MAT1830 - Discrete Mathematics for Computer Science
Assignment #7 Solutions

1. (a) A ternary string of length 9 is an ordered selection with repetition of 9 elements from the set $\{0, 1, 2\}$ with 3 elements. There are 3^9 such selections. **[2]**

   (b) Such a string is an ordered selection without repetition of 6 letters from a set $\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \ldots, \mathrm{z}\}$ with 26 elements. So there are $\frac{26!}{(26-6)!} = \frac{26!}{20!} = 26 \times 25 \times 24 \times 23 \times 22 \times 21$ such strings. **[2]**

   (c) For any $k \in \mathbb{N}$, every binary string of length 100 that contains exactly k 1s corresponds to a subset of $\{1, 2, \ldots, 100\}$ with k elements (recording the positions of the 1s). So there are $\binom{100}{k}$ such strings. Thus there are $\binom{100}{0} + \binom{100}{1} + \binom{100}{2}$ binary strings of length 100 that contain at most two 1s. **[3]**

   (d) Similarly to (c) above, there are $\binom{10}{2}$ ways of choosing 2 positions from the 10 positions in the string to contain 1s. Once these are chosen, there are $\binom{8}{3}$ ways of choosing 3 positions from the remaining 8 positions to contain 2s. So there are $\binom{10}{2} \times \binom{8}{3}$ such strings. (Equally, there are $\binom{10}{3} \times \binom{7}{2}$ such strings.) **[3]**

2. Let B be the set of students on the basketball team, let C be the set of students on the chess team, let N be the set of students on the netball team, and let S be the set of students on the soccer team. We are told that the four-way intersection is empty and that all the three-way intersections except for $C \cap N \cap S$ are empty. So the inclusion/exclusion principle tells us that

   $$|B \cup C \cup N \cup S| = |B| + |C| + |N| + |S| - |B \cap C| - |B \cap N| - |B \cap S| - |C \cap N| - |C \cap S| - |N \cap S| + |C \cap N \cap S|. \quad \textbf{[3]}$$

   Because the school has 14 students and each is on at least one of the teams we know that $|B \cup C \cup N \cup S| = 14$. The size of the teams tells us that $|B| = 5$, $|C| = 4$, $|N| = 7$ and $|S| = 11$. We are also told that $|C \cap N \cap S| = 1$, $|B \cap C| = 1$, and $|B \cap N| = 1$, $|C \cap N| = 1$, $|C \cap S| = 3$ and $|N \cap S| = 5$. Substituting all this information into the equation we see that

   $$14 = 5 + 4 + 7 + 11 - 1 - 1 - |B \cap S| - 1 - 3 - 5 + 1. \quad \textbf{[2]}$$

   So $|B \cap S| = 3$. There are exactly 3 students who play basketball and soccer. **[1]**

3. By the binomial theorem, the terms of this expansion will be $\binom{14}{i}(5x^2)^i(3)^{14-i}$ for $i = 0, 1, \ldots, 14$. **[1]** Because $(5x^2)^i = 5^i x^{2i}$, the relevant term will be for $2i = 10$, so for $i = 5$. This term is

   $$\binom{14}{5}(5x^2)^5(3)^9 = \binom{14}{5}5^5 3^9 x^{10}. \quad \textbf{[2]}$$

   So the coefficient is $\binom{14}{5}5^5 3^9$. **[1]**