MAT1830 - Discrete Mathematics for Computer Science
Assignment #9 Solutions

1. Let X be the number of heads flipped. Then X is a binomial random variable with $p = \frac{3}{7}$ and $n = 80$. **[1]**

   Using the formula for the binomial distribution with $p = \frac{3}{7}$ and $n = 80$,

   $$\Pr(X = 30) = \binom{80}{30}\left(\frac{3}{7}\right)^{30}\left(\frac{4}{7}\right)^{50} \approx 5.71\%$$

   So the probability that heads is flipped exactly 30 times is $\binom{80}{30}(\frac{3}{7})^{30}(\frac{4}{7})^{50}$. **[2]**

2. (a) Let X be the number of cars that pass in this minute. Using the formula for the Poisson distribution with $\mu = 7$,

   $$\Pr(X = 1) = \frac{7^1 e^{-7}}{1!} = 7e^{-7} \approx 0.64\%$$

   So the probability that exactly one car passes through the junction in the minute is $7e^{-7}$. **[2]**

   (b) Let X_1, X_2 and X_3 be, respectively, the number of cars that pass through in the first, second, and third minute. We know that the expected value of the Poisson distribution is $\mu = 7$, so $\mathrm{E}[X_1] = 7$, $\mathrm{E}[X_2] = 7$ and $\mathrm{E}[X_3] = 7$. So

   $$\mathrm{E}[X_1 + X_2 + X_3] = \mathrm{E}[X_1] + \mathrm{E}[X_2] + \mathrm{E}[X_3] = 7 + 7 + 7 = 21.$$

   So the expected number of cars to pass through in three minutes is 21. **[2]**

   (c) Let Y be the number of cars that pass in the three minute period. Note that Y is random variable with a Poisson distribution with $\mu = 21$. Using the formula for the Poisson distribution with $\mu = 21$, we see that

   $$\Pr(Y = 21) = \frac{21^{21} e^{-21}}{21!} \approx 8.67\%$$

   So the probability that exactly 21 cars pass through the junction in the three minute period is $\frac{21^{21}e^{-21}}{21!}$. **[2]**

3. The probability distribution of Y must be given by

y	-10	0	10
$\Pr(Y = y)$	p	q	r

[1]

for some nonnegative real numbers p, q, r such that $p + q + r = 1$.

Because $E(Y) = 0$ we have

$$0 = \mathrm{E}[Y] = p \times (-10) + q \times 0 + r \times 10 = 10r - 10p.$$

So $10p = 10r$ and $p = r$. **[2]**

Because $\mathrm{Var}[Y] = 80$ we have

$$80 = \mathrm{Var}[Y] = p \times (-10 - 0)^2 + q \times (0 - 0)^2 + r \times (10 - 0)^2 = 100p + 100r.$$

So $100p + 100r = 80$ and $p + r = \frac{4}{5}$. **[2]**

Because $p = r$ and $p + r = \frac{4}{5}$, we have $p = \frac{2}{5}$ and $r = \frac{2}{5}$. Because $p + q + r = 1$, we have $q = 1 - \frac{2}{5} - \frac{2}{5} = \frac{1}{5}$.

So the probability distribution of Y is

y	-10	0	10
$\Pr(Y = y)$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$

[2]

4. (a)

$$\begin{aligned}
r_0 &= 2 \\
r_1 &= (r_0)^2 - 1 - 1 &&= 2^2 - 1 - 1 &&= 2 \\
r_2 &= (r_1)^2 - 2 - 1 &&= 2^2 - 2 - 1 &&= 1 \\
r_3 &= (r_2)^2 - 3 - 1 &&= 1^2 - 3 - 1 &&= -3 \\
r_4 &= (r_3)^2 - 4 - 1 &&= (-3)^2 - 4 - 1 &&= 4
\end{aligned}$$

[2]

(b)

$$\begin{aligned}
s_0 &= 2 \\
s_1 &= s_0^2 &&= 2^2 &&= 4 \\
s_2 &= (s_1)^2 + (s_0)^2 &&= 4^2 + 2^2 &&= 20 \\
s_3 &= (s_2)^2 + (s_1)^2 + (s_0)^2 &&= 20^2 + 4^2 + 2^2 &&= 420 \\
s_4 &= (s_3)^2 + (s_2)^2 + (s_1)^2 + (s_0)^2 &&= 420^2 + 20^2 + 4^2 + 2^2 &&= 176820
\end{aligned}$$

[2]