MAT1830 - Discrete Mathematics for Computer Science
Assignment #10 Solutions

1. (a) $2 + \frac{2}{5} + \frac{2}{9} + \frac{2}{13} + \frac{2}{17}$ **[1]**

   (b) $((z+4)^4 - 12)((z+5)^5 - 15)((z+6)^6 - 18)((z+7)^7 - 21)$ **[1]**

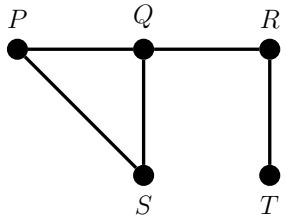
2. (a) $\prod_{i=0}^{30}(x - i^2)$ **[2]**

   (b) $\sum_{i=2}^{11} \frac{1}{(3i)^{i+2}}$ **[2]**

3. (a) $t_3 = 5$ (the strings are aaa, abb, acc, bba, cca) **[1]**
   $t_4 = 11$ (the strings are $aaaa, aabb, aacc, abba, acca, bbaa, bbbb, bbcc, ccaa, ccbb, cccc$) **[1]**

   (b) Call a string of letters "legal" if it can be produced by concatenating copies of "a","bb" and "cc".
   Every legal string of n letters can be written in exactly one of the following ways: **[1]**
   • Xa where X is a legal string of length $n-1$; **[1]**
   • Xbb where X is a legal string of length $n-2$; **[1]**
   • Xcc where X is a legal string of length $n-2$; **[1]**
   So we can see that $t_n = t_{n-1} + 2t_{n-2}$ for all $n \geq 3$. **[2]**

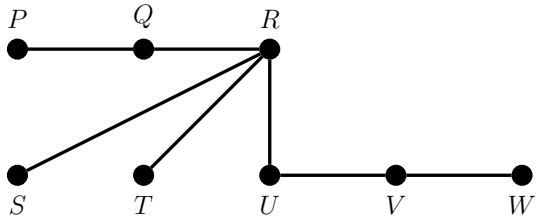
4. (a)



[1]

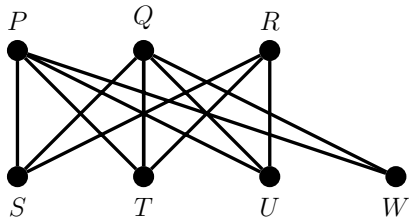
(b) No simple graph has the properties required by (b). The number of unordered pairs of vertices in a graph with 11 vertices is $\binom{11}{2} = 55$ and each of these pairs can be joined by at most one edge. So the maximum number of edges in a simple graph with 11 vertices is 55. **[2]**

(c) Any tree on 8 vertices. Here is one possibility:



[1]

(d) Here is one possibility:



[2]