

MTH 1035 (Semester 1, 2022) Sample “exam-like” questions

Parts of the final exam specific to MTH1035 will be similar to several (but not all) of the questions given below. You are highly encouraged to treat this questions as if it were an assignment. You can submit your solutions to Andy in advance of the final exam in order to get feedback. Please, email or talk to Andy if you have any questions.

- (1) State the definition of a sequence $\{x_n\}$ converging to a limit L . Show, using this definition, that the sequence $\{\frac{\sin(n)}{n^5+n+1}\}$ converges.
- (2) Consider the sequence $\{x_n\}$ defined by $x_1 = 1$ and $x_{n+1} = \sqrt{x_n + 6}$ for $n \geq 1$.
 - (a) Show that $1 \leq x_n < 3$ for all n .
 - (b) Use the Monotone Convergence Theorem to show that the sequence converges.
 - (c) Find the limit of the sequence.
- (3) Show that if $\{x_n\}$ is a convergent sequence and $x_n \geq 0$ for all n , then $\lim_{n \rightarrow \infty} x_n \geq 0$.
- (4) State the definition of a Cauchy sequence. Using this definition, show that $\{\frac{n+2}{3n+4}\}$ is a Cauchy sequence.
- (5) State the definition of a Cauchy sequence. Show that the sum of two Cauchy sequences is a Cauchy sequence.
- (6) Show using the definition of a vector space, that if u and v are elements of a vector space V , then

$$2u + (v + (u + v)) = 3u + 2v.$$

Justify each step by listing the rule used.

Note: if a question of this form is posed, then the definition of an abstract vector space will be provided on the exam.