MONASH University

MAT1830 - Discrete Mathematics for Computer Science - S1 2022

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Started on	Monday, 11 April 2022, 6:19 PM
State	Finished
Completed on	Wednesday, 13 April 2022, 11:55 PM
Time taken	2 days 5 hours
Grade	**3.67** out of 15.00 (**24**%)

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Information

Each answer to a short answer question on this quiz is an **integer**, a **set**, an **ordered pair**, or **NA**. Enter your answers as follows.

Integers: Enter these as numerals, using the minus character where necessary, like on previous quizzes.
For example 16 or 1 or 0 or -12 **BUT NOT** sixteen or 1.0 or zero or minus 12.

Sets: Enter these between curly brackets with elements listed explicitly, separated by commas and **no spaces**. Any ordering of the elements is okay.
For example {1,2,3,4} or {2,3,4} or {2} or {} **BUT NOT** {1,...,4} or {x:1<x<5} or 2 or emptyset. If the answer is {2,3,4} we will also accept {3,4,2} etc.

Ordered pairs: Enter these between round brackets with the coordinates separated by a comma and **no spaces**. The ordering of the coordinates is important, of course.
For example (6,5) or (0,12) **BUT NOT** (six,five) or (zero,12).

Sets and ordered pairs can be nested and combined, for example {(0,1),(1,2)} is a possible answer corresponding to a set of two ordered pairs and {{},{1}} is a possible answer corresponding to a set of two sets.

NA: Enter this just as the two capital letters NA with **no spaces**. The question will tell you when this is an acceptable answer.

Remember:

- **Do not** enter anything other than the answer. For example {1,2} **BUT NOT** z={1,2}.
- **No answer should contain a space**, **equals sign**, **full stop** etc.

The quiz is auto-marked. Answers entered incorrectly will be marked wrong. Failure to follow the above instructions will not be grounds for marks to be adjusted.

Question **1**

Incorrect

Mark 0.00 out of 2.00

Let $f : \mathcal{P}(\mathbb{Z} \times \mathbb{Z}) \to \mathbb{Z}$ be a function. Which of the following correctly gives an example of an element from its domain and an element from its codomain?

- ○ $\{6\}$ is an element of the domain and 9 is an element of the codomain.
- ○ $(3,6)$ is an element of the domain and 15 is an element of the codomain.
- ○ $(\{1,3\},\{7\})$ is an element of the domain and $\{5,7\}$ is an element of the codomain.

- $\{\}$ is an element of the domain and $\{\}$ is an element of the codomain.
- $(\{1,4,6\},\{2,7\})$ is an element of the domain and 30 is an element of the codomain.
- $\{3,29\}$ is an element of the domain and 11 is an element of the codomain.
- $\{(2,4),(7,3)\}$ is an element of the domain and $\{6,9\}$ is an element of the codomain. ✖
- $\{(1,3),(7,2)\}$ is an element of the domain and 62 is an element of the codomain.

Your answer is incorrect.

The domain of f is $\mathcal{P}(\mathbb{Z}\times\mathbb{Z})$. Now $\mathbb{Z}\times\mathbb{Z}$ is the set of all ordered pairs of integers, and so $\mathcal{P}(\mathbb{Z}\times\mathbb{Z})$ is the set of all sets of ordered pairs of integers. So an element of the domain must be a set of ordered pairs of integers.

The codomain of f is $\mathbb{Z}$. Now $\mathbb{Z}$ is the set of all integers. This means that an element of the codomain must be an integer.

Only the correct answer obeys both of these facts.

The correct answer is: $\{(1,3),(7,2)\}$ is an element of the domain and 62 is an element of the codomain.

Question 2

Partially correct

Mark 2.67 out of 4.00

Let $f:\mathcal{P}(\mathbb{N})\to\mathcal{P}(\mathbb{N})$ be the function defined by $f(X)=X\cup\{5,6\}$.

Enter the value $f(\{3,5\})=$ {3,5,6} ✔

Is f one-to-one?

- Yes
- No ✔

Mark 1.00 out of 1.00

The correct answer is: No

What is the image of f?

- $\mathcal{P}(\mathbb{N})$ ✖
- $\mathcal{P}(\{5,6\})$
- $\{5,6\}$
- $\mathbb{N}$
- $\{X : X\in\mathcal{P}(\mathbb{N}) \text{ and } \{5,6\}\subseteq X\}$
- None of the above

Mark 0.00 out of 1.00

The correct answer is: $\{X : X\in\mathcal{P}(\mathbb{N}) \text{ and } \{5,6\}\subseteq X\}$

$f(\{3,5\})=\{3,5\}\cup\{5,6\}=\{3,5,6\}$.

f is not one-to-one. For example $f(\{3,5\})=\{3,5,6\}$ and $f(\{3,6\})=\{3,5,6\}$.

For any element X of $\mathcal{P}(\mathbb{N})$ we have that $f(X)=X\cup\{5,6\}$ contains the elements of 5 and 6. So the image of f must be a subse
of $\{X : X\in\mathcal{P}(\mathbb{N}) \text{ and } \{5,6\}\subseteq X\}$. Also for each Y in $\{X : X\in\mathcal{P}(\mathbb{N}) \text{ and } \{5,6\}\subseteq X\}$ we have $f(Y)=Y$. So the image

of $\{X : X \in \mathcal{P}(\mathbb{N})$ and $\{5,6\} \subseteq X\}$. Also, for each Y in $\{X : X \in \mathcal{P}(\mathbb{N})$ and $\{5,6\} \subseteq X\}$, we have $f(Y) = Y$. So the image of f is $\{X : X \in \mathcal{P}(\mathbb{N}) \text{ and } \{5,6\} \subseteq X\}$.

Question 3

Partially correct

Mark 1.00 out of 6.00

Let $S = \{1,2,3,\ldots,10\}$ and let f and g be the following functions.

$f : \mathcal{P}(S) \to \mathcal{P}(S)$ defined by $f(X) = \{a \in S : a+3 \in X\}$.

$g : \mathcal{P}(S) \to \mathbb{Z}$ defined by $g(X) = |X|$.

If $f \circ f$ exists, then evaluate $f \circ f(\{2,3,8,10\})$. If $f \circ f$ does not exist, then enter NA. {8,9} ✖

If $f \circ g$ exists, then evaluate $f \circ g(\{4,5\})$. If $f \circ g$ does not exist, then enter NA. {7,8} ✖

If $g \circ f$ exists, then evaluate $g \circ f(\{3,8,9\})$. If $g \circ f$ does not exist, then enter NA. NA ✖

If $g \circ g$ exists, then evaluate $g \circ g(\{5,6\})$. If $g \circ g$ does not exist, then enter NA. NA ✔

$f \circ f$ exists because the codomain of f and the domain of f are both $\mathcal{P}(S)$.
$f \circ f(\{2,3,8,10\}) = f(\{5,7\}) = \{2,4\}$.

$f \circ g$ does not exist because $\mathbb{Z}$ is the codomain of g and $\mathcal{P}(S)$ is the domain of f, and these are not equal.

$g \circ f$ exists because the codomain of f and the domain of g are both $\mathcal{P}(S)$.
$g \circ f(\{3,8,9\}) = g(\{5,6\}) = 2$.

$g \circ g$ does not exist because $\mathbb{Z}$ is the codomain of g and $\mathcal{P}(S)$ is the domain of g, and these are not equal.

Question 4

Incorrect

Mark 0.00 out of 3.00

Are the following statements true for all functions f and for all subsets A and B of the domain of f?

(i) if $\{f(x) : x \in A\} \cap \{f(x) : x \in B\} = \emptyset$, then $A \cap B = \emptyset$

(ii) if $A \cap B = \emptyset$, then $\{f(x) : x \in A\} \cap \{f(x) : x \in B\} = \emptyset$

- Yes for (i). Yes for (ii).
- Yes for (i). No for (ii).
- No for (i). Yes for (ii).
- No for (i). No for (ii). ✖

Mark 0.00 out of 1.00

The correct answer is: Yes for (i). No for (ii).

Yes for (i). Let f be any function and A and B be any subsets of the domain of f. We will prove the contrapositive of (i). Suppose that $A \cap B \neq \emptyset$. Then there is an x' such that $x' \in A$ and $x' \in B$. So $f(x')$ is an element of $\{f(x) : x \in A\}$ and $f(x')$ is an element of $\{f(x) : x \in B\}$. So $f(x')$ is an element of $\{f(x) : x \in A\} \cap \{f(x) : x \in B\}$ and hence $\{f(x) : x \in A\} \cap \{f(x) : x \in B\} \neq \emptyset$

We have shown that if $A \cap B \neq \emptyset$, then $\{f(x) : x \in A\} \cap \{f(x) : x \in B\} \neq \emptyset$, which is the same as showing (i).

No for (ii). Let $f : \{-1, 0, 1\} \to \{0, 1\}$ be the function defined by $f(x) = x^2$ and let $A = \{-1\}$ and $B = \{1\}$. Then $A \cap B = \emptyset$ but $\{f(x) : x \in A\} = \{f(x) : x \in B\} = \{1\}$ so $\{f(x) : x \in A\} \cap \{f(x) : x \in B\} \neq \emptyset$.

◄ Assignment 2 solutions

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Assignment 3 ►