

Solutions to Lecture 10 Review Questions**Question 1:**

Honest Al's Used Cars wants to predict how many cars are sold each month. He has collected data for 12 months. He needs your help in analyzing this data using moving averages.

- Calculate the 4-month moving average forecast from Month 5 onwards.
- Calculate the 4-period centred-moving average.
- Compute the mean square error (MSE).

Solution:

(a)			(b)	
Time Period	Number of cars sold	4-Month Moving Average	4-Month Centred- Moving Average	
			Step 1	Step 2
1	70			
2	80		72.50	
3	66		71.00	71.75
4	74		70.00	70.50
5	64	72.50	71.50	70.75
6	76	71.00	73.50	72.50
7	72	70.00	78.00	75.75
8	82	71.50	78.00	78.00
9	82	73.50	81.00	79.50
10	76	78.00	80.50	80.75
11	84	78.00		
12	80	81.00		
13		80.50		
14				
(c)	MSE	40.59		20.25

Question 2:

Honest Al's Used Cars wants to predict how many cars are sold each month. He has collected data for 12 months. He needs your help in analyzing this data using exponential smoothing.

- a) Forecast the car sales using exponential smoothing (using $\alpha = 0.346$).
- b) Compute the mean square error (MSE).

Solution:

(a)

Time Period	Number of cars sold	Exponential Smoothing
1	70	70.00
2	80	70.00
3	66	73.46
4	74	70.88
5	64	71.96
6	76	69.21
7	72	71.56
8	82	71.71
9	82	75.27
10	76	77.60
11	84	77.05
12	80	79.45
13		79.64
14		79.64
15		79.64
16		79.64
(b)	MSE	43.41

$$\alpha = 0.346$$

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t)$$

where $0 \leq \alpha \leq 1$

As there is no actual value in period 13, we use the forecasted value of period 13 as the actual:
 $\text{Forecast}_{14} = 79.64 + 0.346 \cdot (79.64 - 79.64)$
 $= 79.64$
...and all future values will be the same

- (c) Assume the forecasted value for month 13 is 79.64. What is the forecasted value for month 16
- Answer: 79.64

Question 3:

3. A company has built a regression model to predict the number of labor hours (Y_i) required to process a batch of parts (X_i). It has developed the following Excel spreadsheet of the results.

	A	B	C	D	E	F	G
1	<i>Regression Statistics</i>						
2	Multiple R	0.9970					
3	R Square	0.9941					
4	Adjusted R Square	0.9933					
5	Standard Error	0.3679					
6	Observations	10					
7							
8	ANOVA						
9		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	
10	Regression	1	181.5971	181.5971	1341.5500	0.0000	
11	Residual	8	1.0829	0.1354			
12	Total	9	182.6800				
13							
14		<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
15	Intercept	4.8400	0.2513	19.2571	0.0000	4.2604	5.4196
16	X Variable 1	1.4836	0.0405	36.6272	0.0000	1.3902	1.5770

- f) What is the estimated regression function for this problem? Explain what the terms in your equation mean.

ANSWER: $\hat{Y} = 4.8400 + 1.4836 X_{1i}$. The average labor hours for a batch of parts increases by 1.4836 hours for each additional part. A batch of 0 parts will require 4.8400 hours which can be viewed as the setup time for the machine.

- g) Predict the mean number of labor hours for a batch of 5 parts.

ANSWER: $\hat{Y}_5 = 4.8400 + 1.4836 (5) = 12.258$

- h) Provide a rough 95% confidence interval on the number of labor hours for a bath of 5 parts.

ANSWER: $\hat{Y}_5 = 12.258$ and $S_e = 0.3679$, $CI = (11.5222, 12.9982)$

- i) Interpret the meaning of the "Lower 95%" and "Upper 95%" terms in cells F16:G16 of the spreadsheet.

ANSWER: We are 95% confident that $1.3902 \leq \beta_1 \leq 1.5770$.

- j) Interpret the meaning of R Square in cell B3 of the spreadsheet.

ANSWER: The R Square value of 0.9941 means that 99.41% of the variation in the Y values is explained by the X values.

Question 4:

4. The owner of Tim's Toys wants to predict monthly sales. He has collected data for 12 months. He needs your help in analyzing this data using exponential smoothing.
- a) Work out the exponential smoothing prediction in column C.
 - b) If the predicted value for Period 12 is 189.61, what is the forecast for time period 13?
 - c) Compute the Mean Absolute Percentage Error (MAPE) and comment on the accuracy of the prediction.

Solution for a) and c) :

	A	B	C	D	E	F
1	Time Period	Number of Toys Sold	Exponential Smoothing Prediction			
2				APE		
3	1	174	174.00		alpha	0.44
4	2	189	174.00	0.079		
5	3	168	180.60	0.075		
6	4	180	175.06	0.027		
7	5	165	177.23	0.074		
8	6	183	171.85	0.061		
9	7	177	176.76	0.001		
10	8	192	176.86	0.079		
11	9	192	183.52	0.044		
12	10	183	187.25	0.023		
13	11	195	185.38	0.049		
14	12	189	189.61	0.003		
15						
16		MAPE		0.047		

- c) As the MAPE is quite low (~about 4.7%), we can consider that the prediction is fairly accurate.
- b) What is the forecast for time period 13?

ANSWER:

Time Period	Number of Toys Sold	Exponential Smoothing Prediction
12	189	189.61
13		189.34

5. Maryland Confectionary wants to predict their quarterly sales for 2008 based on their sales records for the past three years, from 2005 to 2007. The sales data are tabulated as shown below:

Year	Qtr	Actual Sales
2005	1	55.2
	2	60
	3	86.4
	4	74.4
2006	1	62.4
	2	67.2
	3	115.2
	4	86.4
2007	1	74.4
	2	100.8
	3	127.2
	4	103.2

- a) Using the Multiplicative time series model, perform a forecast for 2008
Hint: Perform the following steps:
- Smooth the data using a centered 4 period moving average.
 - Calculate the seasonal indices.
 - Deseasonalise the data.
 - Fit a least squares regression line to the data.
 - Forecast sales for 2008.
- b) If it turned out that sales in 2008 were: 85.7, 104.5, 150.8 and 121.7, calculate the MAPE and MSE of the forecast.

Answer: Lecture 10 ReviewQ Answers.xls

6. The business analyst in Maryland Confectionary decided to fit a multiple regression to the data. Hence, the following output was generated:

SUMMARY OUTPUT	
<i>Regression Statistics</i>	
Multiple R	0.970885
R Square	0.942618
Adjusted R Square	0.76697
Standard Error	6.918299
Observations	12
<i>Coefficients</i>	
Intercept	43.75
Period	4.05
1	0
2	7.95
3	37.5
4	11.85

- a) Using the sales data in Question 5, as well as the regression output above, predict their quarterly sales for 2008 using the Additive Time Series Model.
- b) If it turned out that sales in 2008 were: 85.7, 104.5, 150.8 and 121.7, calculate the MAPE and MSE of the forecast.

Answer: Lecture 10 ReviewQ Answers.xls