

MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #5 and Additional Practice Questions

Tutorial Questions

1. Let $X = \{a, b, c, d\}$ and $Y = \{a, b, e, f, g, h\}$.
 - (a) Make up and draw the arrow diagram of a function $q : X \rightarrow Y$ whose image contains 3 elements, or explain why no such function exists.
 - (b) Make up and draw the arrow diagram of a function $r : Y \rightarrow X$ which is one-to-one, or explain why no such function exists.
 - (c) Make up and draw the arrow diagram of a function $s : Y \rightarrow X$ such that $s(a) = a$, $s(b) = a$, $s(e) = a$ and the image of s is X , or explain why no such function exists.

2. Let p , q and s be the following functions.

$$p : \mathbb{Z} \rightarrow \mathbb{Z} \text{ defined by } p(x) = \begin{cases} 1, & \text{if } x \text{ is even;} \\ -1, & \text{if } x \text{ is odd.} \end{cases}$$

$$q : \mathbb{Z} \rightarrow \mathbb{R} \text{ given by the set } \{(x, x^2 - \frac{1}{2}) : x \in \mathbb{Z}\}$$

$$s : \{x : x \in \mathbb{R} \text{ and } x \geq 0\} \rightarrow \mathbb{R} \text{ defined by } s(x) = \sqrt{x} + 2$$

$$t : \mathbb{Z} \rightarrow \mathbb{N} \times \{-1, 1\} \text{ defined by } t(x) = (y, z) \text{ where } y = x^2 \text{ and } z = \begin{cases} 1, & \text{if } x \geq 0; \\ -1, & \text{if } x < 0. \end{cases}$$

- (a) What are the images of p , q , s and t ? Which of them are onto?
 - (b) Which of p , q , s and t are one-to-one?
 - (c) Do p and q have inverse functions? If they do, give a formula for the function and give its domain, codomain, and image.
3. Let p , q , s and t be the functions defined in Question 2. State whether each of the following compositions exists and, if they do, give a formula for the function and give its domain, codomain, and image.
 - (a) $p \circ q$
 - (b) $q \circ t$
 - (c) $q \circ p$
4. Let D be the set of all dogs.

- (a) Which of the following subsets of $D \times D$ correspond to functions from D to D ? Why or why not?
 - i. $\{(x, y) : y \text{ is the mother of } x\}$
 - ii. $\{(x, y) : y \text{ is the brother of } x\}$
 - iii. $\{(x, y) : y \text{ is the eldest dog in the same litter as } x\}$
 - iv. $\{(x, y) : y \text{ is the eldest daughter of } x\}$
 - (b) Let $m : D \rightarrow D$ and $e : D \rightarrow D$ be the functions corresponding to i and iii in part (a). What are the following?
 - i. $m \circ m(\text{Rover})$
 - ii. $e \circ m(\text{Rover})$
 - iii. $m \circ e(\text{Rover})$

(See over for practice questions.)

Practice Questions

1. Let $A = \{1, 2\}$ and $B = \{10, 11, 12\}$. Write down the set of ordered pairs which corresponds to the function $f : A \times B \rightarrow \mathbb{N}$ defined by $f((a, b)) = a^2b$.
2. Which of the following rules correspond to functions. If they do, are those functions one-to one?
 - (a) For each banana b , let $\ell(b)$ be the length of b in centimetres (rounded down).
 - (b) For each circle C in the (x, y) -plane whose centre is the origin, let $r(C)$ be the radius of C .
 - (c) For each latitude-longitude pair (x, y) , let $C((x, y))$ be the set of all coffee shops within one kilometer of (x, y) .
 - (d) For each set of integers X , let $m(X)$ be the smallest integer in X .
3. Let P be the set of all propositional logic sentences. Let $d : P \rightarrow \mathbb{R}$ be a function defined by $d(\psi) = \frac{c}{2^n}$ where c is the number of interpretations under which ψ is true and n is the number of different variables in ψ .
 - (a) What is $d(p \wedge q)$? What is $d(p \wedge (q \vee r))$? (where p, q and r are variables)
 - (b) What can we say about ψ if $d(\psi) = 1$?
 - (c) Can you think of a way to think of d as a probability?
 - (d) What do you think the image of d is?
 - (e) Can you prove your answer to (d)?
4.
 - (a) Find a function $f : \mathbb{N} \rightarrow \mathbb{Z}$ which is one-to-one and has image \mathbb{Z} .
 - (b) Do you think your answer to (a) means that there is the “same number” of natural numbers as integers?
 - (c) Do you think your answer to (a) means that the natural numbers are “the same as the integers, just in disguise”?