MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #10 Solutions

1. (a) Note that $a_0 = 2^0 = 1$.
   Also, $a_n = 2^n = 2(2^{n-1}) = 2a_{n-1}$ for $n \geq 1$.
   So "$a_0 = 1$ and $a_n = 2a_{n-1}$ for all integers $n \geq 1$" is a recursive definition for the sequence.

   (b) Note that $b_0 = 0^2 = 0$.
   Also, $b_n = n^2 = (n-1)^2 + 2n - 1 = b_{n-1} + 2n - 1$ for $n \geq 1$.
   So "$b_0 = 0$ and $b_n = b_{n-1} + 2n - 1$ for all integers $n \geq 1$" is a recursive definition the sequence.

2. (a) Vertex set: $\{P, Q, R, S\}$
   Edge set: $\{PQ, PR, PS\}$

   (b) Vertex set: $\{P, Q, R, S, T\}$
   Edge set: $\{PQ, PR, PS, QR, QS, RS\}$

   (c) Vertex set: $\{P, Q, R, S, T\}$
   Edge set: $\{PS, QS, QT, RS, ST\}$

3. There is only one way of writing 1 in this form: "1".

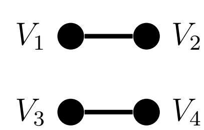
   There are two ways of writing 2: "$1 + 1$" and "2".

   So $s_1 = 1$ and $s_2 = 2$.

   If the first term in such a sum is a 1, then there are s_{n-1} ways to finish it so that it adds to n.
   If the first term in such a sum is a 2, then there are s_{n-2} ways to finish it so that it adds to n.
   These two cases account for every possible sum.

   So $s_1 = 1$, $s_2 = 2$ and $s_n = s_{n-1} + s_{n-2}$ for all integers $n \geq 3$.

4. (a)



   (b) 1. The walk is V_1, V_2, V_1, V_2.

   (c) For any $n \geq 2$:
   - The 1st entry in the top row of M^n is the number of walks of length n from V_1 to V_1, which is 1 for any even $n \geq 2$ (the walk is $V_1, V_2, V_1, V_2, \ldots, V_1$).
   - The 2nd entry in the top row of M^n is the number of walks of length n from V_1 to V_2, which is 0 for any even $n \geq 2$.
   - The 3rd entry in the top row of M^n is the number of walks of length n from V_1 to V_3, which is 0 for any even $n \geq 2$.
   - The 4th entry in the top row of M^n is the number of walks of length n from V_1 to V_4, which is 0 for any even $n \geq 2$.

   So the top row of M^n for any even $n \geq 2$ is "1 0 0 0".

5. One line creates two regions, so $r_1 = 2$.
   The nth line added intersects each of the $n-1$ previous lines exactly once because it is not parallel to any of them. The $n-1$ intersection points created are all different because no three lines meet at a point. These $n-1$ intersection points divide the nth line into n segments and each of these segments splits a region in two. So the new number of regions is n more than the previous number.

   Thus $r_1 = 2$ and $r_n = r_{n-1} + n$ for all integers $n \geq 2$.