MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #5 Solutions

1. When solutions exist there are lots of possibilities. I'll give one possible function as a set of ordered pairs and leave you to draw the diagrams.

   (a) $\{(a,a),(b,a),(c,b),(d,e)\}$.

   (b) No such function exists. Any function from Y (which has 6 elements) to X (which has 4 elements) must obviously map two elements of Y to the same element of X.

   (c) $\{(a,a),(b,a),(e,a),(f,b),(g,c),(h,d)\}$.

2. (a) The image of p is $\{-1,1\}$. So p is not onto.
   The image of q is $\{a^2-\frac{1}{2} : a \in \mathbb{Z}\} = \{-\frac{1}{2}, \frac{1}{2}, 3\frac{1}{2}, 8\frac{1}{2}, \ldots\}$. So q is not onto.
   The image of s is $\{a : a \in \mathbb{R} \text{ and } a \geq 2\}$. So s is not onto.
   The image of t is $\{(0,1)\} \cup (\{a^2 : a \in \mathbb{Z}, a \geq 1\} \times \{-1,1\})$. So t is not onto.

   (b) p is not one-to-one since $p(2) = 1$ and $p(10) = 1$, for example.
   q is not one-to-one since $q(1) = \frac{1}{2}$ and $q(-1) = \frac{1}{2}$, for example.
   s is one-to-one since if $s(x_1) = s(x_2)$ for $x_1, x_2 \in \{x : x \in \mathbb{R} \text{ and } x \geq 0\}$, then $\sqrt{x_1} + 2 = \sqrt{x_2} + 2$ and so $\sqrt{x_1} = \sqrt{x_2}$ and $x_1 = x_2$ (squaring both sides).
   t is one-to-one. Let $x_1, x_2 \in \mathbb{Z}$ and let $t(x_1) = (y_1, z_1)$ and $t(x_2) = (y_2, z_2)$. Because $y_1 = y_2$, $(x_1)^2 = (x_2)^2$ and so $|x_1| = |x_2|$. Because $z_1 = z_2$ either both x_1 and x_2 are at least 0 or both x_1 and x_2 are less than 0. Thus $x_1 = x_2$.

   (c) p cannot have an inverse function because it is not one-to-one.
   q cannot have an inverse function because it is not one-to-one.

3. (a) $p \circ q$ does not exist because $\text{codomain}(q) \neq \text{domain}(p)$.

   (b) $q \circ t$ does not exist because $\text{codomain}(t) \neq \text{domain}(q)$.

   (c) $q \circ p$ exists because $\text{codomain}(p) = \text{domain}(q)$.
   $q \circ p : \mathbb{Z} \to \mathbb{R}$ is defined by $q \circ p(x) = \frac{1}{2}$ because

$$q(p(x)) = \begin{cases} 1^2 - \frac{1}{2}, & \text{if } x \text{ is even;} \\ (-1)^2 - \frac{1}{2}, & \text{if } x \text{ is odd.} \end{cases}$$

   Its image is $\{\frac{1}{2}\}$.

4. (a) i. Yes – each dog has exactly one mother.
   ii. No – some dogs may have more than one brother or no brothers.
   iii. Yes – each dog will have exactly one eldest "sibling" (perhaps itself).
   iv. No – some dogs may have no daughters.

   (b) i. Rover's maternal grandmother. (That is, Rover's mother's mother.)
   ii. The eldest dog in the same litter as Rover's mother.
   iii. Rover's mother (because all the dogs in a litter have the same mother).