MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #6 Solutions

1. There are lots of possibilities. I'll give one possible relation as a set of ordered pairs and leave you to draw the diagrams.

   (a) $\{(a,a),(b,b),(c,c),(d,d),(e,e),(f,f),(a,b),(b,a)(b,c),(c,b)(a,c),(c,a),(d,e)\}$.

   (b) $\{(a,a),(b,b),(c,c),(a,b),(b,a),(b,c),(c,b)\}$.

   (c) $\{(a,a),(b,b),(c,c),(d,d),(e,e),(f,f),(a,b),(b,a)(b,c),(c,b)(a,c),(c,a),(d,e),(e,d)\}$.

2. For $A \in \mathcal{P}(\{1,2,3,4\}) - \{\emptyset\}$, we'll write $\min(A)$ for the smallest number in A.
   S is reflexive because, for all $A \in \mathcal{P}(\{1,2,3,4\}) - \{\emptyset\}$, $\min(A) = \min(A)$ and so ASA.
   S is symmetric because, for all $A, B \in \mathcal{P}(\{1,2,3,4\}) - \{\emptyset\}$ if ASB then $\min(A) = \min(B)$ so $\min(B) = \min(A)$, and so BSA.
   S is not antisymmetric. For example $\{1,2\}S\{1,3\}$ and $\{1,3\}S\{1,2\}$.
   S is transitive because, for all $A, B, C \in \mathcal{P}(\{1,2,3,4\}) - \{\emptyset\}$, if ASB and BSC, then $\min(A) = \min(B)$ and $\min(B) = \min(C)$, and so $\min(A) = \min(C)$ and ASC.

   Let B be the set of finite binary strings.
   T is reflexive because, for all $c \in B$, $c = c$, so cTc.
   T is not symmetric. For example $1T11$ but $11 \not{T} 1$.
   T is antisymmetric because, for all $c, d \in B$, if cTd and dTc then either $c = d$ or c can be obtained from d by deleting some bits and d can be obtained from c by deleting some bits, so $c = d$ because the latter is impossible.
   T is transitive because, for all $c, d, e \in B$, if cTd and dTe then c can be obtained from d by deleting some bits and d can be obtained from e by deleting some bits, so clearly c can be obtained from e by deleting some bits and cTe.

3. S is a equivalence relation. The equivalence classes of S are
   $\{\{4\}\}$,
   $\{\{3\},\{3,4\}\}$,
   $\{\{2\},\{2,3\},\{2,4\},\{2,3,4\}\}$ and
   $\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\}$.

   T is a partial order relation. T is not a total order relation because, for example, $11 \not{T} 00$ and $00 \not{T} 11$. Because T is not a total order relation it cannot be a well-order relation.

4. Let Q, R, S and T be relations on $\{1,2,3,4,5,6,7,8,9\}$.

   (a) Q could be reflexive, could be antisymmetric and could be transitive. It can't be symmetric because $3Q4$ and $4 \not{Q} 3$.

   (b) R could be symmetric. It can't be reflexive because $1 \not{R} 1$, can't be antisymmetric because $1R2$ and $2R1$, and can't be transitive because $1R2$, $2R1$ and $1 \not{R} 1$.

   (c) Transitivity tells us that $5S7$ (because $5S6$ and $6S7$) and $5S8$ (because $5S6$ and $6S8$). Antisymmetry then tells us that $6 \not{S} 5$, $7 \not{S} 6$, $8 \not{S} 6$, $7 \not{S} 5$ and $8 \not{S} 5$ (because, respectively, $5S6$, $6S7$, $6S8$, $5S7$ and $5S8$).

   (d) If T were transitive then $7T4$ (because $7T8$ and $8T4$), but then T could not be antisymmetric because $4T7$ and $7T4$. So T cannot be transitive and antisymmetric.
   T could be an equivalence relation.