MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #10 and Additional Practice Questions

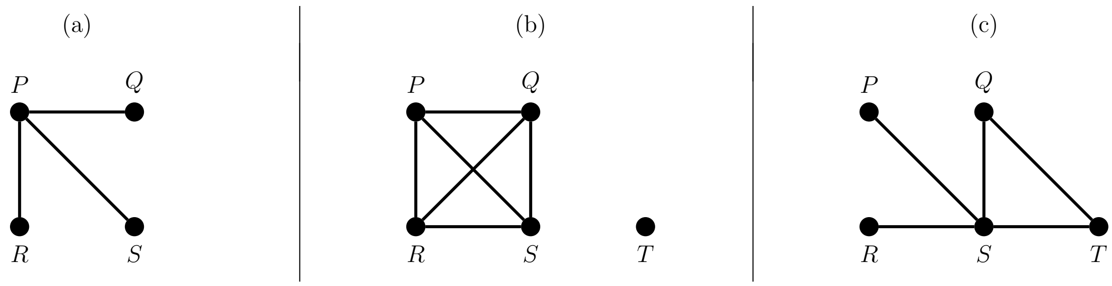
Tutorial Questions

1. Find recursive definitions for the following.

   (a) The sequence $a_0, a_1, a_2, \ldots$ where $a_n = 2^n$ for $n \geq 0$.

   (b) The sequence $b_0, b_1, b_2, \ldots$ where $b_n = n^2$ for $n \geq 0$. (Your recurrence may involve n, but not n^2.)

2. Give the vertex sets and edge sets for the following graphs.



3. Let s_n be the number of ways (order being important) of writing n as a sum of 1s and 2s. For example $s_4 = 5$ because 4 can be written in five ways:

$$1+1+1+1, \quad 1+1+2, \quad 1+2+1, \quad 2+1+1, \quad 2+2.$$

   Find a recurrence for s_n.

4. (a) Draw the simple graph with adjacency matrix

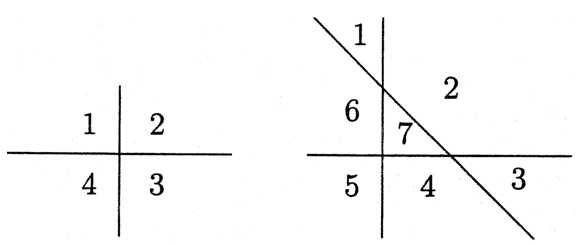
$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

   using V_1, V_2, V_3, V_4 as the names for the vertices corresponding to columns 1, 2, 3, 4 respectively.

   (b) Find the number of walks of length 3 from V_1 to V_2 in the graph.

   (c) Without any calculation show that the top row of M^n for any even $n \geq 2$ is "1 0 0 0".

5. Let r_n be the number of regions created when the plane is divided by n straight (infinite) lines, with no two lines parallel and no three meeting in a single point. For example,



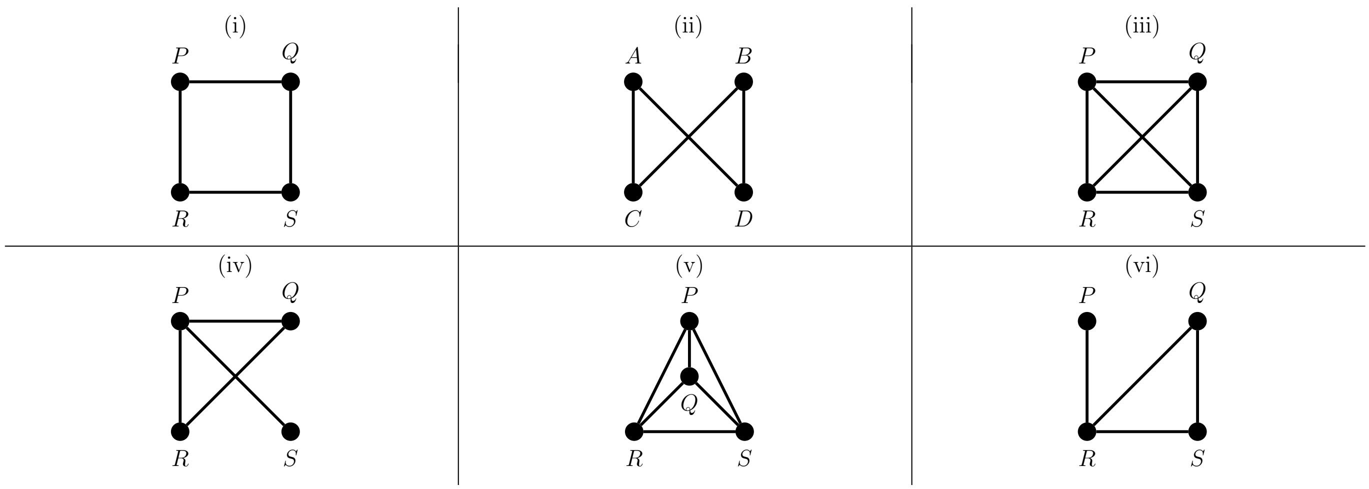
$r_2 = 4$ $\qquad$ $r_3 = 7$

   Find a recurrence for r_n.

(See over for practice questions.)

Practice Questions

1. Two graphs are equal if they have exactly the same vertex and edge sets. They are isomorphic if we can "rename" the vertices of one graph to make it equal to the other. Which of the following graphs are equal? Which are isomorphic? How would you prove this?



2. How would you change the definition of isomorphic graphs given above to make it more formal?

3. (a) What would you do if you saw someone drive past in a car that was isomorphic to yours?
   (b) What would you do if you saw someone drive past in a car that was equal to yours?

4. Suppose you want to network some computers together in such a way that

   - each computer is directly connected to at most three others; and
   - any two computers are either directly connected or are both directly connected to some third computer.

   Can you find a way to network 7 computers like this? 8? 9? 10?

   (For a way to do this for 10 computers, google "Petersen graph")

5. Let $S(n, k)$ be the number of equivalence relations on the set $\{1, 2, \ldots, n\}$ with exactly k (non-empty) equivalence classes. Prove that $S(n, k) = kS(n-1, k) + S(n-1, k-1)$ for all integers n and k such that $n > k > 1$.

   ($S(n, k)$ are sometimes called *Stirling numbers of the second kind.*)