FIT3158 Business Decision Modelling
Tutorial 1 Solution
Linear Programming

Topics covered:

Part A: Linear Programming – Graphical Method
  a. A Maximisation Problem
  b. A Minimisation Problem

Part B: Linear Programming – Graphical Method this week (Spreadsheet modelling next week)

Part A:

Exercise 1: Redundant constraint (Q2.9 Ragsdale 8E/9E)

Consider the following LP problem:

MAX: $3X_1 + 2X_2$

Subject to: $3X_1 + 3X_2 \le 300$

$6X_1 + 3X_2 \le 480$

$3X_1 + 3X_2 \le 480$

$X_1 , X_2 \ge 0$

a) Sketch the feasible region for this model.
b) What is the optimal solution?
c) Identify any redundant constraints in this model.

Exercise 2: Quality Desk Company (Q2.23 Ragsdale 8E/Q2.24 Ragsdale 9E) – A Maximisation Problem

The Quality Desk Company makes two types of computer desks from laminated particle board. The Presidential model requires 30 square feet of particle board, 1 keyboard sliding mechanism, and 5 hours of labour to fabricate. It sells for $149. The Senator model requires 24 square feet of particle board, 1 keyboard sliding mechanism, and 3 hours of labour to fabricate. It sells for $135. In the coming week, the company can buy up to 15,000 square feet of particle board at $1.35 per square foot and up to 600 keyboard sliding mechanisms at a cost of $4.75 each. The company views manufacturing labour as fixed cost and has 3000 labour hours available in the coming week for the fabrication of these desks.

a) Formulate an LP model for this problem.
b) Sketch the feasible region for this problem.
c) Determine the optimal solution to this problem using the graphical method.

Exercise 3: Blacktop Refining (Q2.21 Ragsdale 8E/Q2.22 Ragsdale 9E) – A Minimisation Problem

Blacktop Refining extracts minerals from ore mined at two different sites in Montana. Each ton of ore type 1 contains 20% copper, 20% zinc and 15% magnesium. Each ton of ore type 2 contains 30% copper, 25% zinc and 10% magnesium. Ore type 1 costs $90 per ton and ore type 2 costs $120 per ton. Blacktop would like to buy sufficient ore to extract at least 8 tons of copper, 6 tons of zinc and 5 tons of magnesium in the least costly manner.

a) Formulate an LP model for this problem.
b) Sketch the feasible region for this problem.
c) Determine the optimal solution to this problem using the graphical method.

Additional Questions: Ragsdale 8E, Chapter 2 Questions 6, 7, 8, 11, 13, 20, 23.
or Ragsdale 9E, Chapter 2 Questions 6, ,7, 8, 11, 13, 21, 24.

Part B:

Exercise 1 (Ragsdale 8E, Q3.14/Ragsdale 9E, Q3.13) - Do part a and b this week

A furniture manufacturer produces two types of tables (country and contemporary) using three types of machines. The time required to produce the tables on each machine is given in the following table:

Machine	Country	Contemporary	Total machine time available per week
Router	1.5	2.0	1,000
Sander	3	4.5	2,000
Polisher	2.5	1.5	1,500

Country tables sell for $350 and contemporary tables sell for $450. Management has determined that at least 20% of the tables should be country and at least 30% should be contemporary. How many of each type of table should the company produce if it wants to maximize its revenue?

a) Formulate the LP model for this problem
b) Determine optimal solution using graphical method.
c) Create a spreadsheet model for this problem and solve it using Solver
d) What is the optimal solution? [Compare this with your answer in part (b)]
e) How will your spreadsheet model differ if there are 25 types of tables and 15 machine processes involved in manufacturing them?

Exercise 2: Virginia Tech, (Ragsdale 8E, Q3.32/Ragsdale 9E, Q3.31) – Do part a this week

Virginia Tech operates its own power generating plant. The electricity generated by this plant supplies power to the university and to local businesses and residences in the Blacksburg area. The plant burns three types of coal, which produce steam that drives the turbines that generate the electricity. The Environmental Protection Agency (EPA) requires that for each ton of coal burned, the emissions from the local furnace smoke stacks contain no more than 2,500 parts per million (ppm) of sulphur and no more than 2.8 kilograms (kg) of coal dust. The following table summarizes the amount of sulphur, coal dust, and steam that result from burning a ton of each type of coal.

Coal	Sulphur (in ppm)	Coal Dust (in kg)	Pounds of Steam Produced
1	1,100	1.7	24,000
2	3,500	3.2	36,000
3	1,300	2.4	28,000

The three types of coal can be mixed and burned in any combination. The resulting emission of sulphur or coal dust and the pounds of steam produced by any mixture are given as the weighted average of the values shown in the table for each type of coal. For example, if the coals are mixed to produce a blend that consists of 35% of coal 1, 40% of coal 2, and 25% of coal 3, the sulphur emission (in ppm) resulting from burning one ton of this blend is:

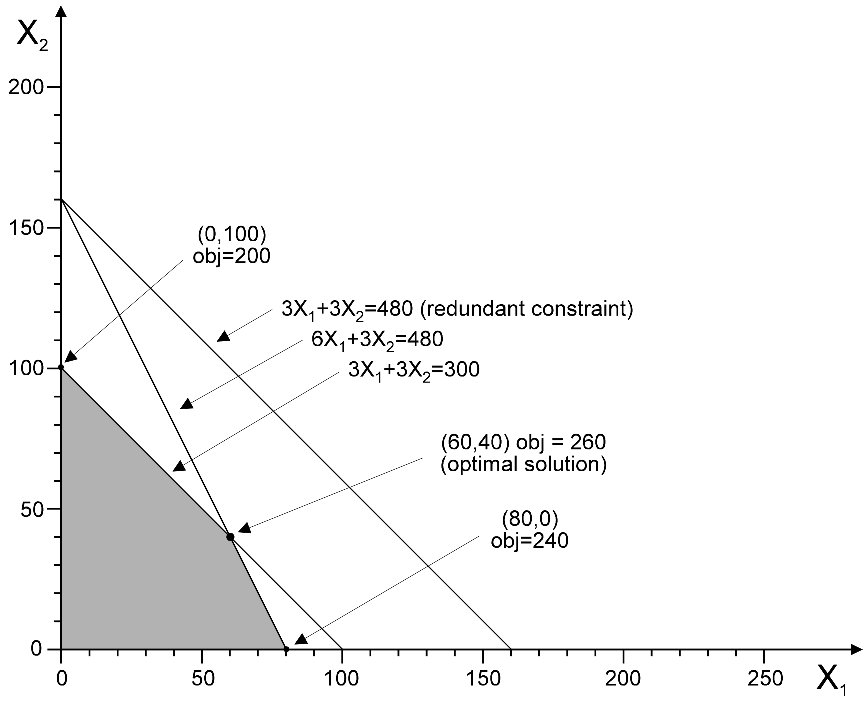
$$0.35 \times 1{,}100 + 0.40 \times 3{,}500 + 0.25 \times 1{,}300 = 2{,}110$$

The manager of this facility wants to determine the blend of coal that will produce the maximum pounds of steam per ton without violating the EPA requirements.

a) Formulate an LP model for this problem.
b) Create a spreadsheet model for this problem and solve it using Solver.
c) What is the optimal solution?
d) If the furnace can burn up to 30 tons of coal per hour, what is the maximum amount of steam that can be produced per hour?

Solutions: ***Part A***

Ex 1 : (Q2.9 Ragsdale 8E/9E)



Ex 2 (Q2.23 Ragsdale 8E/Q2.24 Ragsdale 9E)

P = number of Presidential desks produced, S = number of Senator desks produced

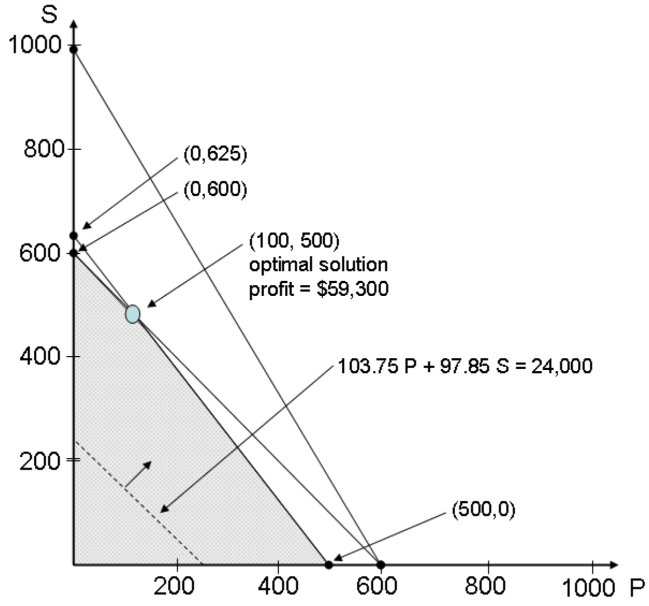
MAX 103.75 P + 97.85 S

ST 30 P + 24 S ≤ 15,000

1 P + 1 S ≤ 600

5 P + 3 S ≤ 3000

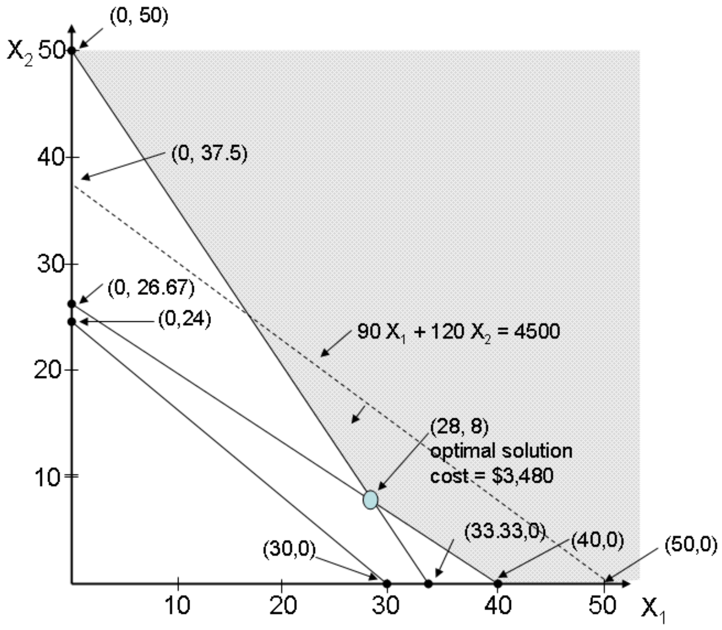
P, S ≥ 0



Ex 3 (Q2.21 Ragsdale 8E/Q2.22 Ragsdale 9E)

X_1 = tons of ore purchased from mine 1, X_2 = tons of ore purchased from mine 2

MIN	$90 X_1 + 120 X_2$	(cost)
ST	$0.2 X_1 + 0.3 X_2 \geq 8$	(copper)
	$0.2 X_1 + 0.25 X_2 \geq 6$	(zinc)
	$0.15 X_1 + 0.1 X_2 \geq 5$	(magnesium)
	$X_1, X_2 \geq 0$	



Part B

Exercise 1 (Ragsdale 8E, Q3.14/Ragsdale 9E, Q3.13)

(a) X_1 = Number of country tables to produce
X_2 = Number of contemporary tables to produce

MAX $350 X_1 + 450 X_2$

ST $1.5 X_1 + 2 X_2 \leq 1{,}000$

$3 X_1 + 4.5 X_2 \leq 2{,}000$

$2.5 X_1 + 1.5 X_2 \leq 1{,}500$

$X_1 / (X_1 + X_2) \geq 0.20$

$X_2 / (X_1 + X_2) \geq 0.30$

$X_i \geq 0$

Exercise 2 (Ragsdale 8E, Q3.32/Ragsdale 9E, Q3.31)

(a) P_i = proportion of coal i to include in the mix

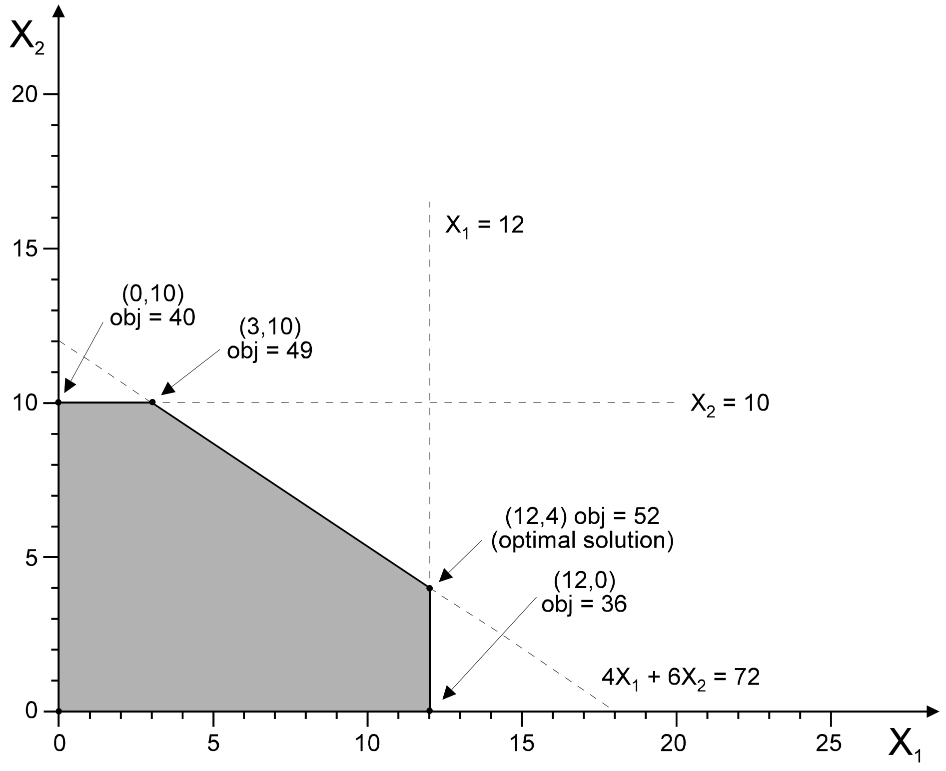
MAX $24{,}000 P_1 + 36{,}000 P_2 + 28{,}000 P_3$

ST $1{,}100 P_1 + 3{,}500 P_2 + 1{,}300 P_3 \leq 2{,}500$

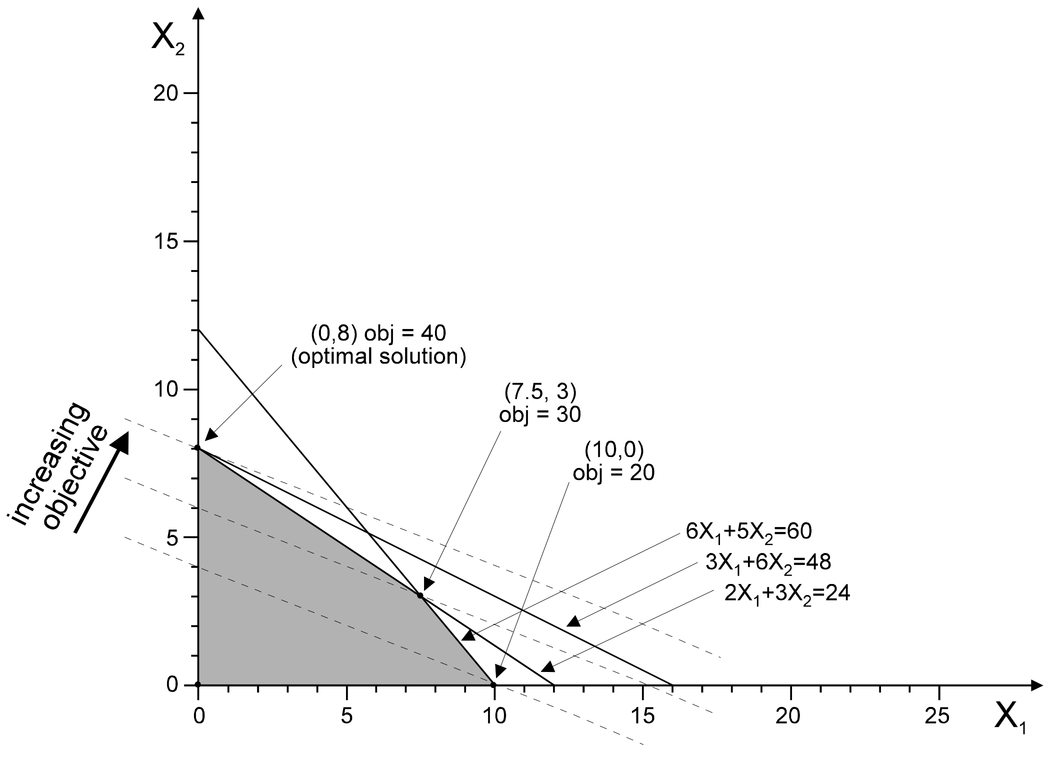
$$1.7\ P_1 + 3.2\ P_2 + 2.4\ P_3 \leq 2.8$$
$$P_1 + P_2 + P_3 = 1.0$$
$$P_i \geq 0$$

Additional Questions

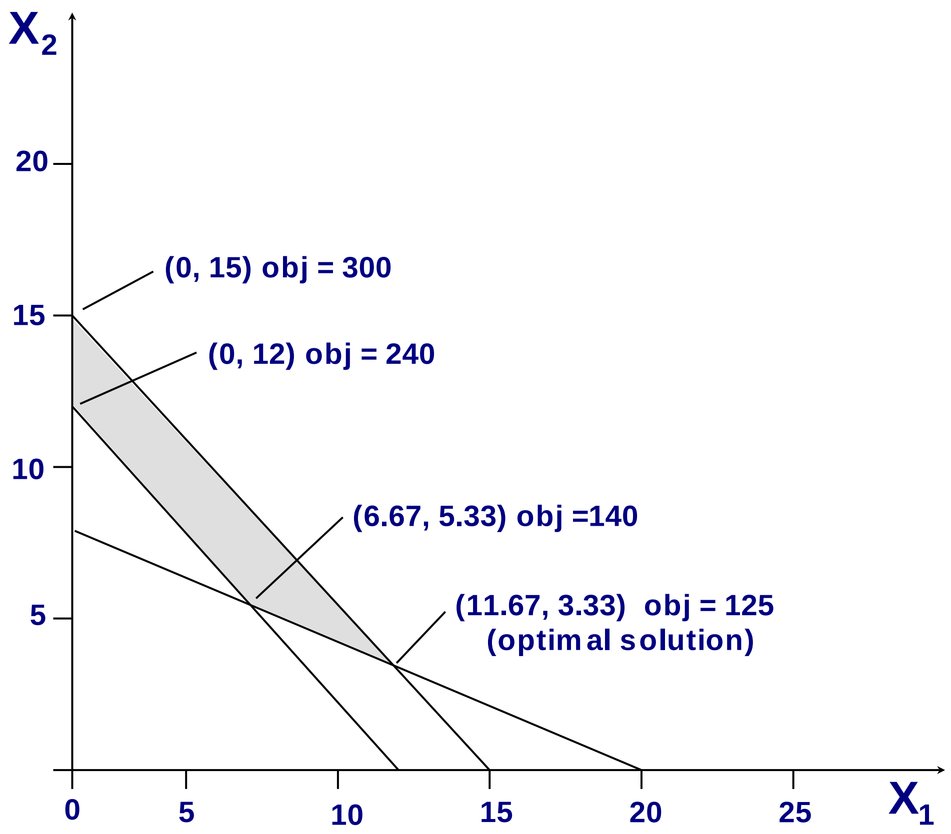
(Q2.6 Ragsdale 8E/9E).



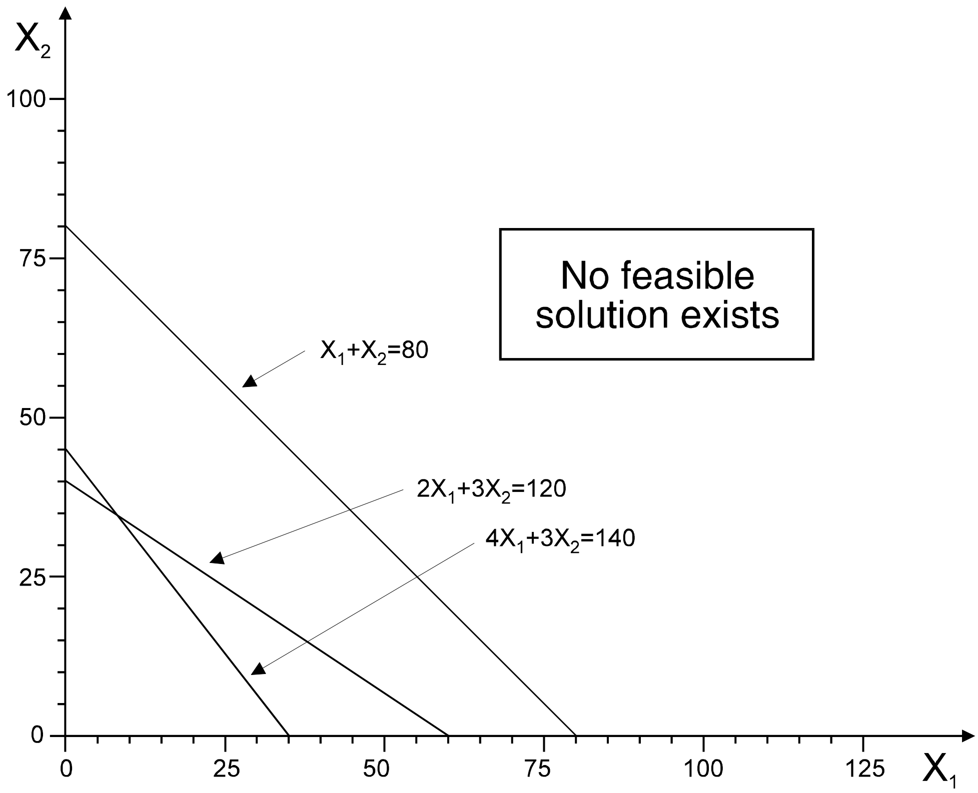
(Q2.7 Ragsdale 8E/9E).



(Q2.8 Ragsdale 8E/9E).



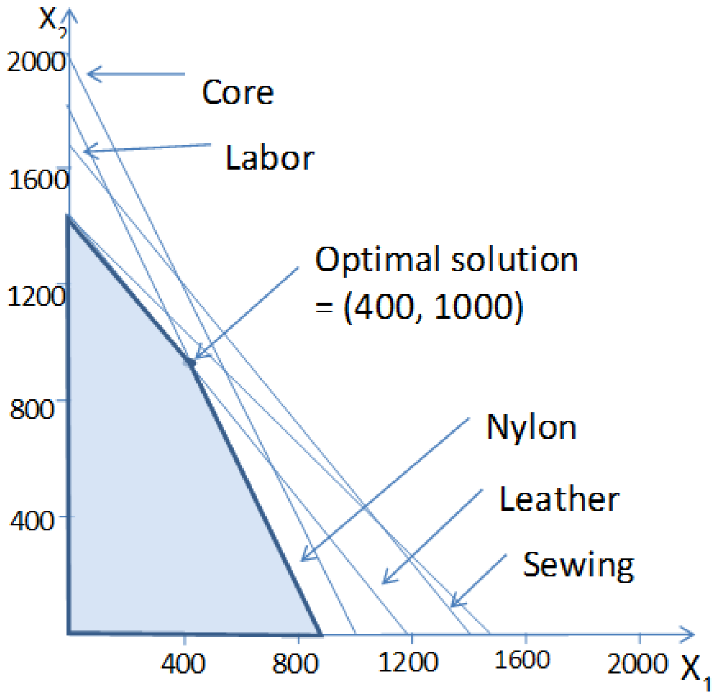
(Q2.11 Ragsdale 8E/9E)



(Q2.13 Ragsdale 8E/9E)

13. X_1 = number of softballs to produce, X_2 = number of baseballs to produce

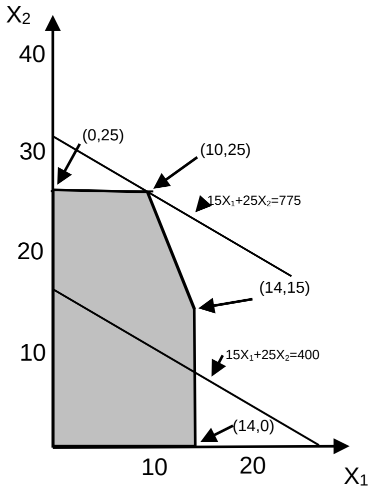
$$
\begin{array}{ll}
\text{MAX} & 6\ X_1 + 4.5\ X_2 \\
\text{ST} & 5X_1 + 4\ X_2 \le 6000 \\
 & 6\ X_1 + 3\ X_2 \le 5400 \\
 & 4\ X_1 + 2\ X_2 \le 4000 \\
 & 2.5\ X_1 + 2\ X_2 \le 3500 \\
 & 1\ X_1 + 1\ X_2 \le 1500 \\
 & X_1, X_2 \ge 0
\end{array}
$$



(Q2.20 Ragsdale 8E/Q2.21 Ragsdale 9E)

20. X_1 = # of TV spots, X_2 = # of magazine ads

$$
\begin{array}{lll}
\text{MAX} & 15\ X_1 + 25\ X_2 & \text{(profit)} \\
\text{ST} & 5\ X_1 + 2\ X_2 \le 100 & \text{(ad budget)} \\
 & 5\ X_1 + 0\ X_2 \le 70 & \text{(TV limit)} \\
 & 0\ X_1 + 2\ X_2 \le 50 & \text{(magazine limit)} \\
 & X_1, X_2 \ge 0 &
\end{array}
$$



X_1 = # of TV spots, X_2 = # of magazine ads

(Q2.23 Ragsdale 8E/Q2.24 Ragsdale 9E)

23. P = number of Presidential desks produced, S = number of Senator desks produced

MAX 103.75 P + 97.85 S
ST 30 P + 24 S ≤ 15,000
1 P + 1 S ≤ 600
5 P + 3 S ≤ 3000
P, S ≥ 0