MAT1830 - Discrete Mathematics for Computer Science
Assignment #5 Solutions

1. (i) $f(\{2,3,4\}) = \{2,3,4\} \cap \{3,4\} = \{3,4\}$. **[1]**

   (ii) $g(\{1,2,3\}) = 3$. **[1]**

   (iii) $h(4) = \{1,2,3,4\} - \{4\} = \{1,2,3\}$. **[1]**

   (iv) $f \circ h(3) = f(h(3)) = f(\{1,2,4\}) = \{1,2,4\} \cap \{3,4\} = \{4\}$. **[1]**

   (Answer only required.)

2. (i) f is not one-to-one. For example, $f(\{1,2\}) = \{\}$ and $f(\{1\}) = \{\}$. **[2]**

   (ii) g is not one-to-one. For example, $g(\{1,3\}) = 3$ and $g(\{1,2,3\}) = 3$. **[2]**

   (iii) h is one-to-one because

   $$h(x_1) = h(x_2) \Rightarrow \{1,2,3,4\} - \{x_1\} = \{1,2,3,4\} - \{x_2\} \Rightarrow x_1 = x_2. \quad \textbf{[2]}$$

   (The last step of this argument works because if $x_1 \neq x_2$ then $x_2 \in \{1,2,3,4\} - \{x_1\}$ and so $\{1,2,3,4\} - \{x_1\} \neq \{1,2,3,4\} - \{x_2\}$.)

   (No marks without an attempt at justification.)

3. (i) $\text{range}(f) = \mathcal{P}(\{3,4\}) = \{\{\},\{3\},\{4\},\{3,4\}\}$. **[1]**
   $\text{range}(f) \subseteq \mathcal{P}(\{3,4\})$ because $f(X) \subseteq \{3,4\}$ for all $X \in A$.
   $\mathcal{P}(\{3,4\}) \subseteq \text{range}(f)$ because $f(X) = X$ for all $X \in \mathcal{P}(\{3,4\})$. **[1]**

   (ii) $\text{range}(g) = \{-1,1,2,3,4\}$ **[1]**
   $\text{range}(g) \subseteq \{-1,1,2,3,4\}$ clearly.
   $\{-1,1,2,3,4\} \subseteq \text{range}(g)$ because $g(\{\}) = -1$ and $g(\{x\}) = x$ for all $x \in \{1,2,3,4\}$. **[1]**

   (iii) $\text{range}(h) = \{h(1),h(2),h(3),h(4)\} = \{\{2,3,4\},\{1,3,4\},\{1,2,4\},\{1,2,3\}\}$ **[2]**

4. (i) No because $\text{codomain}(g) \neq \text{domain}(f)$. **[1]**

   (ii) Yes because $\text{codomain}(f) = \text{domain}(g)$. $g \circ f : A \to \mathbb{Z}$. **[1]**
   $\text{range}(g \circ f) = \{-1,3,4\}$. **[1]**
   This is because

   $$\begin{aligned}
   \text{range}(g \circ f) &= \{g(f(X)) : X \in A\} \\
   &= \{g(Y) : Y \in \text{range}(f)\} \\
   &= \{g(Y) : Y \in \mathcal{P}(\{3,4\})\} \\
   &= \{g(\{\}), g(\{3\}), g(\{4\}), g(\{3,4\})\} \\
   &= \{-1,3,4\}. \quad \textbf{[1]}
   \end{aligned}$$