MTH1030/35: Assignment 1, Solutions 2021

Modelling cities in 3d and balancing chemical equations done right

Marking scheme and solutions

Question 1

Real world rules apply to a take-home assignment like this - you don't get paid if you don't get things right! Most importantly this means basically no consequential marks for those parts in which it is easy to check answers for correctness. And, there is NO excuse for messy submissions.

- Question not done at all: 0 marks. Otherwise,...
- Start by assigning 50 marks.
- Go straight to the list of results at the end and deduct 2 marks for every incorrect answer. To be counted as correct an answer needs to be accurate to at least two decimal places (e.g. 12.56743 m $\approx$12.57 m). The area and the volume only have to be accurate to one decimal place.
- Skim over the explanations. Three different outcomes: a further 0, 5, or 10 marks deduction. 0 marks deduction if essentially self-contained, complete, easy to follow, well-presented; 5 marks deductions: if any of those four qualifications is definitely not given. 10 marks deduction: a mess.
- Obviously overall some free marks here for just doing something, but happy to be generous in this respect since we are applying real world rules otherwise.
- feedback in the form: 50 - deductions for incorrect answers - deductions for presentation, e.g., 50-4-5=41.

Solution (50 marks)

Summarizing the results we have:

$$
\begin{gathered}
A = (78.4322, 38.9183, 9.44024) \\
B = (58.6047, 30.4488, 13.8151) \\
C = (67.8834, 10.9869, 18.19) \\
D = (87.7108, 19.4564, 13.8151) \\
A' = (77.4547, 36.3302, 0) \\
B' = (57.1743, 26.6613, 0) \\
C' = (66, 6, 0) \\
D' = (86.2804, 15.6689, 0) \\
|AA'| = 9.83726\ m \\
|BB'| = 14.3961\ m \\
|CC'| = 18.955\ m \\
|DD'| = 14.3961 m \\
sidelength = 22\ m \\
area(A'B'C'D') = 504.355\ m^2 \\
volume\ = 6967.73\ m^3
\end{gathered}
$$

Question 2 (30 marks)

a) *3 marks for getting the right system of equations, 4 marks for solving it, 3 marks for getting the right balanced equation.*

$$wNH_3 + xO_2 \to yN_2 + zH_2O$$

Extracting one equation for each type of atom we get: $w = 2y$ (Nitrogen), $3w = 2z$ (Hydrogen), $2x = z$ (Oxygen). Writing as a homogeneous system of linear equations and then reducing we get

$$
\begin{array}{rrrrcl}
w & & -2y & & = & 0 \\
3w & & & -2z & = & 0 \\
& 2x & & -z & = & 0
\end{array}
\longrightarrow
\left[\begin{array}{rrrr|r}
1 & 0 & -2 & 0 & 0 \\
3 & 0 & 0 & -2 & 0 \\
0 & 2 & 0 & -1 & 0
\end{array}\right]
\longrightarrow
\left[\begin{array}{rrrr|r}
1 & 0 & 0 & -\frac{2}{3} & 0 \\
0 & 1 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 1 & -\frac{1}{3} & 0
\end{array}\right]
$$

So, $w = \frac{2}{3}z, x = \frac{1}{2}z, y = \frac{1}{3}z$. The smallest positive value of z that will give integer values for all four variables is the least common multiple of the denominators of the three fractions, namely 6. Therefore the balanced equation is

$$4NH_3 + 3O_2 \rightarrow 2N_2 + 6H_2O$$

b) *3 marks for getting the right system of equations, 4 marks for solving it, 3 marks for getting the right balanced equation.*

$$vC_2H_2Cl_4 + wCa(OH)_2 \rightarrow xC_2HCl_3 + yCaCl_2 + zH_2O$$

$$2v - 2x = 0\ (C), 2v + 2w - x - 2z = 0\ (H), 4v - 3x - 2y = 0\ (Cl), w - y = 0\ (Ca)$$

$$\begin{array}{cccccc} 2v & & -2x & & & = 0 \\ 2v & +2w & -x & & -2z & = 0 \\ 4v & & -3x & -2y & & = 0 \\ & w & & -y & & = 0 \end{array} \longrightarrow \left[\begin{array}{ccccc|c} 2 & 0 & -2 & 0 & 0 & 0 \\ 2 & 2 & -1 & 0 & -2 & 0 \\ 4 & 0 & -3 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array}\right]$$

The smallest positive value of z that will give integer values for all five coefficients is the least common multiple of the denominators of the four fractions, namely 2. Therefore the balanced equation is

$$2C_2H_2Cl_4 + Ca(OH)_2 \rightarrow 2C_2HCl_3 + CaCl_2 + 2H_2O$$

c) It is possible to make up up a fantasy unbalanced equation that cannot be balanced! Any equation that has some type of atom on one side but not on the other cannot be balanced, since you can only balance with positive integers. E.g.,

$$A + B \rightarrow C$$

cannot be balanced. Basically, the alchemists' dream of turning silver into gold is not possible. *This is pretty much a "You either get 5 or 0 marks question." Give some marks if somebody did not understand the question correctly, came up with a reasonable interpretation of the question and then answered whatever they thought was asked here correctly.*

An interesting real-world example of an equation that obviously also cannot be balanced is this one.

$$H_2O \rightarrow H_2O_2$$

(H_2O_2 is hydrogen peroxide.)

d) It is also possible to make up a fantasy equation that can be balanced in two fundamentally different ways. The simplest way to achieve this to have the same stuff on both sides. E.g.,

$$A + B \rightarrow A + B$$

Putting any number in front of the As and any other number in front of the Bs will balance this equation. E.g.,

$$5A + 3B \rightarrow 5A + 3B$$

and

$$8A + 7B \rightarrow 8A + 7B$$

are both balanced. However, they are not not both multiples of the same smallest solution. More complex examples can be made up by adding together several different balanced equations. In fact, any equation that can be balanced in multiple essentially different ways is a sum of "prime" equations that can only be balanced in essentially one way. *Again, just like b) this is pretty much a "You either get 5 or 0 marks question." Give some marks if somebody did not understand the question correctly, came up with a reasonable interpretation of the question and then answered whatever they thought was asked here correctly.*

An interesting real-world example of an equation that can be balanced in many ways is

$$CH_4 + C_2H_6 + O_2 \rightarrow H_2O + CO_2$$

Burning methane (CH_4) and ethane (C_2H_6) simultaneously can be balanced any number of ways depending on how much of each there is. Just combine different multiples of the following two balanced equations:

$$CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$$

$$2C_2H_6 + 7O_2 \rightarrow 6H_2O + 4CO_2$$

Finally, go over the explanations. Three different outcomes: 0, 3, or 6 marks deduction. 0 marks deduction if essentially self-contained, complete, easy to follow, well-presented; 3 marks deductions if any of those four qualifications is definitely not given. 6 marks deduction: a mess.

Feedback in the form: points for a+points for b+points for c+points for d-deductions for presentation, e.g. 10+10+5+5-0=30.