# MAT1830

Lecture 33: Trees, queues and stacks

To search a graph $G$ systematically, it helps to have a spanning tree $T$, together with an ordering of the vertices of $T$.

**Queues** Things go in one end and out the other ("first in first out")

Here's one way you could process the letters of STRING with a queue:

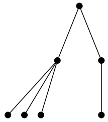
$\underline{\text{S}}$<br>
$\underline{\text{S}}$T<br>
$\underline{\text{S}}$TR<br>
$\underline{\text{T}}$R<br>
$\underline{\text{T}}$RI<br>
$\underline{\text{R}}$I<br>
$\underline{\text{I}}$<br>
$\underline{\text{I}}$N<br>
$\underline{\text{N}}$<br>
$\underline{\text{N}}$G<br>
$\underline{\text{G}}$

The most important letter is the leftmost one: the head of the queue.

## 33.1 Breadth first ordering

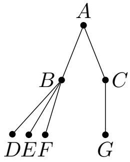
The easiest ordering to understand is called *breadth first*, because it orders vertices "across" the tree in "levels."

Level 0 is a given "root" vertex.
Level 1 is the vertices one edge away from the root.
Level 2 are the vertices two edges away from the root,
. . . and so on.



**Example.**

$A, B, C, D, E, F, G$
is a breadth first ordering of



## 33.2 Queues

Breadth first ordering amounts to putting vertices in a *queue* - a list processed on a "first come, first served" or "first in, first out" basis.

- The root vertex is first in the queue (hence first out).

- Vertices adjacent to the head vertex $v$ in the queue go to the tail of the queue (hence they come out after $v$), if they are not already in it.

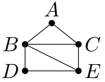
- The head vertex $v$ does not come out of the queue until all vertices adjacent to $v$ have gone in.

## 33.3 Breadth first algorithm

For any connected graph $G$, this algorithm not only orders the vertices of $G$ in a queue $Q$, it also builds a spanning tree $T$ of $G$ by attaching each vertex $v$ to a "predecessor" among the adjacent vertices of $v$ already in $T$. An arbitrary vertex is chosen as the root $V_0$ of $T$.

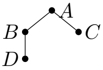
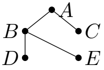
1. Initially, $T =$ tree with just one vertex $V_0$, $Q =$ the queue containing only $V_0$.
2. While $Q$ is nonempty
   - 2.1. Let $V$ be the vertex at the head of $Q$
   - 2.2. If there is an edge $e = VW$ in $G$ where $W$ is not in $T$
     - 2.2.1. Add $e$ and $W$ to $T$
     - 2.2.2. Insert $W$ in $Q$ (at the tail).
   - 2.3. Else remove $V$ from $Q$.

**Example.**

If $G =$ 

, with root vertex $A$.

Then $Q$ and $T$ grow as follows:

| Step | $Q$ | $T$ |
|---|---|---|
| 1 | $A$ |  |
| 2 | $AB$ |  |
| 3 | $ABC$ |  |
| 4 | $BC$ | |
| 5 | $BCD$ |  |
| 6 | $BCDE$ |  |
| 7 | $CDE$ | |
| 8 | $DE$ | |
| 9 | $E$ | |

## 33.3 Breadth first algorithm

For any connected graph $G$, this algorithm not only orders the vertices of $G$ in a queue $Q$, it also builds a spanning tree $T$ of $G$ by attaching each vertex $v$ to a "predecessor" among the adjacent vertices of $v$ already in $T$. An arbitrary vertex is chosen as the root $V_0$ of $T$.

1. Initially, $T =$ tree with just one vertex $V_0$, $Q =$ the queue containing only $V_0$.
2. While $Q$ is nonempty
   - 2.1. Let $V$ be the vertex at the head of $Q$
   - 2.2. If there is an edge $e = VW$ in $G$ where $W$ is not in $T$
     - 2.2.1. Add $e$ and $W$ to $T$
     - 2.2.2. Insert $W$ in $Q$ (at the tail).
   - 2.3. Else remove $V$ from $Q$.

**Remarks**

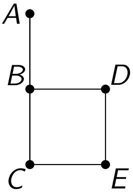
1. If the graph $G$ is not connected, the algorithm gives a spanning tree of the *connected component* containing the root vertex $A$, the part of $G$ containing all vertices connected to $A$.

2. Thus we can recognise whether $G$ is connected by seeing whether all its vertices are included when the algorithm terminates.

3. Being able to recognise connectedness enables us, e.g., to recognise bridges.

# Question

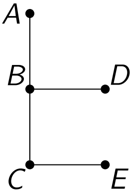
**32.2** Construct a breadth first spanning tree for the graph



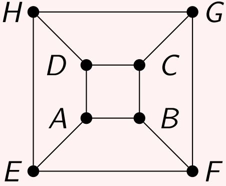

## Answer

The queue follows the steps shown, producing the spanning tree shown far right.

A
AB
B
BC
BCD
CD
CDE
DE
E

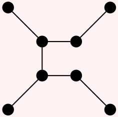


## Flux Exercise (LQMTZZ)

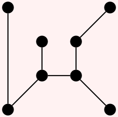



The spanning tree constructed by a breadth-first search from vertex A is

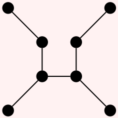
A:



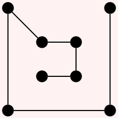
C:



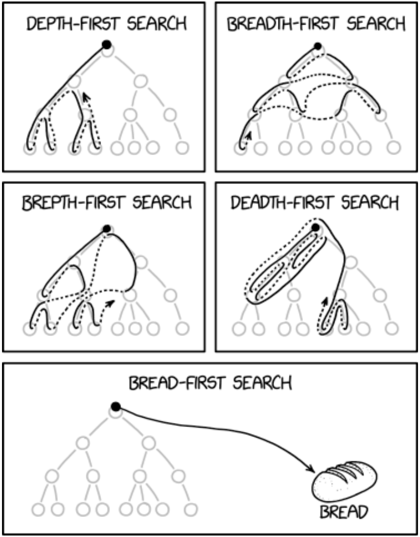
B:



D:



**Answer** B


DEPTH-FIRST SEARCH
BREADTH-FIRST SEARCH
BREPTH-FIRST SEARCH
DEADTH-FIRST SEARCH
BREAD-FIRST SEARCH
BREAD

**Stacks** Things go in and out the same end ("last in first out")

Here's one way you could process the letters of STRING with a stack:

$\underline{\text{S}}$
$\text{S}\underline{\text{T}}$
$\text{ST}\underline{\text{R}}$
$\text{S}\underline{\text{T}}$
$\text{ST}\underline{\text{I}}$
$\text{S}\underline{\text{T}}$
$\underline{\text{S}}$
$\text{S}\underline{\text{N}}$
$\underline{\text{S}}$
$\text{S}\underline{\text{G}}$
$\underline{\text{S}}$

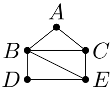
The rightmost letter is the most important: the top of the stack.

## 33.4 Depth first algorithm

This is the same except it has a *stack* $S$ instead of a queue $Q$. $S$ is "last in, first out," so we insert and remove vertices from the same end of $S$ (called the *top* of the stack).

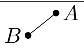
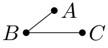
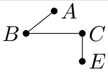
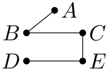
1. Initially, $T =$ tree with just one vertex $V_0$, $S =$ the stack containing only $V_0$.
2. While $S$ is nonempty
   - 2.1. Let $V$ be the vertex at the top of $S$
   - 2.2. If there is an edge $e = VW$ in $G$ where $W$ is not in $T$
     - 2.2.1. Add $e$ and $W$ to $T$
     - 2.2.2. Insert $W$ in $S$ (at the top).
   - 2.3. Else remove $V$ from $S$.

**Remark.** The breadth first and depth first algorithms give two ways to construct a spanning tree of a connected graph.

$G =$  , with root vertex $A$.



**Example.**
We use the same $G$, and take the top of $S$ to be its right hand end.

| Step | $S$ | $T$ |
|---|---|---|
| 1 | $A$ |   |
| 2 | $AB$ |   |
| 3 | $ABC$ |   |
| 4 | $ABCE$ |   |
| 4 | $ABCED$ |   |
| 6 | $ABCE$ | |
| 7 | $ABC$ | |
| 8 | $AB$ | |
| 9 | $A$ | |

## 33.4 Depth first algorithm

This is the same except it has a *stack* $S$ instead of a queue $Q$. $S$ is "last in, first out," so we insert and remove vertices from the same end of $S$ (called the *top* of the stack).

1. Initially, $T =$ tree with just one vertex $V_0$, $S =$ the stack containing only $V_0$.

2. While $S$ is nonempty

   2.1. Let $V$ be the vertex at the top of $S$

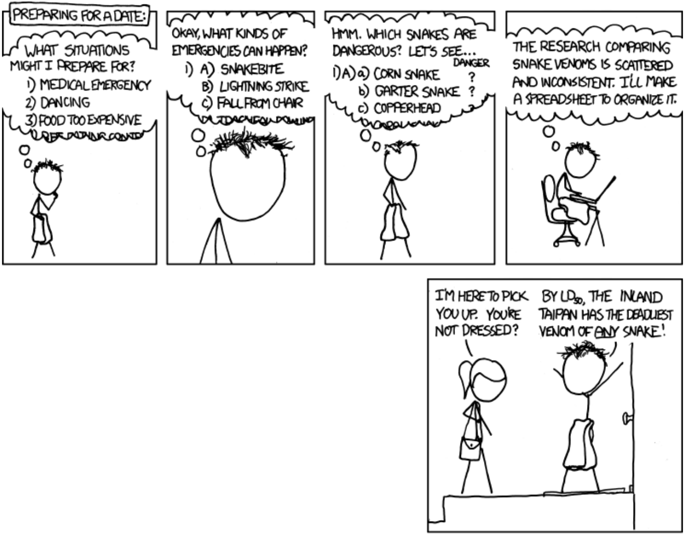
   2.2. If there is an edge $e = VW$ in $G$ where $W$ is not in $T$

      2.2.1. Add $e$ and $W$ to $T$

      2.2.2. Insert $W$ in $S$ (at the top).

   2.3. Else remove $V$ from $S$.

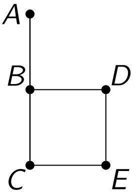
**Remark.** The breadth first and depth first algorithms give two ways to construct a spanning tree of a connected graph.




I REALLY NEED TO STOP USING DEPTH-FIRST SEARCHES.

# Question

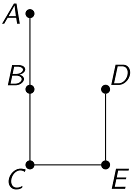
**32.3** Construct a depth first spanning tree for the graph



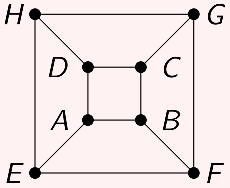

## Answer

The stack trace is as shown, producing the spanning tree shown far right.

A
AB
ABC
ABCE
ABCED
ABCE
ABC
AB
A

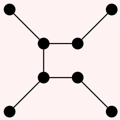


## Flux Exercise (LQMTZZ)

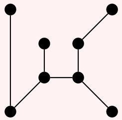



The spanning tree constructed by a depth-first search from vertex A is

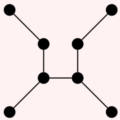
A:



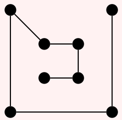
C:



B:



D:



**Answer** D

# Question

**32.1** The following list gives the state, at successive stages, of either a queue or a stack.

A
AB
ABC
BC
BCD
CD
D

Which is it: a queue or a stack?

**Answer** It is a queue. Notice that items are entering at the right hand end and leaving at the left hand end, which is characteristic of a queue.

In a stack, things would enter and leave at the same end.

Roughly speaking:

- breadth first search thinks "go from the first place visited"
- depth first search thinks "go from the last place I visited"

Spanning trees generated by breadth first search have the property that each path from the root to another vertex is one of the shortest paths from the root to the vertex.

This can be incredibly useful in analysing "distances" in networks.

On the other hand if you're looking for solutions that you know are far away from the root, then depth first search can be preferable.