

MAT1830 - Discrete Mathematics for Computer Science
Assignment #4

To be handed in at the beginning of your support class in week 6 (3–7 April)

Show your working and give full explanations for all questions.

- (1) Define a sequence of integers a_1, a_2, a_3, \dots by

$$a_1 = 4, \quad a_2 = 12, \quad \text{and} \quad a_n = 10a_{n-1} - 12a_{n-2} \text{ for each integer } n \geq 3.$$

Prove by strong induction that 2^n divides a_n for all integers $n \geq 1$.

- (2) Let R, S and T be sets defined as follows

$$R = \{x : x \in \mathbb{Z} \text{ and either } x \leq -4 \text{ or } x \geq 3\}$$

$$S = \{-6, -5, -3, 3, 4, 5\}$$

$$T = \{x : x \in \mathbb{Z} \text{ and } x \geq 0\}.$$

Find the following.

- (i) $R \cap S$
- (ii) $R - T$
- (iii) $R \Delta S$
- (iv) $\mathcal{P}(R) \cap \{\{-6, -5, 3\}, \{2, 3\}, \{5\}, \{\}, \{-5, 1, 4\}\}$
- (v) $|\mathcal{P}(\mathcal{P}(S \cap T))|$

No explanation is required for (i)–(iv). Give some explanation for (v).

- (3) (i) Is $(A \cup B) \times C = (A \times C) \cup (B \times C)$ true for all sets A, B and C ? If so, prove it. If not, give an example of sets A, B and C for which it is false.
- (ii) Is $\mathcal{P}(A) \Delta \mathcal{P}(B) = \mathcal{P}(A \Delta B)$ true for all sets A and B ? If so, prove it. If not, give an example of sets A and B for which it is false.

Hint: A good way to prove that two sets are equal is to use laws of logic to show that they have exactly the same elements.