MAT1830 - Discrete Mathematics for Computer Science
Assignment #3 Solutions

(1) (a) This is true. For any integer, there is another integer which is three times the first. **[2]**
(b) This is false. It is not the case that every integer is three times an integer. For example, 4 is not three times any integer. **[2]**
(c) This is false. It is clearly not the case that there is a single integer which is equal to $3x$ for each integer x. **[2]**

(2) Yes.

$$\begin{aligned}\neg(\exists x A(x) \to \forall x \exists y B(x,y)) &\equiv \neg(\neg\exists x A(x) \vee \forall x \exists y B(x,y))\\ &\equiv \neg\neg\exists x A(x) \wedge \neg\forall x \exists y B(x,y)\\ &\equiv \exists x A(x) \wedge \exists x \neg\exists y B(x,y)\\ &\equiv \exists x A(x) \wedge \exists x \forall y \neg B(x,y)\end{aligned}$$

[4]

(3) It's not. Think of the interpretation where $Q(x)$ is "x is even", $R(x)$ is "x is odd" and x ranges over the integers. Then $\exists x Q(x) \wedge \exists x R(x)$ is true because $\exists x Q(x)$ is true (there is an even integer) and $\exists x R(x)$ is true (there is an odd integer). But $\exists x(Q(x) \wedge R(x))$ is false (there does not exist an integer that is both even and odd). So the statement $\exists x Q(x) \wedge \exists x R(x) \leftrightarrow \exists x(Q(x) \wedge R(x))$ is false under this interpretation. **[4]**

(4) Let $P(n)$ be the statement "$7 + 7^2 + 7^3 + \cdots + 7^n = \frac{7^{n+1}-7}{6}$".

Base step. The left hand side of $P(1)$ is 7 and the right hand side of $P(1)$ is $\frac{7^2-7}{6} = 7$. So $P(1)$ is true. **[1]**

Induction step. For some integer $k \geq 1$, assume that $P(k)$ is true, that is, assume

$$7 + 7^2 + 7^7 + \cdots + 7^k = \frac{7^{k+1}-7}{6}.$$ **[1]**

Now we need to prove that $P(k+1)$ is true. So we must prove

$$7 + 7^2 + 7^7 + \cdots + 7^{k+1} = \frac{7^{k+2}-7}{6}.$$ **[1]**

Working with the left hand side of this equation we see that

$$\begin{aligned}7 + 7^2 + 7^7 + \cdots + 7^{k+1} &= (7 + 7^2 + 7^7 + \cdots + 7^k) + 7^{k+1}\\ &= \left(\frac{7^{k+1}-7}{6}\right) + 7^{k+1} \quad \text{(using our assumption)}\\ &= \frac{7^{k+1}-7}{6} + \frac{6(7^{k+1})}{6}\\ &= \frac{7(7^{k+1})-7}{6}\\ &= \frac{7^{k+2}-7}{6},\end{aligned}$$

which is the right hand side we required.

So we have proved by induction that $P(n)$ is true for each integer $n \geq 1$. **[3]**