MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #2 Solutions

1. (a)

b	p	$\neg b$	$\neg p$	$\neg b \to \neg p$	$b \vee p$	$(\neg b \to \neg p) \wedge (b \vee p)$	$(\neg b \to \neg p) \wedge (b \vee p) \wedge \neg b$
T	T	F	F	T	T	T	F
T	F	F	T	T	T	T	F
F	T	T	F	F	T	F	F
F	F	T	T	T	F	F	F

   (b) A contradiction (because it's column in the truth table is all Fs).

   (c) Heaps of possible answers. For example, $p \to p$ is a tautology and $(p \vee q) \wedge \neg p \wedge \neg q$ is a contradiction.

2. (a) $\neg b \to \neg p$ (if no broccoli then no potatoes)
   $b \vee p$ (eat at least one of broccoli or potatoes)
   $\neg b$ (no broccoli)

   (b) It's impossible to follow these rules.

3. (a) "Her car isn't blue and her car it isn't red." ($\neg$(blue $\vee$ red) $\equiv \neg$blue $\wedge\ \neg$red)

   (b) "The integer I am thinking of is even or not prime." ($\neg$(odd$\wedge$prime) $\equiv \neg$odd$\vee\neg$prime)

   (c) "If we can't do anything we want with your data, then you don't use our app."
   (use app $\to$ control data $\equiv \neg$control data $\to \neg$use app)

4. Yes.

$$\begin{aligned} \neg p \vee (\neg q \to \neg r) &\equiv \neg p \vee (q \vee \neg r) && \text{(by the implication law)} \\ &\equiv \neg(p \wedge \neg(q \vee \neg r)) && \text{(by DeMorgan's laws)} \\ &\equiv \neg(p \wedge (\neg q \wedge r)) && \text{(by DeMorgan's laws)} \end{aligned}$$

5. (a) Could be true or false (for example, it's true when p, q, r, s are all F, and it's false when p, q are F and r, s are T).

   (b) Definitely false. This is the negation of the original statement (after applying DeMorgan's law to $\neg(r \wedge s)$).

   (c) Definitely true. This is the contrapositive of the original statement (after applying DeMorgan's law to both sides).

   (d) Definitely true. The original statement being true means that if either p or q are T, then both r and s must be T. So certainly if p is T then r will be T.

   (e) Definitely false. The original statement being true means that if either p or q are T, then both r and s must be T. So certainly if q is T then both r and s will be T. This means that $q \wedge (\neg r \vee \neg s)$ cannot be true.