MAT1830 - Discrete Mathematics for Computer Science
Assignment #2 Solutions

1.

p	q	$p \to q$	$\neg(p \to q)$	$p \vee \neg(p \to q)$	$\neg(p \vee \neg(p \to q))$
T	T	T	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	F	F	T

[3]

So this is a not tautology (because the final column in the truth table contains some Ts and some Fs). **[1]**

2.

a	b	c	$\neg b$	$a \wedge \neg b$	$b \vee c$	$(a \wedge \neg b) \to (b \vee c)$	$\neg a \vee b$	$\neg a \vee b \vee c$
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	T	T
T	F	T	T	T	T	T	F	T
T	F	F	T	T	F	F	F	F
F	T	T	F	F	T	T	T	T
F	T	F	F	F	T	T	T	T
F	F	T	T	F	T	T	T	T
F	F	F	T	F	F	T	T	T

[3]

So the two statements are logically equivalent (because the corresponding columns in the truth table are identical). **[1]**

OR **[OR]**

$$\begin{aligned}
(a \wedge \neg b) \to (b \vee c) &\equiv \neg(a \wedge \neg b) \vee (b \vee c) && \text{(using the implication law)} \\
&\equiv (\neg a \vee b) \vee (b \vee c) && \text{(using DeMorgan's laws)} \\
&\equiv \neg a \vee (b \vee b) \vee c && \text{(using the associative laws)} \\
&\equiv \neg a \vee b \vee c && \text{(using the idempotent laws)}
\end{aligned}$$

[3]

So the two statements are logically equivalent. **[1]**

3. "If it doesn't have feathers, then it's not a bird." **[1]**
"If the function returns 0, then the input is valid." **[1]**

4.
- $r \veebar s$ **[2]**
- $(p \wedge r) \to \neg s$ **[2]**
- $(p \wedge \neg r) \vee (q \wedge \neg s)$ **[2]**

(Other answers are possible for these.)

5.

$$\begin{aligned}
(\neg p \vee \neg q) \to \neg q &\equiv \neg(\neg p \vee \neg q) \vee \neg q && \text{(using the implication law)} \\
&\equiv (p \wedge q) \vee \neg q && \text{(using DeMorgan's laws)} \\
&\equiv (p \vee \neg q) \wedge (q \vee \neg q) && \text{(using the distributive laws)} \\
&\equiv (p \vee \neg q) \wedge \mathrm{T} && \text{(using the inverse laws)} \\
&\equiv p \vee \neg q && \text{(using the identity laws)}
\end{aligned}$$

[4]

(You don't have to name-check the logic laws if you don't want to. I do it to help you see what's going on.)