MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #9 Solutions

1. (a) Geometric.
   (b) This is $\Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2)$. Using the geometric distribution formula this is
   $$\tfrac{1}{10} + \tfrac{1}{10}(\tfrac{9}{10})^1 + \tfrac{1}{10}(\tfrac{9}{10})^2 = \tfrac{1}{10} + \tfrac{9}{100} + \tfrac{81}{1000} = \tfrac{271}{1000} = 27.1\%.$$
   (c) The Romulan ship is destroyed by one of Kirk's first three torpedoes if and only if one of the following (mutually exclusive) events occurs.
   - The first torpedo destroys the ship. The probability of this is $\frac{1}{10}$.
   - The first torpedo doesn't destroy the ship, but the second does. The probability of this is $\frac{9}{10} \times \frac{1}{10} = \frac{9}{100}$.
   - The first two torpedoes don't destroy the ship, but the third does. The probability of this is $\frac{9}{10} \times \frac{9}{10} \times \frac{1}{10} = \frac{81}{1000}$.

   (d) All else equal we'd expect X to be smaller (Spock is a better marksman so we'd expect he'd waste fewer missiles).

2. (a)
   $$\begin{array}{llll} r_1 & = 2r_0 - 1 & = 2(3) - 1 & = 5 \\ r_2 & = 2r_1 - 1 & = 2(5) - 1 & = 9 \\ r_3 & = 2r_2 - 1 & = 2(9) - 1 & = 17 \\ r_4 & = 2r_3 - 1 & = 2(17) - 1 & = 33 \end{array}$$
   (b)
   $$\begin{array}{llll} t_3 & = t_2 t_0 & = (-2)(1) & = -2 \\ t_4 & = t_3 t_1 & = (-2)(1) & = -2 \\ t_5 & = t_4 t_2 & = (-2)(-2) & = 4 \\ t_6 & = t_5 t_3 & = (4)(-2) & = -8 \end{array}$$

3. (a) $\frac{1}{13} + \frac{1}{15} + \frac{1}{17} + \frac{1}{19} + \frac{1}{21}$
   (b) $(\frac{x^4}{8} + 4)(\frac{x^5}{10} + 5)(\frac{x^6}{12} + 6)$
   (c) $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$

4. (a) Binomial.
   (b) Using the formulas for the expected value and variance of the binomial distribution with $n = 400$ and $p = 0.025$, $\mathrm{E}[Y] = np = 10$ and $\mathrm{Var}[Y] = np(1-p) = 9.75$.
   (c) Using the formula for the binomial distribution $\Pr(Y = 10) = \binom{400}{10}(0.025)^{10}(0.975)^{390} \approx 12.67\%$.
   (d) $\displaystyle\sum_{i=5}^{20} \binom{400}{i} (0.025)^i (0.975)^{400-i}$
   (e) Using the formulas for the expected value and variance of the binomial distribution with $n = 25$ and $p = 0.4$, $\mathrm{E}[Z] = np = 10$ and $\mathrm{Var}[Z] = np(1-p) = 6$.
   (f) Both Y and Z have expected value 10, but Z has a smaller variance than Y so we would expect the values of Z to cluster a little more closely around 10 than those of Y. Based on this and eyeballing the table, I'd guess that the first row is Aperture and the second Umbrella. Have a think about various ways you might formalise this.