Formula Sheet

Expected Value of a Project

If a decision has a number of outcomes, i, each having a payoff x_i, with probability $p(x_i)$ then the expected value of the decision is given by $\sum_i x_i p(x_i)$

Bayes' Theorem

To find the posterior probability that event A_i will occur given that event B has occurred

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \ldots + P(A_n)P(B|A_n)}$$

Tabular form for calculations

States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
A1	$P(A_1)$	$P(B \mid A_1)$	$P(B \cap A_1)$	$P(A_1 \mid B)$
A2	$P(A_2)$	$P(B \mid A_2)$	$P(B \cap A_2)$	$P(A_2 \mid B)$
			$P(B)$	

Least Squares Regression

For bivariate data consisting of n pairs of observations (x, y), the Least Squares Line of Best Fit is $y = mx + c$, where $m = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$ and $c = \bar{y} - m\bar{x}$.

Simple exponential smoothing

$\hat{y}_{t+1} = \hat{y}_t + a(y_t - \hat{y}_t),$

where : $\hat{y}_t$: forecast value, y_t : observed value, α : smoothing factor, t : period (time) index

Mean Squared Error

$$MSE = \frac{\sum_{i=1}^{n}(Y_i - F_i)^2}{n}$$

Mean Absolute Percent Error

$$MAPE = \frac{\sum_{i=1}^{n}\frac{|F_i - Y_i|}{Y_i}}{n}$$

Y_i *are the actual (observed) values*

F_i *are the fitted (forecast) values*

n *is the number of forecast values*

Queuing, Probability and Simulation

Service and waiting times for a single server queue, Poisson arrivals, Exponential service:

$\lambda =$ the average number of arrivals per time period (arrival rate)

$\frac{1}{\lambda} =$ the average time between arrivals

$\mu =$ the average number of services per time period (service rate)

$\frac{1}{\mu} =$ the average time taken for each service

$P_0 = 1 - \frac{\lambda}{\mu}$ the probability that no units are in the system

$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$ the average number of units in the waiting line

$L = L_q + \frac{\lambda}{\mu}$ the average number of units in the system

$W_q = \frac{L_q}{\lambda}$ the average time a unit spends in the waiting line

$W = W_q + \frac{1}{\mu}$ the average time a unit spends in the system

$P_w = \frac{\lambda}{\mu}$ the probability that an arriving unit has to wait for service

$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ the probability of n units in the system

Probability distributions:

The Poisson distribution

$f(x) = \frac{\theta^x e^{-\theta}}{x!}$ for a distribution having mean $\theta, (e = 2.71828...)$

The exponential distribution

$f(x) = \frac{1}{\theta} e^{-x/\theta}$ for a distribution having mean $\theta, (e = 2.71828...)$

$P(x \le x_0) = 1 - e^{-x_0/\theta}$

$P(x \ge x_0) = e^{-x_0/\theta}$ for a given value of x_0

Linear congruential generation of uniform random variables

Let X_0 be an integer chosen at random (the random seed) then uniformly distributed integers are generated as $X_{n+1} = AX_n \bmod B$ where A and B are large co prime integers. Random numbers between 0 and 1 are calculated as $r_n = \frac{X_n - 1}{B - 2}$.

Generation of Exponentially distributed random variables

Exponential variates with mean b are generated from uniform [0,1] random numbers, r_n, by the transformation $t_n = -b \log_e(r_n)$.

Inventory Models: Deterministic Demand

$\text{Holding cost}(per\ item) = h \quad \text{Order cost} = k \quad \text{Item cost} = c \quad \text{Back order cost}(per\ item) = p$

$\text{Annual demand} = A \quad \text{Production Rate} = B$

Economic Order Quantity

$$\text{Optimal order quantity}: Q^* = \sqrt{\frac{2Ak}{ch}}$$

$$\text{Number of orders per year} = \frac{A}{Q^*}$$

$$\text{Time between orders (cycle time)} = \frac{Q^*}{A} \text{ years}$$

$$\text{Total annual cost} = \text{ordering cost} + \text{holding cost} = \frac{Ak}{Q} + \frac{Qch}{2}$$

Economic Production Quantity

$$\text{Optimal production lot size}: Q^* = \sqrt{\frac{2Ak}{ch}}\sqrt{\frac{B}{B-A}}$$

$$\text{Number of production runs per year} = \frac{A}{Q^*}$$

$$\text{Time between setups (cycle time)} = \frac{Q^*}{A} \text{ years}$$

$$\text{Total annual cost} = \text{setup cost} + \text{holding cost} = \frac{Ak}{Q} + \frac{chQ}{2}\left(\frac{B-A}{B}\right)$$

EOQ with back orders

$$\text{Optimal order quantity, } Q^* = \sqrt{\frac{2Ak}{ch}\left(\frac{p+ch}{p}\right)}$$

$$\text{Quantity at the beginning of each cycle}, S^* = \sqrt{\frac{2Ak}{ch}\left(\frac{p}{p+ch}\right)}$$

$$\text{Maximum number of backorders} = Q^* - S^*$$

$$\text{Number of orders per year} = \frac{A}{Q^*}$$

$$\text{Time between orders (cycle time)} = \frac{Q^*}{A} \text{ years}$$

$$\text{Total annual cost} = \text{setup} + \text{holding} + \text{backorder}$$

$$= \frac{Ak}{Q} + \frac{chS^2}{2Q} + \frac{p(Q-S)^2}{2Q}$$

Formula for total cost using Quantity discounts.

$$\text{Total annual cost} = \text{purchase cost} + \text{holding cost} + \text{item cost}$$

$$= \frac{Ak}{Q} + \frac{chQ}{2} + Ac$$

Inventory models under random demand, assumed to be normally distributed with mean μ and standard deviation σ.

Single Period Order Quantity	Reorder Point Model
$P(demand < Q^*) = \frac{C_u}{C_u + C_v}$, $Q^* = \mu + z\sigma$ Cost of overestimating demand: $C_v = h_E + c$ Cost of underestimating demand: $C_u = P_S + P_R - c$ Where: Unit Cost $= c$ Penalty for item held at end of inventory cycle $= h_E$ Penalty for each item short (goodwill etc.) $= P_S$ Selling Price $= P_R$	Reorder Point: $r = \mu + z\sigma$ Average Inventory: $\frac{Q}{2} + z\sigma$ Total Annual Cost: $\left(\frac{Q}{2} + z\sigma\right)ch + \frac{Ak}{Q}$

Cumulative Probabilities for the Standard Normal Distribution

Table gives P(Z<z) for Z = N(0,1)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000