

## MAT1830 - Discrete Mathematics for Computer Science

### Assignment #2

Submit by uploading a pdf to moodle by 11:55pm Wednesday in week 6

Assessment questions/solutions for this unit must not be posted on any website.

For questions 1 and 2, make sure you set out and explain your proofs clearly. To receive marks for question 3, your answers must be exactly right and use correct notation.

- (1) Prove by simple induction that, for each integer  $n \geq 1$ ,

$$6 + 6^2 + 6^3 + \cdots + 6^n = \frac{6^{n+1} - 6}{5}. \quad [6]$$

- (2) Let  $S_1, S_2, S_3, S_4, \dots$  be the sequence of sets defined by  $S_1 = \{0, 1, 2\}$ ,  $S_2 = \{0, 2, 3\}$ ,  $S_3 = \{0, 3, 4\}$  and

$$S_i = (S_{i-3} \triangle S_{i-2}) \triangle (S_{i-1} \cup \{i-3, i+1\}) \text{ for each integer } i \geq 4.$$

Prove by strong induction that  $S_n = \{0, n, n+1\}$  for each integer  $n \geq 1$ . [7]

- (3) Let  $R$ ,  $S$  and  $T$  be sets defined as follows.

$$\begin{aligned} R &= \{2, 4, 6, 7, 8\} \\ S &= \{\{2\}, \{2, 3, 4\}, \{2, 4, 6\}, \{6, 7\}\} \\ T &= \{x \in \mathbb{Z} : x \leq 4 \text{ or } x \geq 8\} \end{aligned}$$

Find the following.

- (i)  $R - T$
- (ii)  $S - \mathcal{P}(R)$
- (iii)  $\mathcal{P}(R) \cap \mathcal{P}(T)$
- (iv)  $(R \cap T) \times (S - \{\{2, 3, 4\}, \{2, 4, 6\}, \{6\}\})$
- (v)  $|(\mathcal{P}(R) - S) \times S|$

[No explanation required.] [5]

- (4) Let  $A$  and  $B$  be finite sets and let  $a = |A|$ ,  $b = |B|$  and  $c = |A \cap B|$ . Write an expression in terms of  $a$ ,  $b$  and  $c$  that is equal to  $|(A \times B) \cup (B \times A)|$  for every choice of  $A$  and  $B$ .

[No explanation required.] [2]