MAT1830 - Discrete Mathematics for Computer Science
Assignment #6 Solutions

1. E is reflexive and symmetric and transitive.
   E is not antisymmetric (for example, aEb and bEa). **[2]**

   F is reflexive and antisymmetric and transitive.
   F is not symmetric (for example, aFb and $b \not{F} a$). **[2]**

   G is not reflexive (for example, $f \not{G} f$).
   G is not symmetric (for example, aGb and $b \not{G} a$).
   G is not antisymmetric (for example, fGi and iGf).
   G is not transitive (for example, fGi and iGh but $f \not{G} h$). **[2]**

2. E is an equivalence relation. The equivalence classes of E are $\{a, b, d, e\}$, $\{f, h, i\}$, $\{c\}$ and $\{g\}$. **[1]**

3. F is a partial order relation. F is not a total order relation because, for example, $b \not{F} e$ and $e \not{F} b$. **[1]**

4. R is not reflexive. For example, $|\{1, 2, 3\} \cap \{1, 2, 3\}| = |\{1, 2, 3\}| = 3$ and so $\{1, 2, 3\} \not{R} \{1, 2, 3\}$. **[2]**
   R is symmetric because, for all $A, B \in \mathcal{P}(\mathbb{N})$, if ARB, then $|A \cap B| \leq 2$, and so $|B \cap A| = |A \cap B| \leq 2$ and BRA. **[2]**
   R is not antisymmetric. For example $\{1\}R\{2\}$ and $\{2\}R\{1\}$. **[2]**
   R is not transitive. For example $\{1, 2, 3, 4\}R\{9\}$ and $\{9\}R\{2, 3, 4, 5\}$ but $\{1, 2, 3, 4\} \not{R} \{2, 3, 4, 5\}$. **[2]**
   [No marks for just an answer without explanation.]

5. The equivalence classes are $\{0\}$, $\{x : x \in \mathbb{R} \text{ and } 0 < x \leq 1\}$, $\{x : x \in \mathbb{R} \text{ and } 1 < x \leq 2\}$, $\{x : x \in \mathbb{R} \text{ and } 2 < x \leq 3\}$, $\{x : x \in \mathbb{R} \text{ and } 3 < x \leq 4\}$, and $\{x : x \in \mathbb{R} \text{ and } 4 < x \leq 5\}$. **[2]**

6. T is not a total order relation because, for example, $(1, 2) \not{T} (2, 1)$ and $(2, 1) \not{T} (1, 2)$. **[2]**