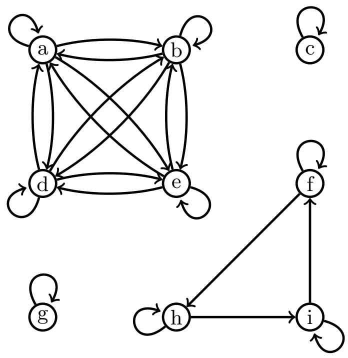
MAT1830 - Discrete Mathematics for Computer Science
Assignment #3
Submit by uploading a pdf to moodle by 11:55pm Wednesday in week 8

Assessment questions/solutions for this unit must not be posted on any website.

For Question 1, each 'no' needs an accompanying example. For Question 2, full justifications are required.

(1) Let E be the binary relation on the set $\{a, b, c, d, e, f, g, h, i\}$ pictured below. Write down whether E is reflexive, symmetric, antisymmetric, transitive. When E does not have one of these properties give an example of why not.



[Each 'no' needs an accompanying example.] **[4]**

ANS: E is reflexive.
E is not symmetric (for example, iEf but $f \not{E} i$).
E is not antisymmetric (for example, aEb and bEa).
E is not transitive (for example, fEh and hEi but $f \not{E} i$).
[No marks without examples for the last three.]

(2) Let R be a binary relation on $\mathcal{P}(\{1, \ldots, 50\}) - \{\emptyset\}$ defined by XRY if and only if $\text{sum}(X) \leq \text{sum}(Y)$, where $\text{sum}(X)$ is the sum of the elements of X and $\text{sum}(Y)$ is the sum of the elements of Y.
Is R reflexive? Is R symmetric? Is R antisymmetric? Is R transitive?
[Fully justify each answer.] **[8]**

ANS: R is reflexive. For each $X \in \mathcal{P}(\{1, \ldots, 50\}) - \{\emptyset\}$ we have XRX because $\text{sum}(X) \leq \text{sum}(X)$.
R is not symmetric. For example $\{1, 2\}R\{2, 3\}$ and $\{2, 3\} \not{R} \{1, 2\}$.
R is not antisymmetric. For example $\{1, 4\}R\{2, 3\}$ and $\{2, 3\}R\{1, 4\}$ but $\{2, 3\} \neq \{1, 4\}$.
R is transitive. For all $X, Y, Z \in \mathcal{P}(\{1, \ldots, 50\}) - \{\emptyset\}$ if XRY and YRZ, then $\text{sum}(X) \leq \text{sum}(Y)$ and $\text{sum}(Y) \leq \text{sum}(Z)$. So $\text{sum}(X) \leq \text{sum}(Z)$ and hence XRZ.
[No marks for just an answer without explanation.]

(3) Let S be the equivalence relation on $\{-1,0,1\} \times \{0,1,2,3\}$ defined by $(a,b)S(c,d)$ if and only if $a^2+b = c^2+d$. Write down the equivalence classes of S.

[Answer only required.] **[4]**

ANS: The equivalence classes are
$\{(0,0)\}$,
$\{(-1,0),(1,0),(0,1)\}$,
$\{(-1,1),(1,1),(0,2)\}$,
$\{(-1,2),(1,2),(0,3)\}$, and
$\{(-1,3),(1,3)\}$.

(One way to get the equivalence classes is to begin by picking any element of $\{-1,0,1\} \times \{0,1,2,3\}$, finding every element related to it and putting all of those elements together in a set which becomes the first equivalence class. Now pick an element not in the first equivalence class and repeat the process to find the second equivalence class and so on. When every element is in one of the classes, we're finished.

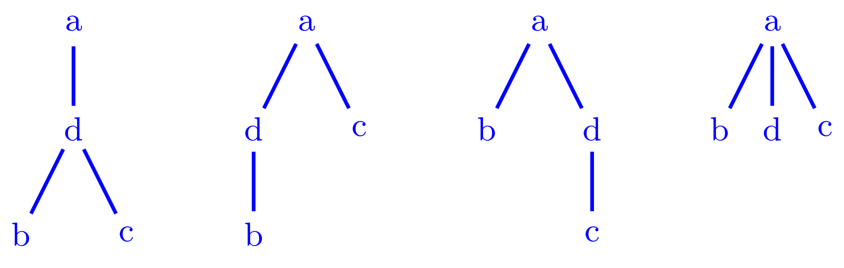
For example, if we first pick $(1,2)$ we find that the elements related to it are the pairs $(a,b) \in \{-1,0,1\} \times \{0,1,2,3\}$ such that $a^2+b=3$: $(1,2),(-1,2),(0,3)$. These form our first class. Next we pick $(1,3)$ and find that the elements related to it pairs $(a,b) \in \{-1,0,1\} \times \{0,1,2,3\}$ such that $a^2+b=4$: $(1,3),(-1,3)$. These form our second class. And so on.)

(4) State how many possible partial order relations U there are on the set $\{a,b,c,d\}$ such that

- bUa, cUa, dUa; and
- $b \not{U} c$, $c \not{U} b$, $d \not{U} b$ and $d \not{U} c$.

Draw a Hasse diagram for each possible relation. **[Answer only required.]** **[4]**

ANS: There are four. Their Hasse diagrams are as follows.



Because U is a partial order relation, it must be reflexive, antisymmetric and transitive. Reflexivity means that xUx for each $x \in \{a,b,c,d\}$ and antisymmetry means that $a \not{U} b$, $a \not{U} c$ and $a \not{U} d$. So the only relationships we don't know about are whether bUd and whether cUd.

- If bUd and cUd, then U must be given by the leftmost diagram.
- If bUd and $c \not{U} d$, then U must be given by the 2nd diagram from the left.
- If $b \not{U} d$ and cUd, then U must be given by the 2nd diagram from the right.
- If $b \not{U} d$ and $c \not{U} d$, then U must be given by the rightmost diagram.