MONASH University

MAT1830 - Discrete Mathematics for Computer Science - S1 2022

Dashboard / My units / MAT1830 - S1 2022 / Assessments / Quiz 4

Started on	Sunday, 1 May 2022, 3:54 PM
State	Finished
Completed on	Wednesday, 4 May 2022, 11:55 PM
Time taken	3 days 8 hours
Grade	**6.00** out of 15.00 (**40**%)

Print friendly format

Information

Each answer to a short answer question on this quiz is an **integer** or a **fraction**. Enter your answers as follows.

Integers: Enter these as numerals, using the minus character where necessary, like on previous quizzes.
For example 16 or 1 or 0 or -12 **BUT NOT** sixteen or 1.0 or zero or minus 12.

Fractions: Enter a non-integer rational number as [numerator]/[denominator] where the fraction is **in lowest terms,** [numerator] and [denominator] are integers entered as above. For example, 1/2, 5/4 or 11/16 **BUT NOT** half, 1.25 or 22/32.

Remember:

- **Do not** enter anything other than the answer. For example 1/3 **BUT NOT** z=1/3.
- **No answer should contain a letter, a space**, **equals sign**, **full stop** etc.

The quiz is auto-marked. Answers entered incorrectly will be marked wrong. Failure to follow the above instructions will not be grounds for marks to be adjusted.

Question **1**

Partially correct

Mark 2.00 out of 8.00

All four parts of this question refer to the following scrabble tiles. You do not need to understand the game of scrabble to answer the question. All you need to know is that it is played with tiles. Each tile has one letter on it, and a point value in the bottom right hand corner (e.g. "M" is worth 3 points).

Note that there are some repeated tiles. For example the two M tiles are identical. For the first 3 parts of the question we place all of the above tiles into a bag.

Two tiles showing **different** letters are taken from the bag and a 2-element **set** is formed containing the two letters on the tiles. How many sets can be formed in this way? 28 ✔

(Examples of sets that can be formed in this way include $\{I, M\}$ and $\{A, H\}$.)

Two tiles are selected from the bag and a 2-element **multiset** is formed containing the letters on the tiles. How many multisets can be formed in this way? 55 ✖

(Examples of multisets that can be formed in this way include $\{I, M\}$ and $\{M, M\}$.)

A tile is taken from the bag and left aside, and then another tile is taken from the bag. An ordered pair is formed whose first coordinate is the letter on the first tile and whose second coordinate is the letter on the second tile. How many possible ordered pairs can be formed in this way? 30 ✖

(Examples of ordered pairs that can be formed in this way include (T, A) and (M, I).)

Suppose that the pictured tiles get split between two bags. Which of the following statements follows from the pigeonhole principle?

- Both bags will have the same number of tiles in them.
- Both bags must contain a tile with the letter M on it.
- One bag will contain at least 4 tiles worth 1 point, the other bag will have at least 3 tiles worth 1 point.
- One bag will contain at least 4 tiles worth 1 point, the other bag will have at most 3 tiles worth 1 point.
- One bag will have more points on its tiles than the other bag. ✖
- Both bags will contain at least 3 tiles worth 1 point.

Mark 0.00 out of 1.00

The correct answer is: One bag will contain at least 4 tiles worth 1 point, the other bag will have at most 3 tiles worth 1 point.

For the first part: There are 8 different letters and we need to choose 2 of them (unordered, without repetition), so the answer is the binomial coefficent $\binom{8}{2} = 28$.

For the second part we use that we know from the previous part that there are 28 sets of 2 different letters. The only other possibility is that we choose a multiset consisting of two of the same letter, which we can do in 3 ways: {M,M}, {A,A}, {T,T}. So there are $28 + 3 = 31$ possible multisets.

Another way to answer this part of the question is using the formula for unordered selection with repetition. If we had two copies of all 8 letters then the number of multisets available would be $\binom{8+2-1}{2} = \binom{9}{2} = 36$. However we only have one copy of H,E,I,C,S, so for these 5 letters we cannot choose the letter twice. Hence the answer is $36 - 5 = 31$.

One way to answer the third part is to use the multisets from the previous part, and think about how many ways we can order them. The sets of two different letters each give us two different ordered pairs, but the multisets consisting of two copies of the same letter only give us one ordered pair. Hence the answer is $2 \times 28 + 1 \times 3 = 59$.

Another way to approach this question is to split into two cases. In case 1 we choose one of H,E,I,C,S first. In that case we have only 7 different options for the second letter, so this case contributes $5 \times 7 = 35$ to the final answer.

In case 2 we choose one of M,A,T for the first letter. Since there is another copy of each of these letters, we still have 8 choices available for the second letter. That means this case contributes $3 \times 8 = 24$ to the final answer.

As these cases cover every possibility we have $35 + 24 = 59$ options in total.

For the last part of the question, there are 7 tiles worth 1 point so the average number per bag is 3.5.

The pigeonhole principle says that one bag will have at least the average, and that one bag will have at most the average. Of course a bag can't have half a tile in it, so at least 3.5 means at least 4. Likewise at most 3.5 means at most 3.

Question **2**

Correct

Mark 1.00 out of 1.00

What is the coefficient of x^3y^6 when $(\frac{x}{9} + 6y)^9$ is expanded?

Answer: 5376

The best way to answer this is to use the binomial theorem, which tells us that the coefficient of x^3y^6 in $(x+y)^9$ is $\binom{9}{3} = 84$. However, here we need to substitute $\frac{x}{9}$ for x and $6y$ for y. So rather than $84x^3y^6$ we get $84(\frac{x}{9})^3(6y)^6 = 84 \times 64x^3y^6 = 5376x^3y^6$.

The correct answer is: 5376

Question **3**

Incorrect

Mark 0.00 out of 3.00

32 travellers meet in a hostel. Among them, 20 speak French, 15 speak Spanish and 19 speak English. These counts include 12 travellers who speak Spanish and at least one of French and English. All the travellers speak at least one of the three languages and 3 of the travellers speak all three. How many travellers speak French and also speak English?

Answer: 19

Let F, S, E denote the sets of travellers who speak French, Spanish and English, respectively. We are given that $|F \cup S \cup E| = 32$, $|F| = 20$, $|S| = 15$, $|E| = 19$, $|S \cap (F \cup E)| = 12$ and $|F \cap S \cap E| = 3$.

By inclusion-exclusion we know that

$$\begin{aligned} 32 &= |F \cup S \cup E| \\ &= |F| + |S| + |E| - |F \cap S| - |F \cap E| - |S \cap E| + |F \cap S \cap E| \end{aligned}$$

Rearranging, we find that

$$\begin{aligned} |F \cap E| &= |F| + |S| + |E| - |F \cap S| - |S \cap E| + |F \cap S \cap E| - 32 \\ &= 20 + 15 + 19 - |F \cap S| - |S \cap E| + 3 - 32 \\ &= 25 - |F \cap S| - |S \cap E| \end{aligned}$$

By inclusion-exclusion again,

$$\begin{aligned} 12 = |S \cap (F \cup E)| &= |F \cap S| + |S \cap E| - |F \cap S \cap E| \\ &= |F \cap S| + |S \cap E| - 3, \end{aligned}$$

so $|S \cap F| + |S \cap E| = 15$.

Finally, we have $|F \cap E| = 25 - |F \cap S| - |S \cap E| = 10$. So the answer is 10.

The correct answer is: 10

Question **4**

Correct

Mark 3.00 out of 3.00

The numbers 1 to 11 are arranged in a line (uniformly at random). What is the probability that the first number and last number are both odd?

[Reminder that for all quizzes in this unit you should enter probabilities as **fractions** (not decimals), and you must reduce them to their lowest terms. eg 3/4 but not 0.75 or 9/12.]

Answer: 3/11

To satisfy the requirements we have 6 choices for the first number and then 5 choices for the last number. The other 9 numbers can be arranged in 9! ways, so all up there are $6 \times 5 \times 9!$ favourable arrangements.

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The number of all arrangements is 11! so the likelihood of success is $\dfrac{6 \times 5 \times 9!}{11!} = \dfrac{3}{11}$.

The correct answer is: 3/11

◄ Assignment 3 solutions

Jump to...

Assignment 4 ►