Lecture 3: Sensitivity Analysis

1. Given the following Solver output what range of values can the objective function coefficient for variable X1 assume without changing the optimal solution?

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Number to make: X1	9.49	0	5	1.54	1
C4	Number to make: X2	1.74	0	6	1.5	1.47

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D8	Used	42	0	48	1E+30	6
D9	Used	132	0.24	132	12	12
D10	Used	24	1.24	24	1.33	2

ANSWER: 4 - 6.54

2. What is the optimal objective function value if X1 is at its lower limit in the following Solver output?

Cell	Target Name	Value
E5	Unit profit: OBJ. FN. VALUE	58

Cell	Adjustable Name	Value
B4	Number to make: X1	6
C4	Number to make: X2	4

Lower Limit	Target Result
0	16
0	42

Upper Limit	Target Result
6	58
4	58

ANSWER: 16

3. Which of the constraints are binding at the optimal solution for the following problem and Solver output?

MAX: $7 X_1 + 4 X_2$

Subject to:

$2 X_1 + X_2 \leq 16$

$X_1 + X_2 \leq 10$

$2 X_1 + 5 X_2 \leq 40$

$X_1, X_2 \geq 0$

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Number to make: X1	6	0	7	1	3
C4	Number to make: X2	4	0	4	3	0.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D8	Used	16	3	16	4	2.67
D9	Used	10	1	10	1	2
D10	Used	32	0	40	1E+30	8

ANSWER:

$X_1 = 6$, $X_2 = 4$

$2 * 6 + 4 = 16$ binding

$6 + 4 = 10$ binding

$2 * 6 + 5 * 4 = 32$ non-binding

4. Consider the following linear programming model and Solver output. What is the optimal objective function value if the RHS of the first constraint increases to 18?

MAX: $7 X_1 + 4 X_2$

Subject to: $2 X_1 + X_2 \le 16$

$X_1 + X_2 \le 10$

$2 X_1 + 5 X_2 \le 40$

$X_1, X_2 \ge 0$

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Number to make: X1	6	0	7	1	3
C4	Number to make: X2	4	0	4	3	0.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D8	Used	16	3	16	4	2.67
D9	Used	10	1	10	1	2
D10	Used	32	0	40	1E+30	8

ANSWER: Shadow price of first constraint is 3 with an allowable increase of 4. A 2-unit increase in RHS value increases objective function by 6. New objective function value is $6 * 7 + 4 * 4 + 2 * 3 = 64$.

5. A farmer is planning his spring planting. He has 20 acres on which he can plant a combination of Corn, Pumpkins and Beans. He wants to maximize his profit but there is a limited demand for each crop. Each crop also requires fertilizer and irrigation water which are in short supply. The following table summarizes the data for the problem.

Crop	Profit per Acre ($)	Yield per Acre (lb)	Maximum Demand (lb)	Irrigation (acre ft)	Fertilizer (pounds/acre)
Corn	2,100	21,000	200,000	2	500
Pumpkin	900	10,000	180,000	3	400
Beans	1,050	3,500	80,000	1	300

Suppose the farmer can purchase more fertilizer for $2.50 per pound, should he purchase it and how much can he buy and still be sure of the value of the additional fertilizer? Base your response on the following Solver output.

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Acres of Corn	9.52	0	2100	1E+30	350
C4	Acres of Pumpkin	0	-500.01	899.99	500.01	1E+30
D4	Acres of Beans	10.79	0	1050	210	375.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E8	Corn demand Used	200000	0.017	200000	136000	152000
E9	Pumpkin demand Used	0	0	180000	1E+30	180000
E10	Bean demand Used	37777.78	0	80000	1E+30	42222.22
E11	Water Used	29.84	0	50	1E+30	20.15
E12	Fertilizer Used	8000	3.5	8000	3619.04	3238.09

ANSWER: Yes, because the cost of $2.50 is less than the shadow price of $3.50. The allowable increase is 3619.04 pounds.

6. Jones Furniture Company produces beds and desks for college students. The production process requires carpentry and varnishing. Each bed requires 6 hours of carpentry and 4 hour of varnishing. Each desk requires 4 hours of carpentry and 8 hours of varnishing. There are 36 hours of carpentry time and 40 hours of varnishing time available. Beds generate $30 of profit and desks generate $40 of profit. Demand for desks is limited so at most 8 will be produced.

   Suppose the company can purchase more varnishing time for $3.00, should it be purchased and how much can be bought before the value of the additional time is uncertain? Base your response on the following Solver output.

Let X_1 = Number of Beds to produce
X_2 = Number of Desks to produce

The LP model for the problem is

MAX: $30 X_1 + 40 X_2$

Subject to:
$6 X_1 + 4 X_2 \leq 36$ (carpentry)
$4 X_1 + 8 X_2 \leq 40$ (varnishing)
$X_2 \leq 8$ (demand for X_2)
$X_1, X_2 \geq 0$

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Number to make: Beds	4	0	30	30	10
C4	Number to make: Desks	3	0	40	20	20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D8	Carpentry Used	36	2.5	36	24	16
D9	Varnishing Used	40	3.75	40	26.67	16
D10	Desk demand Used	3	0	8	1E+30	5

ANSWER: Yes, because the cost of $3.00 is less than the shadow price of $3.75. The allowable increase is 26.67 hours

7. Solve this problem graphically. What is the optimal solution and what constraints are binding at the optimal solution?

MAX: $8 X_1 + 4 X_2$
Subject to: $5 X_1 + 5 X_2 \leq 20$
$6 X_1 + 2 X_2 \leq 18$
$X_1, X_2 \geq 0$

ANSWER:
Obj = 26
$X_1 = 2.5$
$X_2 = 1.5$
Both constraints are binding.