FIT3158 Business Decision Modelling

Tutorial 8 Solution

Stochastic Inventory Modelling

Topics covered:

- **Single-period Inventory Decision - Newsvendor Problem**
- **Continuous Probability Distribution for Demand**
- **Optimal Inventory Policy with Backordering**

Solutions

1. $P_r = 1.95$

   $P_s = 0$ (assuming no loss of good will)

   $C = 0.35$

   $h_E = 0$ (no disposal or salvage cost)

   c_u = cost of underestimation = $P_s + P_r - c = 1.95 - 0.35 = 1.6$,

   c_v = cost of overestimation = $h_E + c = 0.35$

   $$P(\text{demand} < Q^*) = \frac{C_u}{C_u + C_v}$$

   P(demand < Q*) = 1.6/1.95 = .8205 → z = 0.92

   From the normal table, a cumulative probability of 0.8205 corresponds to z = 0.9

   Thus,

   $Q^* = \mu + 0.92\sigma = 800{,}000 + (0.92)(60{,}000) = 855{,}200$ magazines.

2. $P_r = 750$, $P_s = 0$ (assuming no loss of good will)

   $C = 410$

   $h_E = 340$ (salvage)

   $c_u = P_s + P_r - c = 750 - 410 = 340$

   $c_v = h_E + c = -340 + 410 = 70$

   $$P(\text{demand} < Q^*) = \frac{C_u}{C_u + C_v}$$

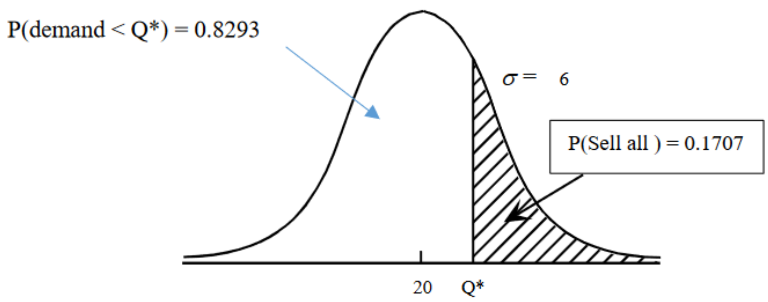
   P(demand < Q*) = 340/410 = .8293 → z = 0.95

   From the normal table, a cumulative probability of 0.8293 corresponds to z = 0.95

   Thus,

   a. $Q^* = \mu + 0.95\sigma = 20 + (0.95)(6) = 25.7$ so order 26.

   b. P(Sell all) = P(demand ≥ Q*) = 1 – 0.8293 = 0.1707



3. Formula:

- Reorder point: $r = \mu + z\sigma$
- Safety stock: $z\sigma$
- Average inventory: $1/2(Q) + z\sigma$
- Total annual cost: $[(1/2)Q^{*}ch] + [zs\ ch] + [Ak/Q^{*}]$

a. $Q^* = \sqrt{\frac{2Ak}{ch}} = \sqrt{\frac{2*(50*360)*42}{7.2*0.24}} = 935.4143$

b. DDLT is N(200,10)

Working: Daily: $\mu = 50$; $\sigma = 5$

4 days (lead time): $\mu = 50 * 4 = 200$;

$\sigma^2 = 25 * 4 \rightarrow \sigma = 10$

To have no more than 5% stock-outs: (this is equivalent to Z = 1.645)

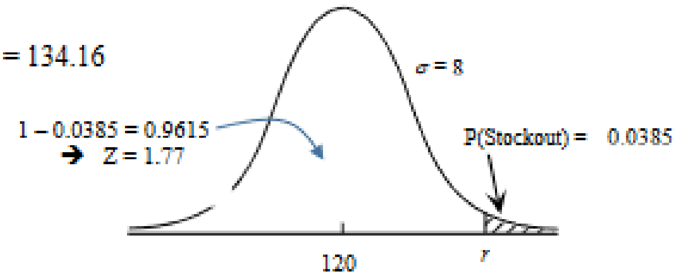
r = 200 + 1.645(10) = 216.45

c. Safety stock = 216.45 – 200 = 16.45

4. a. As the store uses a one-week periodic review system and will allow two stock-outs per year, probability of a stock out = 2/52 = 0.0385

b. DDLT is N(120,8)

M = 120 + 1.77 (8) = 134.16



c. As they have 42 in stock, quantity to order is: 134 - 42 = 92

5.

Given: Mean weekly demand, $\mu = 120$

Weekly standard deviation, $\sigma = 25$

Weekly variance, $\sigma^2 = 25 \times 25$

Total review period plus lead time = 5 weeks

Demand for 5 weeks is normally distributed with:

Mean demand over 5 weeks, $\mu = 5 \times 120 = 600$

Variance of demand over 5 weeks, $\sigma^2 = 5 \times 25 \times 25$

Standard deviation over 5 weeks, $\sigma = 56$

$M = 600 + 1.88(56) = 705$ doggy bags

Answer: As the store currently has 150 bags in stock, they should order:

705 - 150 = 555 bags

The safety stock is: $z\sigma = (1.88)(56) = 105$ bags