MAT1830 - Discrete Mathematics for Computer Science
Assignment #1
Submit by uploading a pdf to moodle by 11:55pm Wednesday in week 3

Assessment questions/solutions for this unit must not be posted on any website.

Show your working and give explanations for questions (1) and (2). For questions (3) and (4) only answers are required.

(1) Give a truth table for $\neg a \to ((a \wedge \neg b) \leftrightarrow c)$. State whether the sentence is a tautology, a contradiction or neither and briefly explain why. **[4]**

ANS:

a	b	c	$\neg b$	$a \wedge \neg b$	$(a \wedge \neg b) \leftrightarrow c$	$\neg a$	$\neg a \to ((a \wedge \neg b) \leftrightarrow c)$
T	T	T	F	F	F	F	T
T	T	F	F	F	T	F	T
T	F	T	T	T	T	F	T
T	F	F	T	T	F	F	T
F	T	T	F	F	F	T	F
F	T	F	F	F	T	T	T
F	F	T	T	F	F	T	F
F	F	F	T	F	T	T	T

[3]

So $\neg a \to ((a \wedge \neg b) \leftrightarrow c)$ is neither a tautology nor a contradiction, because it is true in some rows (eg the first) but false in others (eg the second last). **[1]**

(2) Use laws of logic to show that $(p \wedge \neg(\neg p \wedge q)) \vee \neg(p \to r)$ is logically equivalent to p. Explain each step fully. **[4]**

ANS:

$$
\begin{aligned}
& (p \wedge \neg(\neg p \wedge q)) \vee \neg(p \to r) \\
\equiv\ & (p \wedge (p \vee \neg q)) \vee \neg(p \to r) && \text{(because } \neg(\neg p \wedge q) \equiv p \vee \neg q \text{ by DeMorgan's laws)} \\
\equiv\ & p \vee \neg(p \to r) && \text{(because } p \wedge (p \vee \neg q) \equiv p \text{ by the absorption laws)} \\
\equiv\ & p \vee \neg(\neg p \vee r) && \text{(because } p \to r \equiv \neg p \vee r \text{ by the implication law)} \\
\equiv\ & p \vee (p \wedge \neg r) && \text{(because } \neg(\neg p \vee r) \equiv p \wedge \neg r \text{ by DeMorgan's laws)} \\
\equiv\ & p && \text{(using the absorption laws)}
\end{aligned}
$$

[4]

So the original statement is logically equivalent to p.

(3) For the statement "If n is not prime, then $2^n - 1$ is not prime", write down the statement's contrapositive and then write down the statement's negation. **[2]**

[No explanation required but no partial marks for incorrect answers.]

ANS: Contrapositive: "If $2^n - 1$ is prime, then n is prime." **[1]**
Negation: "n is not prime and $2^n - 1$ is prime." **[1]**

[Other slightly different answers are possible.]

(4) Anita and Omar want to ride motorbikes. Where they live, anyone aged 18 or more is allowed ride a motorbike, people aged 16 or more are allowed to ride a motorbike if they have a permit, and people aged less than 16 are not allowed to ride a motorbike. (But assume someone can have a permit at any age.)

Let p be the proposition that Anita is aged 16 or more.

Let q be the proposition that Anita is aged 18 or more.

Let r be the proposition that Anita has a permit.

Let s be the proposition that Omar is aged 16 or more.

Let t be the proposition that Omar is aged 18 or more.

Let u be the proposition that Omar has a permit.

Write down propositions (using just p, q, r, s, t, u and logical connectives) corresponding to the following statements.

(i) Omar is allowed to ride a motorbike. **[2]**

(ii) If at least one of Anita and Omar is aged less than 18 then they both have permits. **[2]**

(iii) Anita does not have a permit but is allowed to ride a motorbike. **[2]**

(iv) Exactly one of Anita and Omar is allowed to ride a motorbike. **[2]**

(v) Both Anita and Omar are aged less than 18, but at least one of them is allowed to ride a motorbike. **[2]**

[No explanation required but no partial marks for incorrect answers.]

ANS: (i) $t \vee (s \wedge u)$ **[2]**

To be allowed to ride a motorbike Omar must be aged at least 18 or must be aged at least 16 and have a permit. "Omar is aged at least 18" translates to t, and "Omar is aged at least 16 and has a permeit" translates to $s \wedge u$.

(ii) $(\neg q \vee \neg t) \rightarrow (r \wedge u)$ **[2]**

"at least one of Anita and Omar is aged less than 18" translates to $\neg q \vee \neg t$, and "both Anita and Omar have permits" translates to $r \wedge u$.

(iii) $\neg r \wedge q$ **[2]**

If Anita does not have a permit but is allowed to ride a motorbike, she must be aged at least 18. "Anita does not have a permit" translates to $\neg r$, and "Anita is aged at least 18" translates to q.

(iv) $(q \vee (p \wedge r)) \veebar (t \vee (s \wedge u))$ **[2]**

From (i), "Omar is allowed to ride a motorbike" translates to $t \vee (s \wedge u)$ and similarly "Anita is allowed to ride a motorbike" translates to $q \vee (p \wedge r)$.

(v) $\neg q \wedge \neg t \wedge ((p \wedge r) \vee (s \wedge u))$ **[2]**

If Anita and Omar are aged less than 18, then for either of them to be allowed ride a motorbike they must be aged at least 16 and have a permit. "Both Anita and Omar are aged less than 18" translates to $\neg q \wedge \neg t$, and "at least one of them is aged at least 16 and has a permit" translates to $(p \wedge r) \vee (s \wedge u)$.

[Other answers are possible.]