

Bicycle Wheel
System
Identification
and Optimal
Truing
Algorithm

Aaron Hunter

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Bicycle Wheel System Identification and Optimal Truing Algorithm

Aaron Hunter

October 7, 2019

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- Wheel ‘truing’ is the process of adjusting spoke tension to minimize lateral, radial, and tension variations
- Commercial robotic bicycle wheel truing machines employ an open loop algorithm that operates iteratively
- This method may not achieve wheel specifications requiring human intervention

The work that follows demonstrates an optimal approach to wheel truing using system identification techniques and feedback control to achieve wheel alignment.

The Bicycle Wheel

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- The bicycle wheel is a structure consisting of a rim, a hub, and spokes connecting the hub to the rim
- The spokes are under tension to provide the stiffness to wheel structure
- The rim is under compression
- The wheel can be considered a system of springs in parallel and series combinations
- Spoke tension must be high enough to support the load (bicycle and rider) but not so high that the rim warps laterally
- Spokes patterns vary from radial to nearly tangential

Bicycle Wheel Geometry

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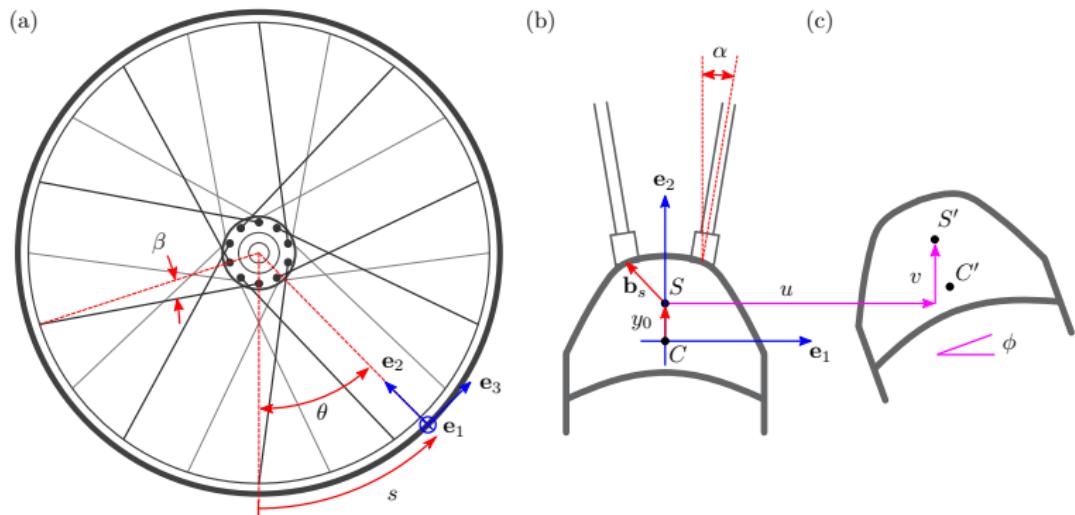


Image credit: Ford, Matthew, *Reinventing the Wheel: Stress Analysis, Stability, and Optimization of the Bicycle Wheel*, PhD. Dissertation Northwestern University, December 2018.

Wheel Truing

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- Spoke tension is adjusted by tightening or loosening the spoke via a threaded nipple seated in the rim
- Spoke tension consists of lateral, radial, and tangential components at the rim
- Spoke tension is adjusted such that rim is ‘true’ in both lateral and radial dimensions, and desired mean tension is achieved
- Conventional truing algorithm:
 - 1 Adjust mean tension
 - 2 Minimize lateral variations
 - 3 Minimize radial variations
 - 4 Repeat until all desired specifications are met

Apparatus

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- Centrimaster Comfort Wheel Truing Stand
- WheelFanatyk Digital Tension Gauge
- Canon EOS M, prime 22mm lens
- Wheel: Stans ZTR Alpha Rim, DT Swiss Competition spokes, White Industries MI5 hub



System Identification Methodology

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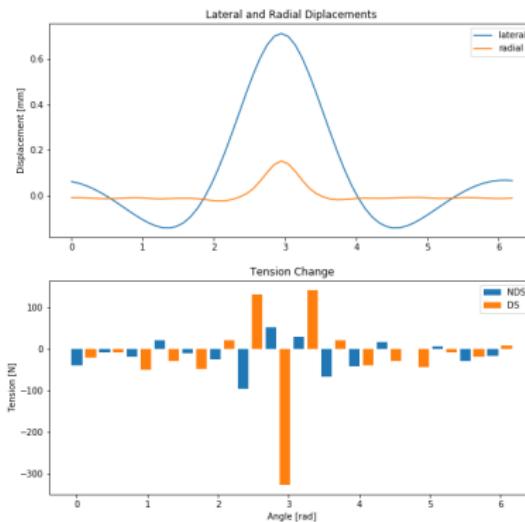
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Identify the lateral, radial and tension changes ('gain curves') induced by a unit spoke adjustment for each spoke



Theoretical gain curves derived for a generic wheel using <https://github.com/dashdotrobot/bike-wheel-calc>

Measurements Using Computer Vision

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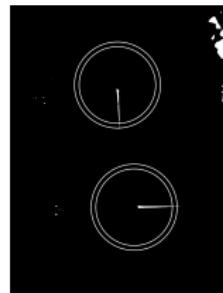
(a)



(b)



(c)



(d)

Algorithm to interpret gauge displacements:

- ① Reference image (a)
- ② Measurement (or zero value) image (b)
- ③ Subtract measurement from reference (c)
- ④ Binary threshold and mask (d)
- ⑤ Calculate angle of centroid from gauge center
- ⑥ Calculate displacement measurement from angle

Spoke Tension Measurements



WheelFanatyk spoke tension meter. Image credit: <https://www.wheelfanatyk.com>

- Digital spoke tension measurements
- Data collected through USB to PC
- Meter collects displacement of spoke by calibrated spring
- Reference measurement accounts for variation of spoke thickness
- Tension values interpolated from calibration table

Measurement Data Vector

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Measurement data collected for a wheel under test:

$$\mathbf{U} = \begin{bmatrix} u(\theta = 1\frac{2\pi}{2n_s}) \\ u(\theta = 2\frac{2\pi}{2n_s}) \\ \vdots \\ u(\theta = 2n_s\frac{2\pi}{2n_s}) \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} v(\theta = 1\frac{2\pi}{2n_s}) \\ v(\theta = 2\frac{2\pi}{2n_s}) \\ \vdots \\ v(\theta = 2n_s\frac{2\pi}{2n_s}) \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} t(\theta = 1\frac{2\pi}{n_s}) \\ t(\theta = 2\frac{2\pi}{n_s}) \\ \vdots \\ t(\theta = n_s\frac{2\pi}{n_s}) \end{bmatrix}$$
$$\mathbf{Y} = \begin{bmatrix} \mathbf{U} \\ \mathbf{V} \\ \mathbf{T} - \bar{T} \end{bmatrix}$$

- n_s = number of spokes
- θ = rim angle measurement location where $\theta = 0$ is taken to be the valve hole
- $u(\theta), v(\theta), t(\theta)$ = lateral, radial, and tension measurements taken at rim angle θ
- \bar{T} = mean spoke tension

Gain Matrices

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Let $\mathbf{u}_s = u_s(\theta)$ be the lateral displacement vector (gain curve) for every discrete θ around the rim induced by turning spoke s by one rotation. The matrix of all \mathbf{u}_s gain curves is defined to be the lateral 'gain' matrix Φ_u . The radial and tension gain matrices are similarly defined.

$$\Phi_u = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \dots \quad \mathbf{u}_{n_s}]$$

$$\Phi_v = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \dots \quad \mathbf{v}_{n_s}]$$

$$\Phi_t = [\mathbf{t}_1 \quad \mathbf{t}_2 \quad \dots \quad \mathbf{t}_{n_s}]$$

Measurement Prediction

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Let \mathbf{d} be a vector of spoke rotations. Let \mathbf{Y}_b be the baseline wheel measurements, that is, the state of the wheel prior to the application of \mathbf{d} . The predicted lateral, radial, and tension measurements, $\hat{\mathbf{Y}}$, after applying \mathbf{d} to the spokes is given by:

$$\Phi = \begin{bmatrix} \Phi_u \\ \Phi_v \\ \Phi_t \end{bmatrix}$$

$$\hat{\mathbf{Y}} = \mathbf{Y}_b + \Phi\mathbf{d}$$

Multi-objective Least Squares

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Compute the optimal vector of spoke rotations, \mathbf{d}_{ls} , from the set of measurements, $\{\mathbf{u}, \mathbf{v}, \mathbf{T}\}$ given the weighting factors μ_v and μ_t :

$$\tilde{\Phi} = \begin{bmatrix} \Phi_u \\ \Phi_v\sqrt{\mu_v} \\ \Phi_t\sqrt{\mu_t} \end{bmatrix} \quad \tilde{\mathbf{Y}} = \begin{bmatrix} \mathbf{u} \\ \mathbf{v}\sqrt{\mu_v} \\ (\mathbf{T} - \bar{\mathbf{T}})\sqrt{\mu_t} \end{bmatrix}$$
$$\mathbf{d}_{ls} = \tilde{\Phi}^\dagger \tilde{\mathbf{Y}}$$

Where $\tilde{\Phi}^\dagger$ is the pseudo-inverse of $\tilde{\Phi}$. The weighting factors represent the tradeoff between the lateral, radial, and tension variables and are found through evaluation of the wheel specification, as well as the noise and accuracy of the measurements.

Tension Targeting

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- Targeting a different tension, T_d , can be accomplished by substituting T_d for \bar{T} into \mathbf{Y}
- However:
 - T_d is weighted when calculating \mathbf{d} —not desirable
 - Non-ideality in tension gain curves is amplified when T_d is significantly different than \bar{T}
- Note: *uniform tension changes do not affect lateral and radial displacements.*
- Therefore uniform tension changes (due to average of $\bar{\mathbf{d}} = \mathbf{d}$) are separable from non-uniform tension changes ($\mathbf{d} - \bar{\mathbf{d}}$)

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A better alternative is to calculate the constant factor, d_{cm} that induces the mean tension change separately from \mathbf{d} :

$$d_{cm} = (T_d - \bar{T})/c$$

where c is the proportionality constant between mean tension and mean adjustment. The predicted tension vector, $\hat{\mathbf{T}}$, becomes:

$$\hat{\mathbf{T}} = \Phi_t(\mathbf{d} - \bar{\mathbf{d}}) + \bar{\mathbf{T}} + (d_{cm} + \bar{d})c$$

Truing Algorithm

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- \mathbf{d}_{ls} is the optimal fit of nipple rotations that result in the measured state relative to a perfectly true wheel
- To true the wheel apply $\mathbf{d} = -\mathbf{d}_{ls}$ to the system
- Spoke nipple turns are difficult to apply accurately. Instead predict the state of the system after each adjustment (entry in \mathbf{d})
- Adjust the spoke until the lateral displacement at the spoke location matches the prediction
- After all spokes adjusted, measure state of the system
- Demonstrated graphically in the next slides...

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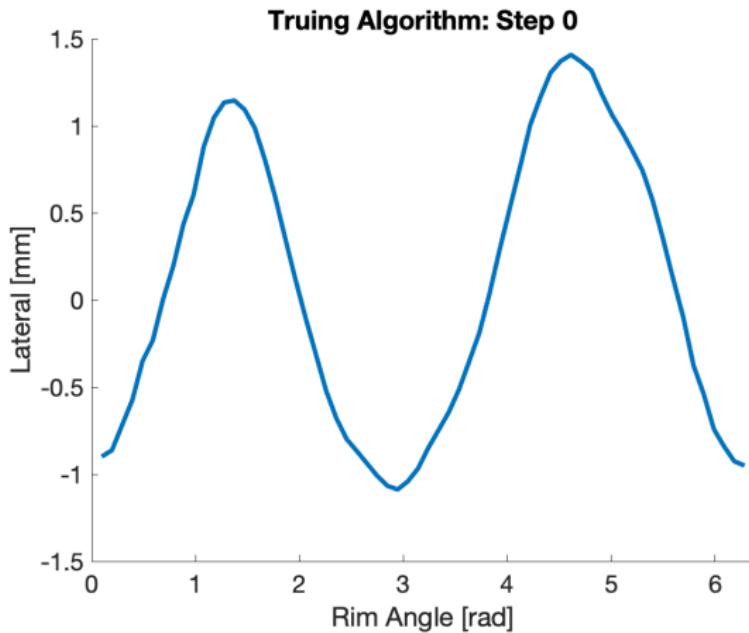
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Thick curve is predicted profile, diamond is lateral target

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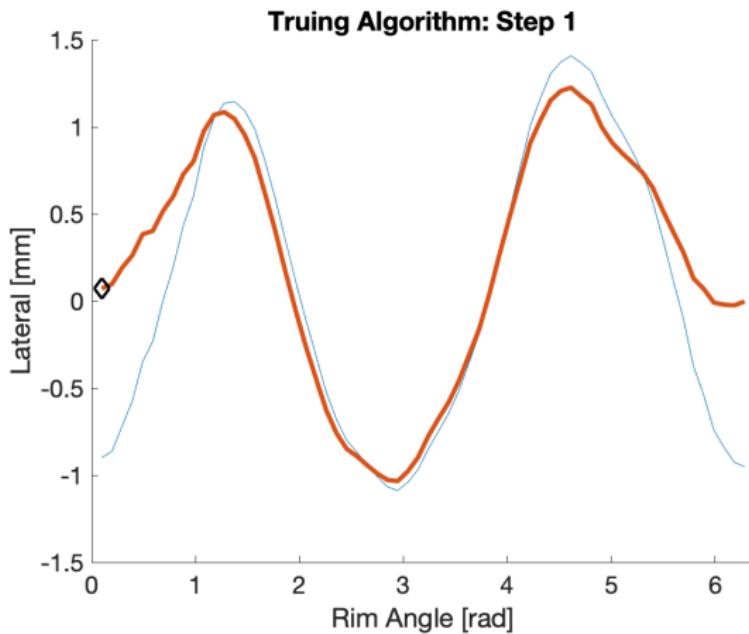
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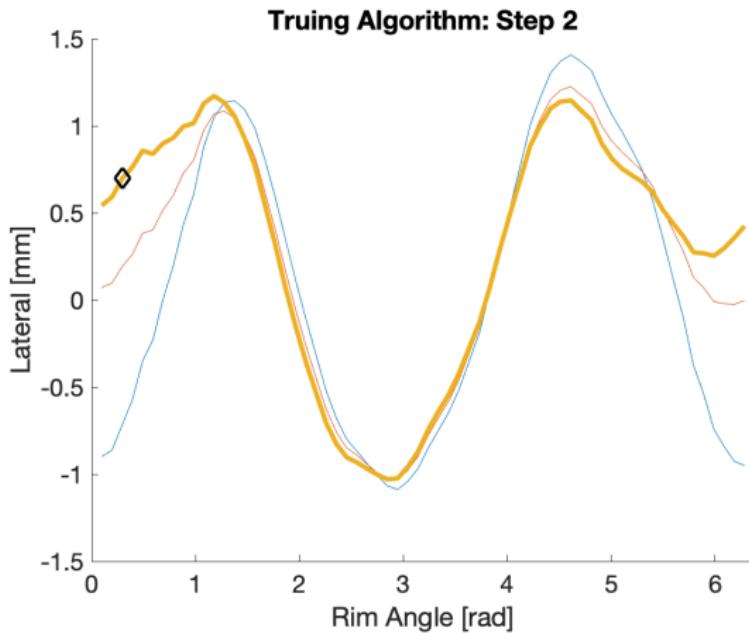
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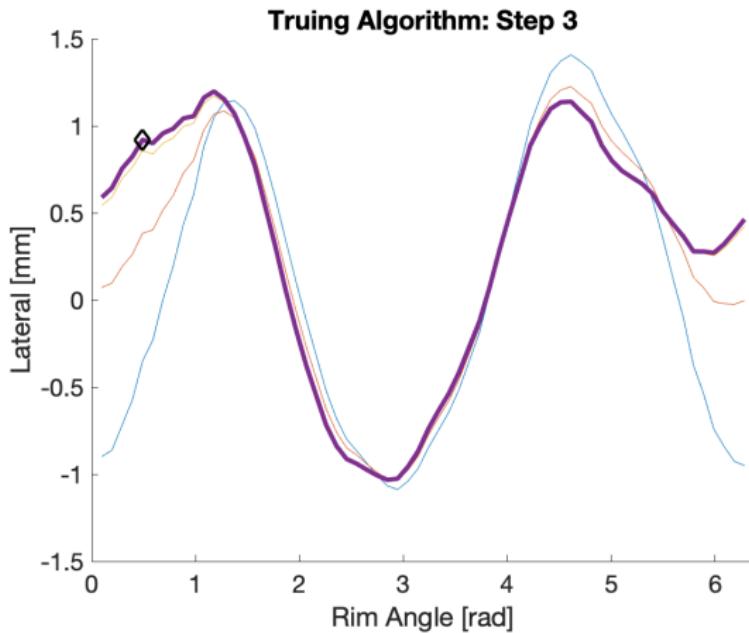
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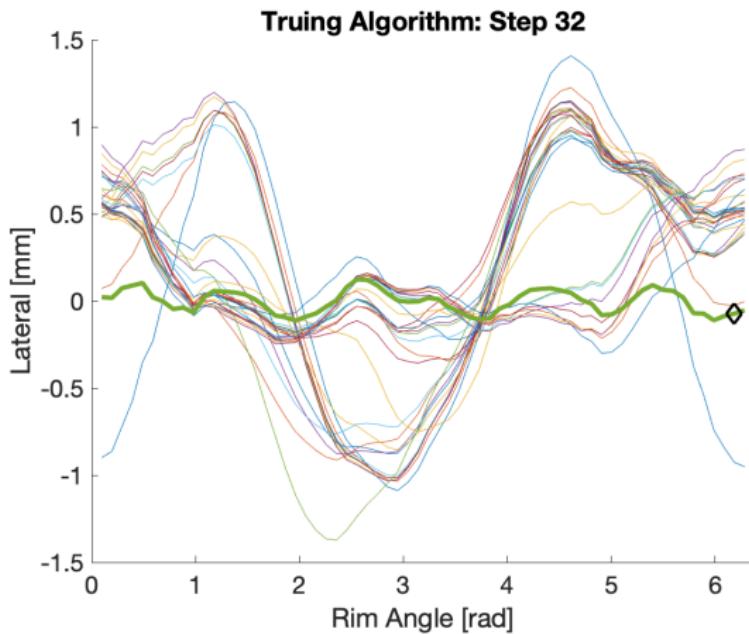
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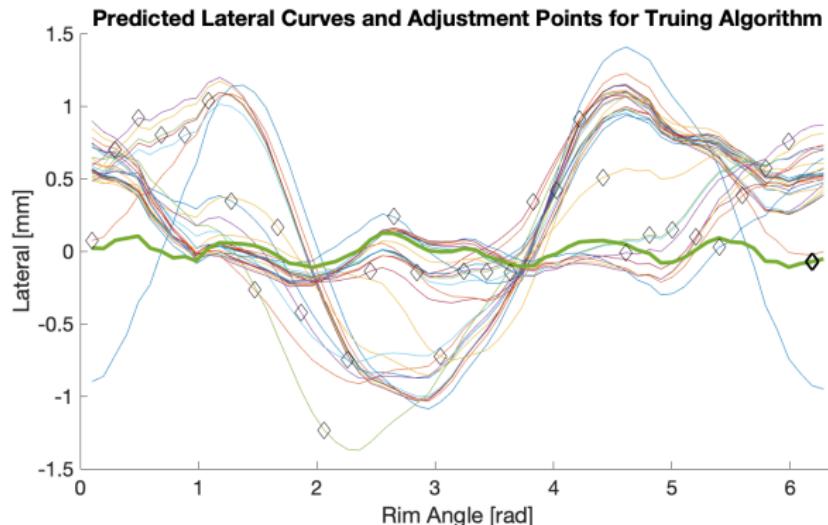
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- Predicted lateral profiles *after* each spoke adjustment
- Diamonds represent lateral targets
- Green represents final (trued) profile

Computer Vision Validation

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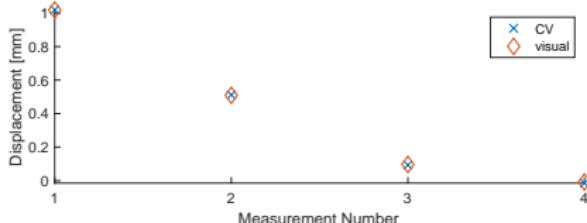
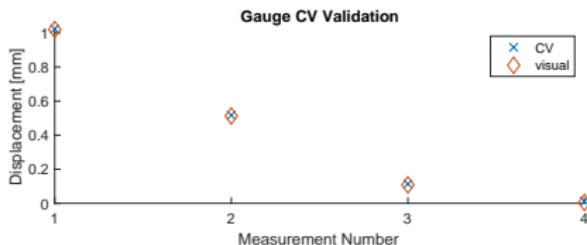
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- Four dual gauge readings were recorded
- Gauges were set to 1mm, 0.5mm, 0.1mm, and 0mm
- Visual analysis and CV algorithm results compared
- Visual analysis resolution is 0.0135mm
- Results agree to $\pm 0.007\text{mm}$

Tension Gain Curves

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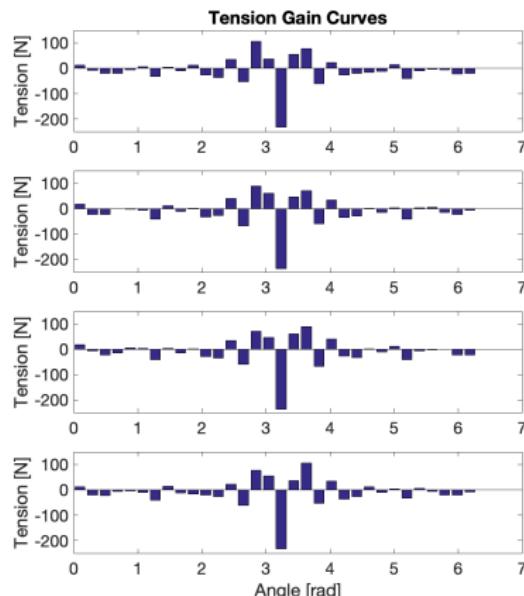
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- 32 tension curves were collected
- Four distinct patterns identified:
 - Non-drive side leading
 - Drive side leading
 - Non-drive side trailing
 - Drive side trailing



Lateral and Radial Gain Curves

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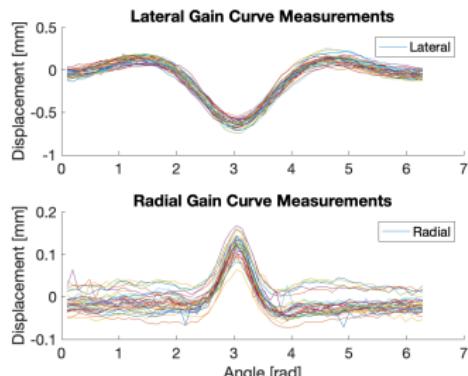
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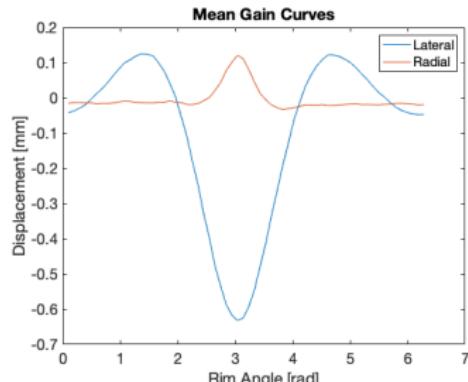
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(a)



(b)

- 32 lateral and radial curves measured (a)
- Mean gain curves used for model (b)
- Curves normalized to same rim angle and side for clarity

Truing Algorithm Simulation

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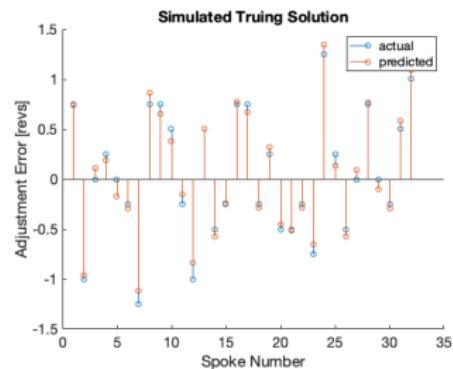
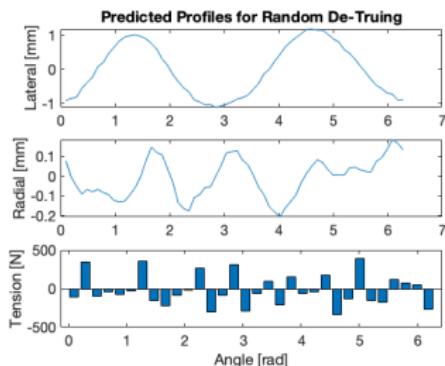
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- Random spoke displacement vector, \mathbf{d} , generated
- Noise added to simulated profile
- Spoke displacement vector, $\hat{\mathbf{d}}$, predicted
- Weighting factors adjusted for performance

Simulation Error

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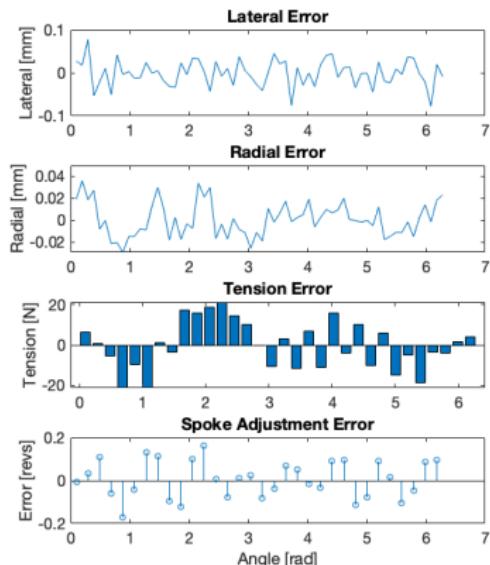
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- Lateral displacement is more sensitive than radial with same spec
- Units comparison of tension to lateral
 $\implies \mu_t < \frac{0.6}{300}$
- Weighting factors yielding satisfactory performance:
 - $\mu_v = 0.5$
 - $\mu_t = 1.0e - 5$



Experiment 1: De-true Test Wheel

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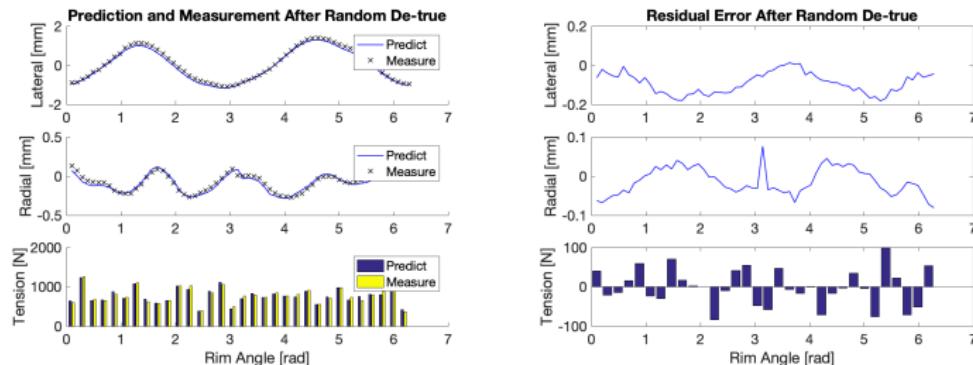
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- Simulation spoke vector \mathbf{d} applied to manually-trued test wheel
- Spokes adjusted using lateral feedback
- The model predicts the experimental results well; some residual structure is evident

Experiment 2: True Test Wheel

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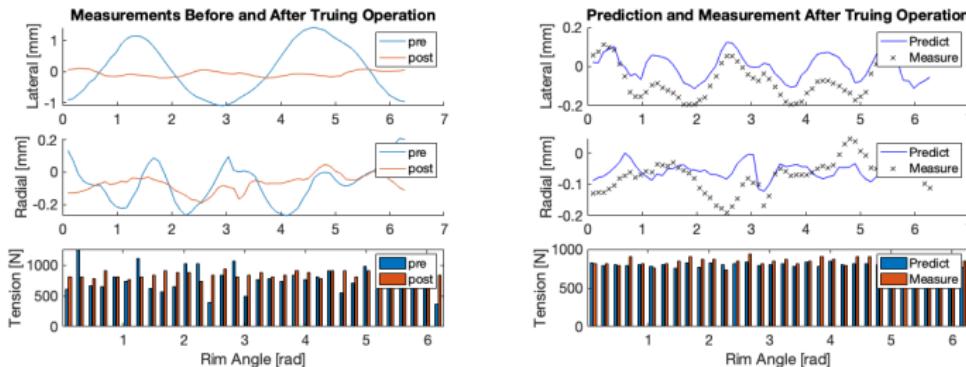
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Wheel measurements ($\mu \pm \sigma$):

Parameter	Initial	Final
Lateral [mm]	0.078 ± 0.820	-0.062 ± 0.091
Radial [mm]	-0.067 ± 0.122	-0.070 ± 0.052
Tension [N]	776 ± 205	849 ± 48

Single iteration effectively trues a wheel

Experiment 3: Iterate Truing Algorithm

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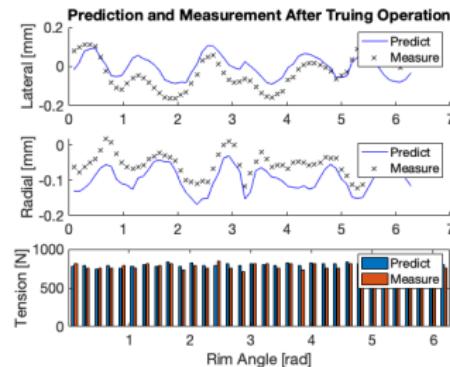
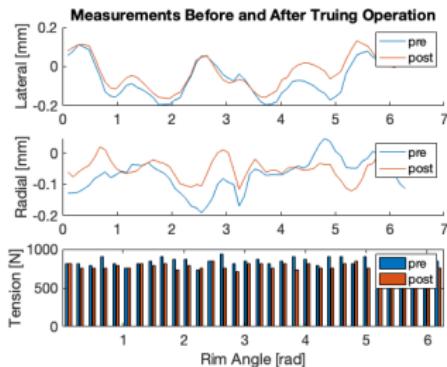
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Wheel measurements ($\mu \pm \sigma$):

Parameter	Initial	Final
Lateral [mm]	-0.062 ± 0.091	-0.029 ± 0.083
Radial [mm]	-0.070 ± 0.052	-0.052 ± 0.032
Tension [N]	849 ± 48	781 ± 33

Small but significant improvement after second iteration

Experiment 4: Target Tension=1000N

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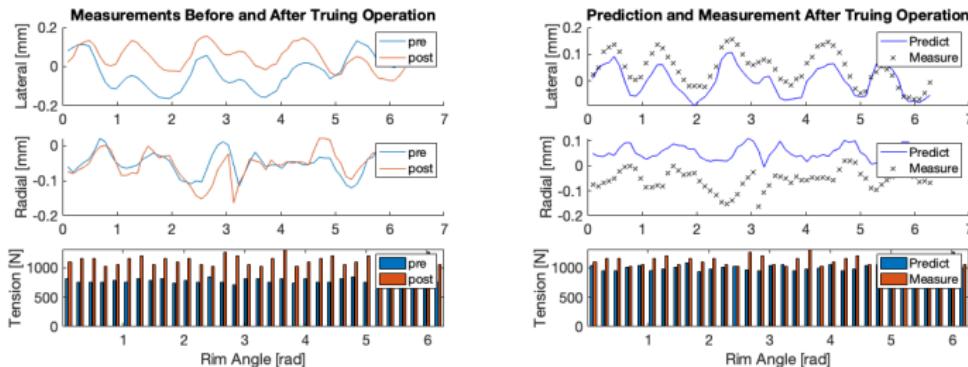
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Wheel measurements ($\mu \pm \sigma$):

Parameter	Initial	Final
Lateral [mm]	-0.029 ± 0.083	0.045 ± 0.063
Radial [mm]	-0.052 ± 0.032	-0.056 ± 0.039
Tension [N]	781 ± 33	1126 ± 76

Tension exceeds target and non-uniform

Experiment 5: Second Iteration

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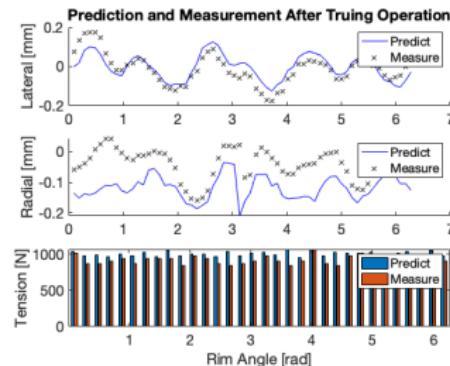
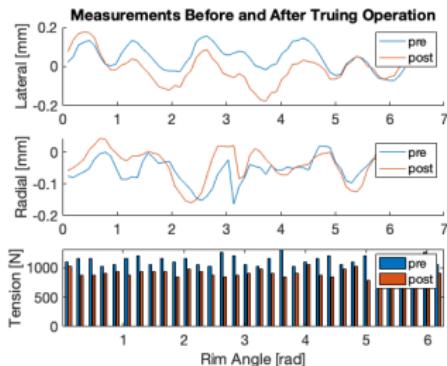
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Wheel measurements ($\mu \pm \sigma$):

Parameter	Initial	Final
Lateral [mm]	0.045 ± 0.063	-0.012 ± 0.079
Radial [mm]	-0.056 ± 0.039	-0.037 ± 0.050
Tension [N]	1126 ± 76	914 ± 63

Tension below target tension and non-uniform!

Experiment 4 & 5: Analysis

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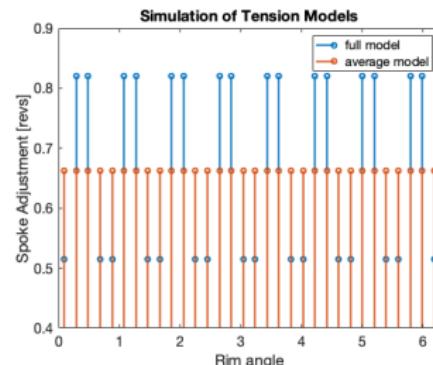
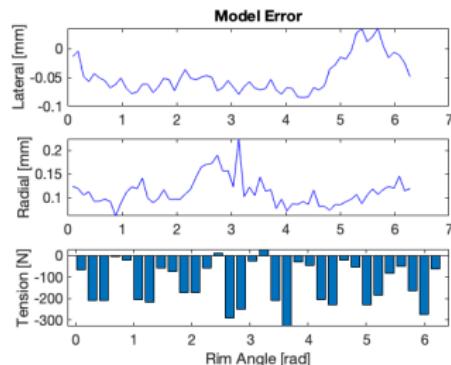
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- Model error demonstrates periodic bias in tension
- A constant change in spoke displacement should result in constant tension change
- Simulation of a 200N tension change on perfectly true wheel highlights the model error

Change tension model to average

Experiment 6: Average Tension Model

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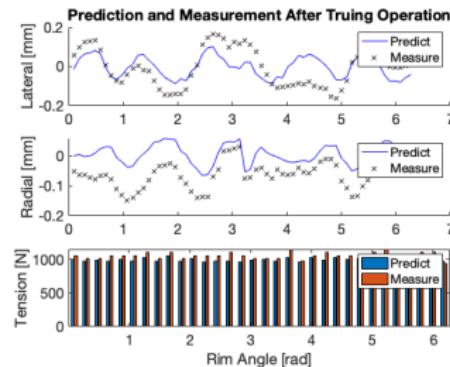
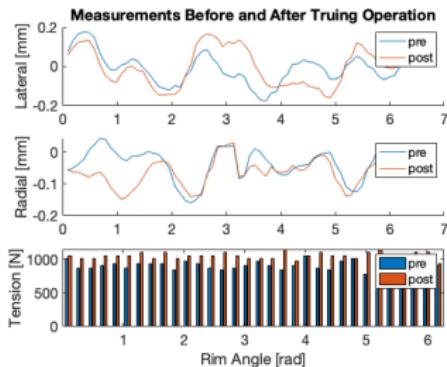
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Wheel measurements ($\mu \pm \sigma$):

Parameter	Initial	Final
Lateral [mm]	-0.012 ± 0.079	-0.004 ± 0.098
Radial [mm]	-0.037 ± 0.050	-0.064 ± 0.042
Tension [N]	914 ± 63	1056 ± 48

Symmetric tension model improves tension uniformity

Conclusions

- System identification techniques used to develop linear model of a wheel subject to spoke tension inputs
- Computer vision techniques provide finer resolution from analog gauges than visual estimations
- Computer vision vastly reduces time for data collection
- The model can be used to predict:
 - Lateral displacement
 - Radial displacement
 - Tension changes
- Multi-objective least squares approximations of these parameters used to find optimal spoke displacements for wheel truing
- Wheel truing algorithm minimizes errors in tension adjustment using lateral feedback

Conclusions

- Truing performance limited by spoke tension adjustment resolution:
 - Spoke twist during adjustment
 - Friction at nipple/spoke thread interface
- Truing performance also limited by tension measurement accuracy and resolution
 - Small effects due to spoke patterns not captured by model

Future Work

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- Comparison with theoretical models
- Extension of model to include tension offset (side to side)
- Improved resolution/accuracy tensiometer
- Mechatronic implementation

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