MONASH University

MAT1830 - Discrete Mathematics for Computer Science - S1 2022

Started on	Monday, 28 March 2022, 8:36 PM
State	Finished
Completed on	Wednesday, 30 March 2022, 11:55 PM
Time taken	2 days 3 hours
Grade	**10.00** out of 15.00 (**67**%)

Print friendly format

Information

Each answer to a short answer question on this quiz is an integer. Enter your answers as follows.

- Enter all integers as numerals. For example 16, 1, or 0 **BUT NOT** sixteen, 1.0, zero.
- Enter negative integers using the minus character -. For example -12 or -4 **BUT NOT** minus 12 or negative four.
- **Do not** enter anything other than the integer. For example 6 **BUT NOT** z=6.
- **No answer should contain a space**, equals sign, comma, full stop etc.

The quiz is auto-marked. Answers entered incorrectly will be marked wrong. Failure to follow instructions is not grounds for an appeal.

Question **1**

Correct

Mark 3.00 out of 3.00

Which of the following sentences is logically equivalent to $\neg\forall x\forall y(P(x,y)\rightarrow Q(x,y))$?

- ○ $\forall x\forall y(P(x,y)\wedge\neg Q(x,y))$
- ○ $\forall x\forall y(\neg Q(x,y)\rightarrow\neg P(x,y))$
- ◉ $\exists x\exists y(P(x,y)\wedge\neg Q(x,y))$ ✔
- ○ $\exists x\exists y(\neg Q(x,y)\rightarrow\neg P(x,y))$

Your answer is correct.

$$\begin{aligned}
&\neg\forall x\forall y(P(x,y)\rightarrow Q(x,y)) &\\
\equiv\ &\exists x\neg\forall y(P(x,y)\rightarrow Q(x,y)) &\\
\equiv\ &\exists x\exists y\neg(P(x,y)\rightarrow Q(x,y)) &\\
\equiv\ &\exists x\exists y\neg(\neg P(x,y)\vee Q(x,y)) &\text{by the implication law}\\
\equiv\ &\exists x\exists y(\neg\neg P(x,y)\wedge\neg Q(x,y)) &\text{by DeMorgan's laws}\\
\equiv\ &\exists x\exists y(P(x,y)\wedge\neg Q(x,y)) &\text{by the double negation law}
\end{aligned}$$

The correct answer is: $\exists x\exists y(P(x,y)\wedge\neg Q(x,y))$

Question **2**

Correct

Mark 2.00 out of 2.00

Consider the sentences $\forall x \exists y P(x,y)$ and $\exists y \forall x P(x,y)$. Are these sentences true or false under the interpretation where x and y range over the positive integers and $P(x,y)$ is "$x+y$ is even"?

- $\forall x \exists y P(x,y)$ is true, $\exists y \forall x P(x,y)$ is true
- $\forall x \exists y P(x,y)$ is true, $\exists y \forall x P(x,y)$ is false
- $\forall x \exists y P(x,y)$ is false, $\exists y \forall x P(x,y)$ is true
- $\forall x \exists y P(x,y)$ is false, $\exists y \forall x P(x,y)$ is false

Your answer is correct.

Under the given interpretation:

$\forall x \exists y P(x,y)$ states that for every positive integer x there is a positive integer y such that $x+y$ is even. This is true as we can take $y=x+2$ for example.

$\exists y \forall x P(x,y)$ states that there is one fixed positive integer y such that, for all positive integers x, we have $x+y$ is even. This is clearly false.

The correct answer is: $\forall x \exists y P(x,y)$ is true, $\exists y \forall x P(x,y)$ is false

Question **3**

Incorrect

Mark 0.00 out of 2.00

Consider the sentences $\forall x \exists y P(x,y)$ and $\exists y \forall x P(x,y)$. Are these sentences true or false under the interpretation where x and y range over the integers and $P(x,y)$ is "$y+4x$ is even"?

- $\forall x \exists y P(x,y)$ is true, $\exists y \forall x P(x,y)$ is true
- $\forall x \exists y P(x,y)$ is true, $\exists y \forall x P(x,y)$ is false
- $\forall x \exists y P(x,y)$ is false, $\exists y \forall x P(x,y)$ is true
- $\forall x \exists y P(x,y)$ is false, $\exists y \forall x P(x,y)$ is false

Your answer is incorrect.

Under the given interpretation:

$\forall x \exists y P(x,y)$ states that for every integer x there is an integer y such that $y+4x$ is even. This is true as we can take $y=2$, for example, for every choice of x.

$\exists y \forall x P(x,y)$ states that there is a fixed integer y such that, for all integers x, we have $y+4x$ is even. This is true as we can take $y=2$ for example.

The correct answer is: $\forall x \exists y P(x,y)$ is true, $\exists y \forall x P(x,y)$ is true

Question **4**

Correct

Mark 3.00 out of 3.00

Consider the sentence $\exists x (P(x) \wedge Q(x)) \rightarrow (\exists x P(x) \wedge \exists x Q(x))$. Which of the following is true?

- A. The sentence is valid.

B. The sentence is not valid. It is false under the interpretation where \(x\) ranges over the positive integers, \(P(x)\) is "\(x \equiv1 \pmod 3\)" and \(Q(x)\) is "\(x \equiv 2 \pmod 6\)".

C. The sentence is not valid. It is false under the interpretation where \(x\) ranges over the positive integers, \(P(x)\) is "\(x \equiv1 \pmod 3\)" and \(Q(x)\) is "\(x \equiv 4 \pmod 6\)".

D. The sentence is not valid. It is false under the interpretation where \(x\) ranges over the positive integers, \(P(x)\) is "\(2x \equiv1 \pmod 3\)" and \(Q(x)\) is "\(2x \equiv 3 \pmod 6\)".

Your answer is correct.

The sentence is valid.

If \(\exists x (P(x) \wedge Q(x))\) is true, then there is a choice for \(x\), \(x'\) say, such that \(P(x')\) and \(Q(x')\) are both true. In this case both \(\exists x P(x)\) and \(\exists x Q(x)\) are true (because \(x'\) exists) and so \(\exists x P(x) \wedge \exists x Q(x)\) is true.

So we have shown that, under any interpretation, if \(\exists x (P(x) \wedge Q(x))\) is true, then \(\exists x P(x) \wedge \exists x Q(x)\) is true. This shows that the overall sentence is true under any interpretation. So it is valid.

The correct answer is: The sentence is valid.

Question 5

Correct

Mark 2.00 out of 2.00

When attempting to prove by simple induction that \(2! + 4! + 6! + \cdots + (2n)! < n^{2n}\) for all integers \(n \geq 4\), the base step would consist of proving which of the following inequalities?

- \(2! + 4! < 4^{8}\)
- \(2! + 4! + 6! +8! < 8^{16}\)
- \(1!+2!+3!+4! > 4^{4}\)
- \(1!+2!+3!+4!+5!+6!+7!+8! < 4^{8}\)
- \(2! + 4! + 6! +8! > 8^{16}\)
- \(2! + 4! + 6! +8! < 4^{8}\)

Your answer is correct.

The base step of the induction would be to prove the statement for \(n=4\). Setting \(n=4\) in the statement we get \(2! + 4! + 6! +8! < 4^{8}\).

The correct answer is: \(2! + 4! + 6! +8! < 4^{8}\)

Question 6

Incorrect

Mark 0.00 out of 3.00

Suppose you know the following about a statement \(P(n)\).

- \(P(3)\), \(P(9)\) and \(P(15)\) are all true.
- \(P(2)\) is false.
- For all integers \(k \geq 6\), if \(P(k)\) is true then \(P(k+1)\) is true.

What is the smallest integer \(x\) for which you can be sure that \(P(n)\) is true for all integers \(n \geq x\)?

Answer: 3

You know $P(9)$ is true and that, for all integers $k \geq 9$, if $P(k)$ is true then $P(k+1)$ is true. This means that $P(n)$ is true for all integers $n \geq 9$ by induction.

You cannot be sure that $P(8)$ is true, however, so the smallest x can be is 9.

The correct answer is: 9

◄ Assignment 1 solutions

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Assignment 2 ►