginia Tech, (Ragsdale, Q3.32 8E/Ragsdale Q3.31 9E) – Do part b, c & d

Virginia Tech operates its own power generating plant. The electricity generated by this plant supplies power to the university and to local businesses and residences in the Blacksburg area. The plant burns three types of coal, which produce steam that drives the turbines that generate the electricity. The Environmental Protection Agency (EPA) requires that for each ton of coal burned, the emissions from the local furnace smoke stacks contain no more than 2,500 parts per million (ppm) of sulphur and no more than 2.8 kilograms (kg) of coal dust. The following table summarizes the amount of sulphur, coal dust, and steam that result from burning a ton of each type of coal.

Coal	Sulphur (in ppm)	Coal Dust (in kg)	Pounds of Steam Produced
1	1,100	1.7	24,000
2	3,500	3.2	36,000
3	1,300	2.4	28,000

The three types of coal can be mixed and burned in any combination. The resulting emission of sulphur or coal dust and the pounds of steam produced by any mixture are given as the weighted average of the values shown in the table for each type of coal. For example, if the coals are mixed to produce a blend that consists of 35% of coal 1, 40% of coal 2, and 25% of coal 3, the sulphur emission (in ppm) resulting from burning one ton of this blend is:

$$0.35 \times 1{,}100 + 0.40 \times 3{,}500 + 0.25 \times 1{,}300 = 2{,}110$$

The manager of this facility wants to determine the blend of coal that will produce the maximum pounds of steam per ton without violating the EPA requirements.

a) Formulate an LP model for this problem (– done last week).
b) Create a spreadsheet model for this problem and solve it using Solver.
c) What is the optimal solution?
d) If the furnace can burn up to 30 tons of coal per hour, what is the maximum amount of steam that can be produced per hour?

Exercise 3: Winery (Ragsdale, Q3.37 8E/Ragsdale Q3.38 9E)

A winery has the following capacity to produce an exclusive dinner wine at either of its two vineyards at the indicated costs:

Vineyard	Capacity	Cost per Bottle
1	3,500 bottles	$23
2	3,100 bottles	$25

Four Italian restaurants around the country are interested in purchasing this wine. Because the wine is exclusive, they all want to buy as much as they need but will take whatever they can get. The maximum amounts required by the restaurants and the prices they are willing to pay are summarized in the following table.

Restaurant	Maximum Demand	Price
1	1,800 bottles	$69
2	2,300 bottles	$67
3	1,250 bottles	$70
4	1,740 bottles	$66

The costs of shipping a bottle from the vineyards to the restaurants are summarised in the following table.

	Restaurant			
Vineyard	1	2	3	4
1	$7	$8	$13	$9
2	$12	$6	$8	$7

The winery needs to determine the production and shipping plan that allows it to maximize its profits on this wine.

a) Formulate an LP model for this problem.
b) Implement a spreadsheet model for this problem and solve it using Solver.
c) What is the optimal solution?

Solution:

Exercise 1 (Ragsdale, Q3.14 8E/Ragsdale, Q3.13 9E)

a) X_1 = Number of country tables to produce
X_2 = Number of contemporary tables to produce

MAX $350 X_1 + 450 X_2$

ST $1.5 X_1 + 2 X_2 \leq 1{,}000$
$3 X_1 + 4.5 X_2 \leq 2{,}000$
$2.5 X_1 + 1.5 X_2 \leq 1{,}500$
$X_1/(X_1 + X_2) \geq 0.20$ (implement as $X_1 \geq 0.2^* (X_1 + X_2)$)
$X_2/(X_1 + X_2) \geq 0.30$ (implement as $X_2 \geq 0.3^* (X_1 + X_2)$)
$X_i \geq 0$

If you attempt to implement the ratio constraints in their original form, it will result in a division by zero error at the null solution and a message from Solver that the model is not linear. The algebraic equivalence of the alternate form of these constraints (given parenthetically above) should be noted.

b. See file: Tutorial 2 Ex1.xlsm
c. $X_1 = 405.80$, $X_2 = 173.91$, Maximum revenue = $220,290
d. It will just have more columns and rows.

Exercise 2 (Ragsdale, Q3.32 8E/Ragsdale Q3.31 9E)

a) P_i = proportion of coal *i* to include in the mix

MAX $24{,}000 P_1 + 36{,}000 P_2 + 28{,}000 P_3$
ST $1{,}100 P_1 + 3{,}500 P_2 + 1{,}300 P_3 \leq 2{,}500$
$1.7 P_1 + 3.2 P_2 + 2.4 P_3 \leq 2.8$
$P_1 + P_2 + P_3 = 1.0$
$P_i \geq 0$

b. See file: Tutorial 2 Ex2.xlsm
c. P1=0.058, P2=0.5507, P3=0.3913
Maximum steam production = 32,174 pounds per ton
d. 32,174 × 30 = 965,217 pounds of steam

Exercise 3 (Ragsdale, Q3.37 8E/Ragsdale Q3.38 9E)

37. a. X_{ij} = number of bottles produced at vineyard *i* sold to restaurant *j*

MAX $39X_{11} + 36X_{12} + 34X_{13} + 34X_{14} + 32X_{21} + 36X_{22} + 37X_{23} + 34X_{24}$
ST $X_{11} + X_{12} + X_{13} + X_{14} \leq 3{,}500$

$X_{21} + X_{22} + X_{23} + X_{24} \leq 3{,}100$
$X_{11} + X_{21} \leq 1800$
$X_{12} + X_{22} \leq 2300$
$X_{13} + X_{23} \leq 1250$
$X_{14} + X_{24} \leq 1750$
$X_{ij} \geq 0$

b. See file: Tutorial 2 Ex3.xlsm
c. X_{11} = 1,800, X_{12} = 450, X_{14} = 1,250, X_{22} = 1,850, X_{23} = 1,250, Maximum profit = $241,750
(Alternate optima exist.)