MAT1830 - Discrete Mathematics for Computer Science
Assignment #4
Submit by uploading a pdf to moodle by 11:55pm Wednesday in week 10

Assessment questions/solutions for this unit must not be posted on any website.

Full justifications are required for Questions 1(i) and 3.

(1) (i) A bag contains one trick coin that always shows heads when flipped, and nine fair coins (that show heads with probability $\frac{1}{2}$ and tails with probability $\frac{1}{2}$ when flipped). A coin is selected uniformly at random from the bag and flipped four times. What is the probability that the coin selected was the trick coin, given that it shows heads all four times? **[5]**

(ii) A bag contains n coins, one of which is a trick coin and the rest of which are fair (the coins are as described in (i)). A coin is selected uniformly at random from the bag and flipped k times. The coin shows heads all k times and, given this, you calculate that there is an exactly 50% probability that the coin selected was the trick coin. What is n?
(Write your answer as an expression in terms of k.) **[2]**

[Fully explain your answer for (i). Answer only required for (ii).]

(2) Let X and Y be independent random variables with distributions

x	0	1	2	4
$\Pr(X=x)$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

y	0	1	2
$\Pr(Y=y)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

Let $W = 8X + 3Y$ and $Z = XY$.

(i) What is $\mathrm{E}[X]$? **[1]**

(ii) What is $\mathrm{Var}[X]$? **[1]**

(iii) What is $\mathrm{E}[W]$? **[2]**

(iv) Write down a table giving the probability distribution for Z. **[3]**

[Answers only required.]

(3) One of the 256 subsets of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ is chosen uniformly at random. Let X be the number of elements of this subset. Let Y be 0 if the subset is empty and be the least element of the subset otherwise.

(i) Are the events "$X = 1$" and "$Y = 3$" independent? **[2]**

(ii) Are the events "$X = 2$" and "$Y = 6$" independent? **[2]**

(iii) What is $\Pr(X = 2 \mid Y \leq 4)$? **[2]**

[Full justification required.]