

**MAT1830 - Discrete Mathematics for Computer Science**  
**Assignment #3**

To be handed in at the beginning of your support class in week 5 (27 – 31 Mar)

Show your working and give full explanations for all questions.

(1) Let  $P(x, y)$  be the predicate “ $y = 3x$ ”. Consider the statements

(a)  $\forall x \exists y P(x, y)$

(b)  $\forall y \exists x P(x, y)$

(c)  $\exists y \forall x P(x, y)$

where  $x$  and  $y$  range over the integers.

Write whether each statement is true or false and give a very short explanation of why.

(2) Are the sentences

$$\neg(\exists x A(x) \rightarrow \forall x \exists y B(x, y)) \quad \text{and} \quad \exists x A(x) \wedge \exists x \forall y \neg B(x, y)$$

logically equivalent? If they are equivalent, prove that they are. If not, give an interpretation under which they have different truth values.

(3) Is the sentence

$$(\exists x Q(x) \wedge \exists x R(x)) \leftrightarrow \exists x (Q(x) \wedge R(x))$$

valid? If it is, explain why. If it isn't, give an interpretation under which it is false.

(4) Prove using simple induction that, for each integer  $n \geq 1$ ,

$$7 + 7^2 + 7^3 + \cdots + 7^n = \frac{7^{n+1} - 7}{6}.$$