MAT1830 - Discrete Mathematics for Computer Science
Assignment #1 Solutions

1. (a) True (because $4 \times 4 = 16$). **[1]**

   (b) False (because 6 does not divide $13 - 24 = -11$). **[1]**

   (c) False (because there is no integer k such that $12k = 3$). **[1]**

   (d) False (for example $\gcd(13, 26) = 13$). **[1]**

   (e) False (for example $3 \times 5 \equiv 6 \pmod 9$ but $5 \not\equiv 2 \pmod 9$).
   Note: We can say definitely that $x \equiv 2 \pmod 3$, however. **[2]**

   (f) False. Suppose such integers x, y, z existed and let $n = 2^x \times 3^y \times 5^z$. Then $n = 14q = 2 \times 7 \times q$ for some integer q. This would mean there were two ways of writing n as a product of primes: one that uses just 2s, 3s and 5s ($2^x \times 3^y \times 5^z$), and one that uses a 7. The fundamental theorem of arithmetic says this can't happen. **[2]**

2. Because $68 \equiv 4 \pmod 8$ and $x \equiv 5 \pmod 8$, we have $68x \equiv 4 \times 5 \equiv 4 \pmod 8$. **[1]**
   Because $y \equiv 3 \pmod 8$ we have $y^2 \equiv 3 \times 3 \equiv 1 \pmod 8$ and so $2y^2 \equiv 2 \times 1 \equiv 2 \pmod 8$. **[1]**
   Using these facts,
   $$68x + 2y^2 \equiv 4 + 2 \equiv 6 \pmod 8. \qquad \textbf{[2]}$$
   So $z = 6$. **[1]**

3. Using the Euclidean algorithm,
   $$\begin{array}{rcrcrcr} 545 & = & 4 & \times & 127 & + & 37 \\ 127 & = & 3 & \times & 37 & + & 16 \\ 37 & = & 2 & \times & 16 & + & 5 \\ 16 & = & 3 & \times & 5 & + & 1 \\ 5 & = & 5 & \times & 1 & + & 0. \end{array}$$
   [2]

   So $\gcd(545, 127) = 1$. Then using the extended Euclidean algorithm,
   $$\begin{array}{rcrcr} 1 & = & 16 - 3 \times 5 & & \\ 1 & = & 16 - 3 \times (37 - 2 \times 16) & = & -3 \times 37 + 7 \times 16 \\ 1 & = & -3 \times 37 + 7 \times (127 - 3 \times 37) & = & 7 \times 127 - 24 \times 37 \\ 1 & = & 7 \times 127 - 24 \times (545 - 4 \times 127) & = & -24 \times 545 + 103 \times 127. \end{array}$$
   [3]

   So $-24 \times 545 + 103 \times 127 = 1$ or, equivalently, $103 \times 127 - 1 = 24 \times 545$. So $127 \times 103 \equiv 1 \pmod{545}$ and $x = 103$ is a solution. **[2]**