# Lecture 5 & 6 Network Modeling
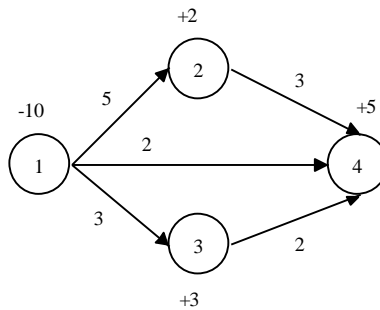
1. Draw the network representation of the following network flow problem.

MIN: $5 X_{12} + 3 X_{13} + 2 X_{14} + 3 X_{24} + 2 X_{34}$

Subject to: $-X_{12} - X_{13} - X_{14} = -10$

$X_{12} - X_{24} = 2$

$X_{13} - X_{34} = 3$

$X_{14} + X_{24} + X_{34} = 5$

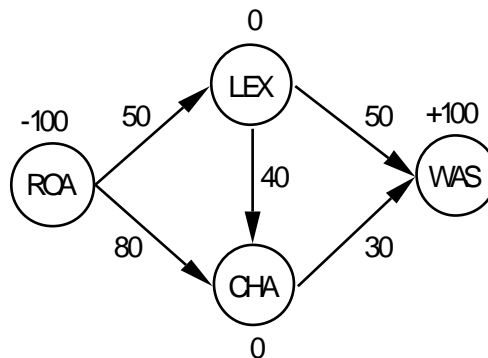$X_{ij} \geq 0$ for all i and j

ANSWER:



2. A company needs to ship 100 units from ROA to WAS at the lowest possible cost. The costs associated with shipping between the cities are:

|        |       | To  |     |
| ------ | ----- | --- | --- |
| From   | LEX   | WAS | CHA |
| ROA    | 50    | -   | 80  |
| LEX    | -     | 50  | 40  |
| CHA    | -     | 30  |     |

Draw the network representation of this problem.

ANSWER:



3. A company needs to ship 100 units from Sydney to Adelaide at the lowest possible cost. The costs associated with shipping between the cities are listed below. Also, the decision variable associated with each pair of cities is shown next to the cost.

| | | To | | |
|---|---|---|---|---|
| From | Melbourne | Darwin | Perth | Adelaide |
| Sydney | 100 ($X_{12}$) | 500 ($X_{13}$) | 600 ($X_{14}$) | - |
| Melbourne | - | 350 ($X_{23}$) | 300 ($X_{24}$) | - |
| Darwin | - | - | 250 ($X_{34}$) | 200 ($X_{35}$) |
| Perth | - | - | - | 200 ($X_{45}$) |

Write out the LP formulation for this problem.

ANSWER:

MIN: $100 X_{12} + 500 X_{13} + 600 X_{14} + 350 X_{23} + 300 X_{24} + 250 X_{34} + 200 X_{35} + 200 X_{45}$

Subject to:

$-X_{12} - X_{13} - X_{14} \geq -100$

$X_{12} - X_{23} - X_{24} = 0$

$X_{13} - X_{34} - X_{35} = 0$

$X_{14} + X_{24} + X_{34} - X_{45} = 0$

$X_{35} + X_{45} \geq 100$

$X_{ij} \geq 0$

4. A manufacturing company has a pool of 50 labor hours. A customer has requested two products, Product A and Product B, and has requested 15 and 20 of each respectively. It requires 2 hours of labor to produce Product A and 3 hours of labor to produce Product B. The company can obtain up to 50 additional hours of labor if required. In-house labor costs $25 per hour while contracted labor costs $45 per hour. Draw the network flow model that captures this problem.

ANSWER:
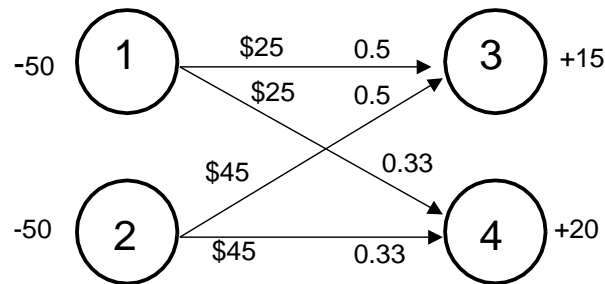


5. Write out the LP formulation for the problem in Question 4.

ANSWER:

MINIMIZE $25X_{13} + 25X_{14} + 45X_{23} + 45X_{24}$

Subject to:

$-X_{13} - X_{14} \geq -50$

$-X_{23} - X_{24} \geq -50$

$0.50X_{13} + 0.50X_{23} \geq 15$

$0.33X_{14} + 0.33X_{24} \geq 20$

$X_{ij} \geq 0$ for i=1,2; j=3,4

6. Carlton Distributing has three plants and four distribution centers. The plants, their supply, the distribution centers, their demands, and the distance between each location is summarized in the following table:

| Distance | Center 1 | Center 2 | Center 3 | Center 4 | Supply |
|---|---|---|---|---|---|
| Plant A | 45 | 60 | 53 | 75 | 500 |
| Plant B | 81 | 27 | 49 | 62 | 700 |
| Plant C | 55 | 40 | 35 | 60 | 650 |
| Demand | 350 | 325 | 400 | 375 | |

Draw the (balanced) transportation network for Carlton's distribution problem.

ANSWER:
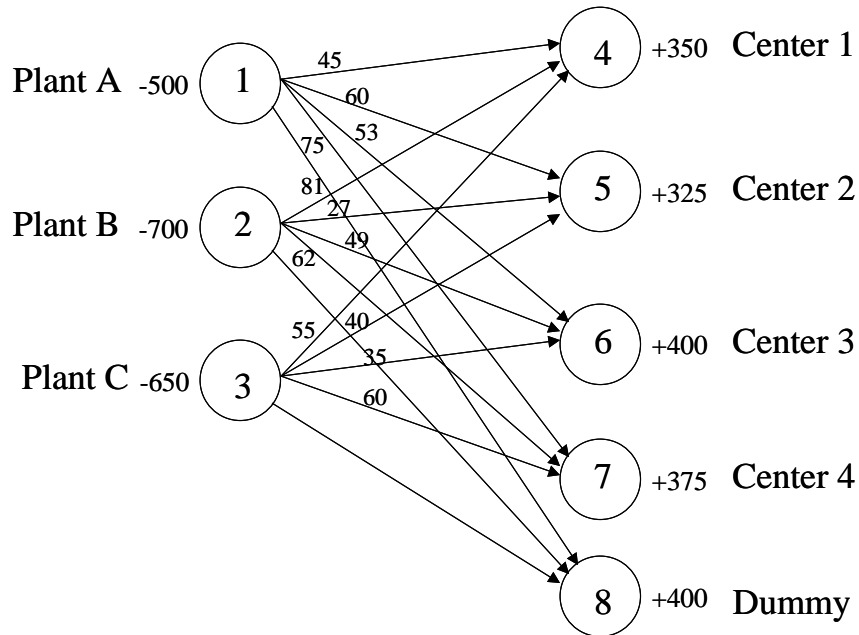


7. Formulate the LP for Carlton (in Question 6) assuming they wish to minimize the total product-miles incurred.

ANSWER:

Let $X_{ij}$ = flow from plant i (A, B, or C) to distribution center j (Center 1, 2, 3, or 4).

MIN: $45 X_{14} + 60X_{15} + 53X_{16} + 75X_{17} + 81X_{24} + 27X_{25} + 49X_{26} + 62X_{27} +$
$55X_{34} + 40X_{35} + 35X_{36} + 60X_{37}$

Subject to:
$-X_{14} - X_{15} - X_{16} - X_{17} - X_{18} = -500$
$-X_{24} - X_{25} - X_{26} - X_{27} - X_{28} = -700$
$-X_{34} - X_{35} - X_{36} - X_{37} - X_{38} = -650$
$X_{14} + X_{24} + X_{34} = 350$
$X_{15} + X_{25} + X_{35} = 325$
$X_{16} + X_{26} + X_{36} = 400$
$X_{17} + X_{27} + X_{37} = 375$
$X_{18} + X_{28} + X_{38} = 400$
All $X_{ij} \geq 0$

8. Draw the network representation of this LP model. What type of problem is it?

MAX          $X_{41}$

Subject to: $\quad X_{41} - X_{12} - X_{13} = 0$
$\quad\quad\quad\quad\quad X_{12} - X_{24} = 0$
$\quad\quad\quad\quad\quad X_{13} - X_{34} = 0$
$\quad\quad\quad\quad\quad X_{24} + X_{34} - X41 = 0$
$\quad\quad\quad\quad\quad 0 \le X_{12} \le 5,$
$\quad\quad\quad\quad\quad 0 \le X_{13} \le 4,$
$\quad\quad\quad\quad\quad 0 \le X_{24} \le 3,$
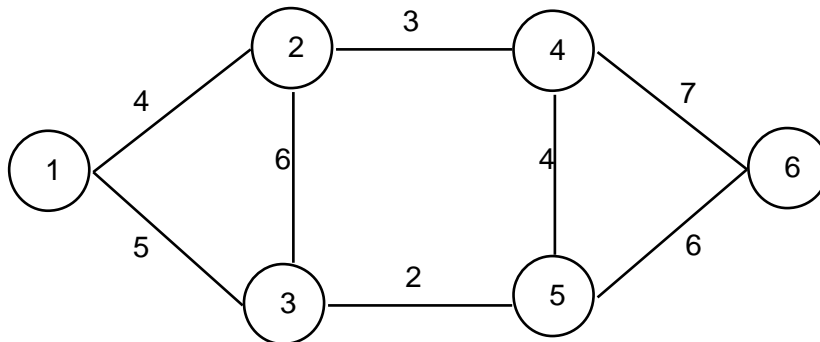$\quad\quad\quad\quad\quad 0 \le X_{34} \le 2,$
$\quad\quad\quad\quad\quad 0 \le X_{41} \le \infty$

ANSWER:
It is a maximal flow problem.
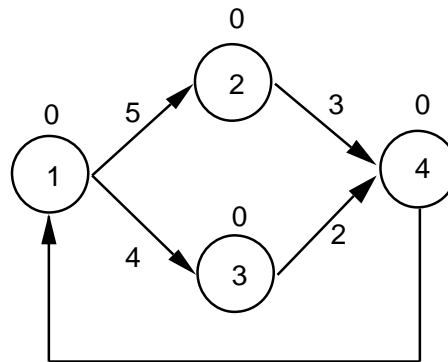


9. Solve the following minimal spanning tree problem starting at node 1.



ANSWER:

| Arc | Value |
|-----|-------|
| 1 - 2 | 4 |
| 2 - 4 | 3 |
| 3 - 5 | 2 |
| 4 - 5 | 4 |
| 5 - 6 | 6 |
| Total | 19 |

10. Consider the following distribution problem for Ace Widgets:

| Plants | Shipping Costs to Warehouses | | | | |
| --- | --- | --- | --- | --- | --- |
| | W1 | W2 | W3 | W4 | Capacity |
| P1 | 2 | 6 | 4 | 12 | 100 |
| P2 | 7 | 3 | 10 | 11 | 250 |
| P3 | 5 | 8 | 9 | 13 | 300 |
| Demand | 50 | 150 | 200 | 250 | |

a) Apply the North-west corner method to determine a starting solution. Compute the total cost.

b) Using the solution obtained in part (a), determine the new shipping schedule according to the closed-loop path (MODI method). Compute the total cost.

c) Draw a network diagram to depict Ace Widgets Shipping Distribution.

d) Formulate an LP formulation for Ace Widgets.

11. The following is a distribution schedule for Ace Widgets warehouses and customers in the Western region:

| Warehouse | Shipping Costs to Customers | | | |
| --- | --- | --- | --- | --- |
| | C1 | C2 | C3 | Capacity |
| W1 | 7 | 2 | 8 | 30 |
| W2 | 5 | 3 | 1 | 50 |
| W3 | 4 | 6 | 7 | 40 |
| Demand | 60 | 40 | 20 | |

e) Using the Vogel's Approximation Method (VAM) determine a starting solution for this problem. Compute the total cost.

f) Using the solution obtained in part (a), determine the new shipping schedule according to the closed-loop path (MODI method). Compute the total cost.

g) Draw a network diagram to depict Ace Widgets Distribution in the Western region.

h) Formulate an LP for this problem.