MAT1830 - Discrete Mathematics for Computer Science
Assignment #8 Solutions

1. (a) $\Pr(A) = \frac{1}{6}$ because one of the six sides is marked 0. **[0.5]**
   $\Pr(B) = \frac{3}{6} = \frac{1}{2}$ because three of the six sides are marked 3. **[0.5]**
   $\Pr(C) = \frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{2} = \frac{1}{3}$ because the only ways the sum of the rolls can be 5 is if the first is 2 and the second is 3 or if the first is 3 and the second is 2. The probability that the first is 2 and the second is 3 is $\frac{1}{2} \times \frac{1}{3}$ because the two rolls are independent. Similarly, the probability that the first is 3 and the second is 2 is $\frac{1}{3} \times \frac{1}{2}$. [1]

   (b) $\Pr(A \cap B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ because the first and second rolls are independent. [1]
   $\Pr(A \cap C) = 0$ because if the first roll is 0 then the sum of the rolls cannot be 5. [1]
   $\Pr(B \cap C) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ because $B \cap C$ can only occur if the first roll is 2 and the second is 3. [1]

   (c) A and C are not independent because $\Pr(A \cap C) \neq \Pr(A)\Pr(C)$.
   From above, $\Pr(A \cap C) = 0$ and $\Pr(A)\Pr(C) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.
   B and C are independent because $\Pr(B \cap C) = \Pr(B)\Pr(C)$.
   From above, $\Pr(B \cap C) = \frac{1}{6}$ and $\Pr(B)\Pr(C) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. [2]

   (d) $\Pr(A \cup C) = \Pr(A) + \Pr(C) - \Pr(A \cap C) = \frac{1}{6} + \frac{1}{3} - 0 = \frac{1}{2}$ using our answers above. [1]
   $\Pr(B \cup C) = \Pr(B) + \Pr(C) - \Pr(B \cap C) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$ using our answers above. [1]

   (e) $\Pr(B|(B \cup C)) = \frac{\Pr(B \cap (B \cup C))}{\Pr(B \cup C)} = \frac{\Pr(B)}{\Pr(B \cup C)} = \frac{1}{2}/\frac{2}{3} = \frac{3}{4}$ using our answers above. [1]

2. Let S be the event that Bond survives.
   Let M be the event that a male scorpion bit Bond.

   (Because Bond was bitten by either a male or female scorpion, $\overline{M}$ is the event that Bond was bitten by a female scorpion.)

   $\Pr(M) = \frac{60}{100} = \frac{3}{5}$ because the biter was chosen uniformly at random from the pit. [1]
   $\Pr(S|M) = 1 - 70\% = \frac{3}{10}$ from the question. [1]
   $\Pr(S|\overline{M}) = 1 - 90\% = \frac{1}{10}$ from the question. [1]

   By Bayes' theorem,

$$\begin{aligned}\Pr(M|S) &= \frac{\Pr(S|M)\Pr(M)}{\Pr(S|M)\Pr(M) + \Pr(S|\overline{M})\Pr(\overline{M})} \\ &= \frac{\frac{3}{10} \times \frac{3}{5}}{(\frac{3}{10} \times \frac{3}{5}) + (\frac{1}{10} \times \frac{2}{5})} \\ &= \frac{9}{11}.\end{aligned}$$ [3]

   So, given that Bond survives, the probability that the scorpion that bit him was male is $\frac{9}{11}$. [1]

3. Each string in $\{\text{BBBBB}, \text{ABBBC}, \text{AACCC}, \text{ABBCC}, \text{BBBBC}\}$ occurs with probability $\frac{1}{5}$. **[1]**

$X = 0$ exactly when the chosen string is BBBBB or BBBBC.
$X = 1$ exactly when the chosen string is ABBBC or ABBCC.
$X = 2$ exactly when the chosen string is AACCC. **[1]**

Thus the probability distribution of X is given by

x	0	1	2
$\Pr(X = x)$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{5}$

. **[1]**