Bicycle Wheel System Identification and Optimal Truing Algorithm

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Introduction

- Wheel ‘truing’ is the process of adjusting spoke tension to minimize lateral and radial variations
- Commercial robotic bicycle wheel truing machines use a heuristic truing method.
- This method mimics the actions a human might take to perform the task.
- This method can be inefficient or ineffective requiring human intervention

The work that follows demonstrates an optimal approach to wheel truing using system identification techniques and feedback control to achieve wheel alignment.

The Bicycle Wheel

- The bicycle wheel is a structure consisting of a rim, a hub, and spokes connecting the hub to the rim
- The spokes are under tension to provide the stiffness to wheel structure
- The rim is under compression
- The wheel can be considered a system of springs in parallel and series combinations
- Spoke tension must be high enough to support the load (bicycle and rider) but not so high that the rim warps laterally
- Spokes patterns vary from radial to nearly tangential

Bicycle Wheel Geometry

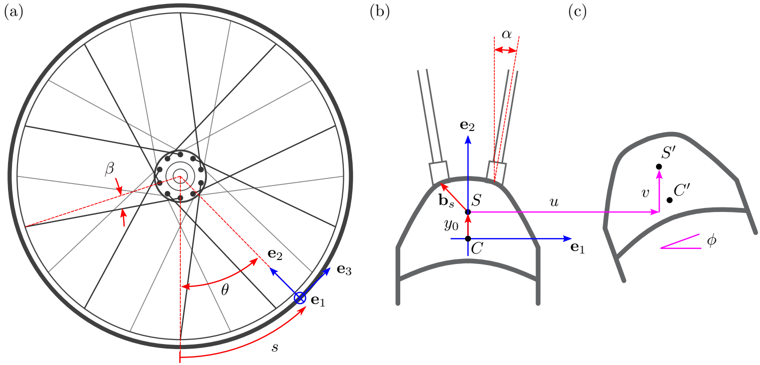


Image credit: Ford, Matthew, *Reinventing the Wheel: Stress Analysis, Stability, and Optimization of the Bicycle Wheel*, PhD. Dissertation Northwestern University, December 2018.

Wheel Truing

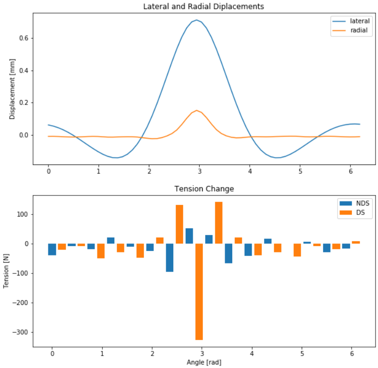
- Spoke tension is adjusted by changing the effective length of the spoke via a threaded nipple seated in the rim
- Spoke tension consists of lateral, radial, and tangential components at the rim
- Spoke tension is adjusted such that rim is ‘true’ in both lateral and radial dimensions, and desired mean tension is achieved
- Conventional truing algorithm:
  1. Adjust mean tension
  2. Minimize lateral variations
  3. Minimize radial variations
  4. Repeat until all desired specifications are met

Apparatus

- Centrimaster Comfort Wheel Truing Stand
- WheelFanatyk Digital Tension Gauge
- Canon EOS M, prime 22mm lens
- Wheel: Stans ZTR Alpha Rim, DT Swiss Competition spokes, White Industries MI5 hub

System Identification Methodology

Identify the lateral, radial and tension changes ('gain curves') induced by a unit spoke adjustment for each spoke



Theoretical gain curves derived for a generic wheel using https://github.com/dashdotrobot/bike-wheel-calc

Measurements Using Computer Vision

(a) (b) (c) (d)

Algorithm to interpret gauge displacements:

1. Reference image (a)
2. Measurement (or zero value) image (b)
3. Subtract measurement from reference (c)
4. Binary threshold and mask (d)
5. Calculate angle of centroid from gauge center
6. Calculate displacement measurement from angle

Spoke Tension Measurements

WheelFanatyk spoke tension meter. Image credit: https://www.wheelfanatyk.com

- Digital spoke tension measurements
- Data collected through USB to PC
- Meter collects displacement of spoke by calibrated spring
- Reference measurement accounts for variation of spoke thickness
- Tension values interpolated from calibration table

Measurement Data Vector

Measurement data collected for a wheel under test:

$$\mathbf{U} = \begin{bmatrix} u(\theta = 1\frac{2\pi}{2n_s}) \\ u(\theta = 2\frac{2\pi}{2n_s}) \\ \vdots \\ u(\theta = 2n_s\frac{2\pi}{2n_s}) \end{bmatrix} \mathbf{V} = \begin{bmatrix} v(\theta = 1\frac{2\pi}{2n_s}) \\ v(\theta = 2\frac{2\pi}{2n_s}) \\ \vdots \\ v(\theta = 2n_s\frac{2\pi}{2n_s}) \end{bmatrix} \mathbf{T} = \begin{bmatrix} t(\theta = 1\frac{2\pi}{n_s}) \\ t(\theta = 2\frac{2\pi}{n_s}) \\ \vdots \\ t(\theta = n_s\frac{2\pi}{n_s}) \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{U} \\ \mathbf{V} \\ \mathbf{T} \end{bmatrix}$$

- n_s = number of spokes
- $\theta =$ rim measurement location where $\theta = 0$ is taken to be the valve hole
- $u(\theta) =$ lateral measurement at θ
- $v(\theta) =$ radial measurement at θ
- $t(\theta) =$ spoke tension measurement at θ

Gain Matrices

Let $\mathbf{u_s}$=$u_s(\theta)$ be the lateral displacement vector (gain curve) for every discrete θ around the rim induced by turning spoke s by one rotation. The matrix of all $\mathbf{u_s}$ gain curves is defined to be the lateral 'gain' matrix Φ_u. The radial and tension gain matrices are similarly defined.

$$\Phi_u = \begin{bmatrix} \mathbf{u_1} & \mathbf{u_2} & \dots & \mathbf{u_{n_s}} \end{bmatrix}$$
$$\Phi_v = \begin{bmatrix} \mathbf{v_1} & \mathbf{v_2} & \dots & \mathbf{v_{n_s}} \end{bmatrix}$$
$$\Phi_t = \begin{bmatrix} \mathbf{t_1} & \mathbf{t_2} & \dots & \mathbf{t_{n_s}} \end{bmatrix}$$

Measurement Prediction

Let $\mathbf{d}$ be a vector of spoke rotations. Let $\mathbf{Y_b}$ be the baseline wheel measurements, that is, the state of the wheel prior to the application of $\mathbf{d}$. The predicted lateral, radial, and tension measurements, $\hat{\mathbf{Y}}$, after applying $\mathbf{d}$ to the spokes is given by:

$$\Phi = \begin{bmatrix} \Phi_u \\ \Phi_v \\ \Phi_t \end{bmatrix}$$

$$\hat{\mathbf{Y}} = \mathbf{Y_b} + \Phi \mathbf{d}$$

Multi-objective Least Squares

Predict the set of spoke rotations, $\hat{\mathbf{d}}$, that yield given set of measurements, $\mathbf{Y}$ given the weighting factors μ_v, μ_t and desired state $\mathbf{Y_d} = [\mathbf{u_d}, \mathbf{v_d}, \mathbf{T_d}]^{\mathbf{T}}$:

$$\tilde{\Phi} = \begin{bmatrix} \Phi_u \\ \Phi_v \sqrt{\mu_v} \\ \Phi_t \sqrt{\mu_t} \end{bmatrix} \Delta\tilde{\mathbf{Y}} = \begin{bmatrix} \mathbf{u} - \mathbf{u_d} \\ (\mathbf{v} - \mathbf{v_d})\sqrt{\mu_v} \\ (\mathbf{T} - \mathbf{T_d})\sqrt{\mu_t} \end{bmatrix}$$

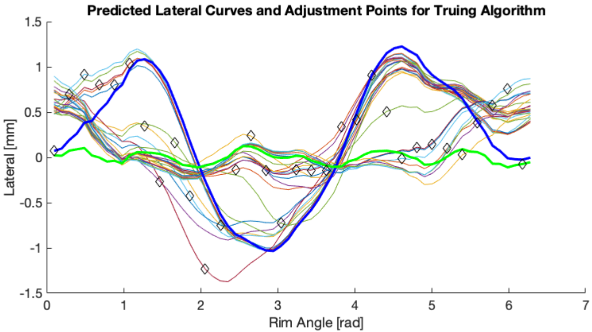
$$\hat{\mathbf{d}} = \tilde{\Phi}^{\dagger} \Delta\tilde{\mathbf{Y}}$$

Where $\tilde{\Phi}^{\dagger}$ is the pseudo-inverse of $\tilde{\Phi}$. The weighting factors represent the tradeoff between the lateral, radial, and tension variables and are found through evaluation of the wheel specification and exhaustive simulation.

Truing Algorithm

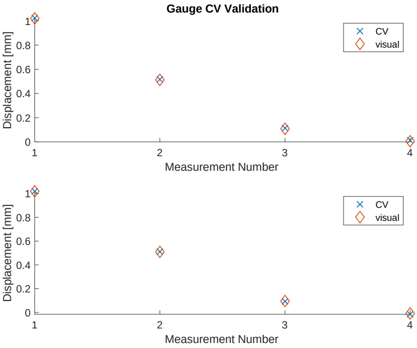
- $\hat{\mathbf{d}}$ is the best fit of spoke rotations that result in the measured state relative to the desired state
- To achieve the desired state therefore apply $-\hat{\mathbf{d}}$ to the system
- Spoke turns are difficult to apply accurately. Instead predict the state of the system after each spoke adjustment (element of $-\hat{\mathbf{d}}$)
- Adjust the spoke until the lateral displacement at the spoke location matches the prediction
- After all spokes adjusted, measure state of the system
- Demonstrated graphically in the next slide...

Truing Algorithm



- Predicted lateral profiles after each spoke adjustment
- Diamonds represent lateral targets
- Blue curve represents the profile *after* first spoke adjustment
- Green represents final (trued) profile

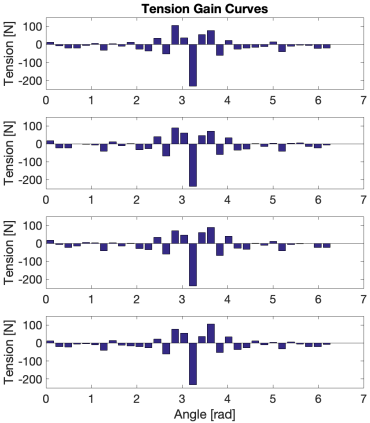
Computer Vision Validation



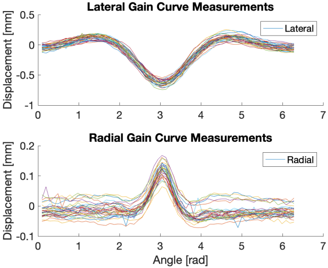
- Four dual gauge readings were recorded
- Gauges were set to 1mm, 0.5mm, 0.1mm, and 0mm
- Visual analysis and CV algorithm results compared
- Visual analysis resolution is 0.0135mm
- Results agree to $\pm$ 0.007mm

Tension Gain Curves

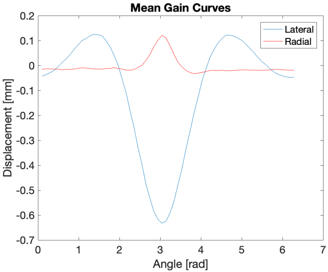
- 32 tension curves were collected
- Four distinct patterns identified:
  - Non-drive side leading
  - Drive side leading
  - Non-drive side trailing
  - Drive side trailing
- Tension meter discretization leads to bias so average curve used for all spokes



Lateral and Radial Gain Curves



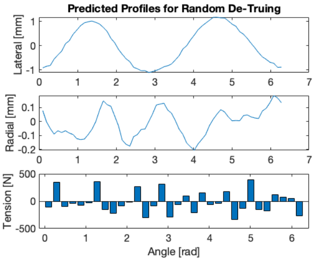
(a)

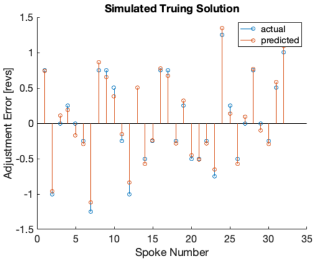


(b)

- 32 lateral and radial curves measured (a)
- Mean gain curves used for model (b)
- Curves normalized to same rim angle and side for clarity

Truing Algorithm Simulation



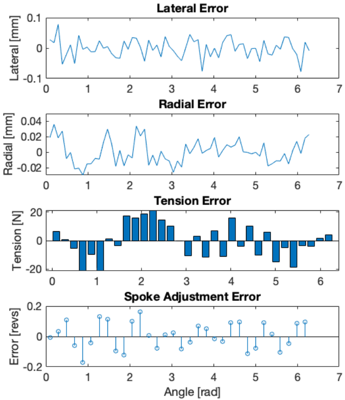


- Random spoke displacement vector, $\mathbf{d}$, generated
- Noise added to simulated profile
- Spoke displacement vector, $\hat{\mathbf{d}}$, predicted
- Weighting factors adjusted for performance

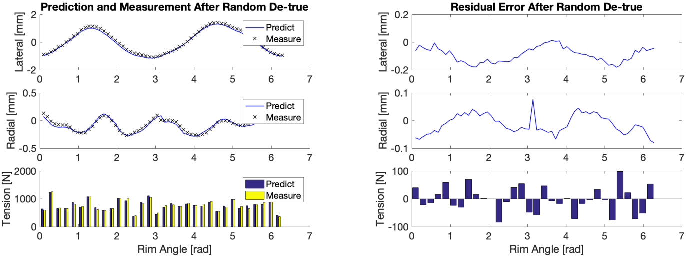
Simulation Error

Weighting factors yielding satisfactory performance:

- $\mu_v = 0.5$
- $\mu_t = 1.0e-5$

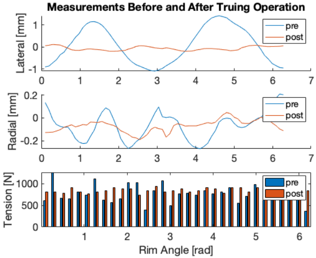


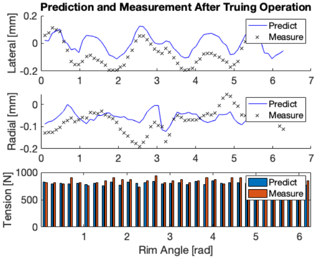
Experiment 1: De-true Test Wheel



- Simulation spoke vector $\mathbf{d}$ applied to manually-trued test wheel
- Spokes adjusted using lateral feedback
- The model predicts the experimental results well; some residual structure is evident

Experiment 2: True Test Wheel



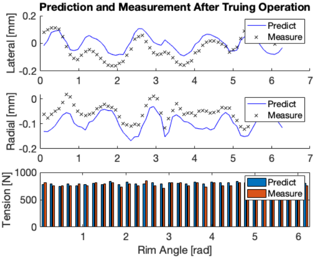


Wheel measurements ($\mu \pm \sigma$):

Parameter	Inital	Final
Lateral	0.078 ± 0.820	-0.062 ± 0.091
Radial	-0.067 ± 0.122	-0.070 ± 0.052
Tension	776 ± 205	849 ± 48

Significant improvement in all parameters

Experiment 3: Iterate Truing Algorithm

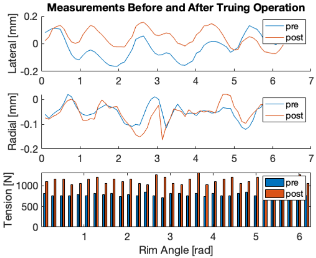


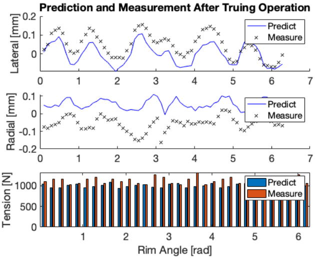
Wheel measurements ($\mu \pm \sigma$):

Parameter	Inital	Final
Lateral	-0.062 ± 0.091	-0.029 ± 0.083
Radial	-0.070 ± 0.052	-0.052 ± 0.032
Tension	849 ± 48	781 ± 33

Small but significant improvement after second iteration

Experiment 4: Target Tension=1000N



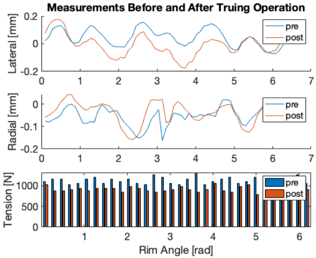


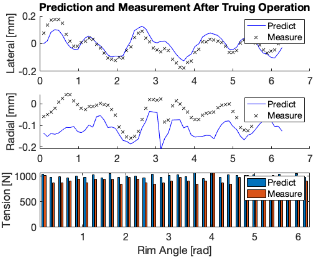
Wheel measurements ($\mu \pm \sigma$):

Parameter	Inital	Final
Lateral	-0.029 ± 0.083	0.045 ± 0.063
Radial	-0.052 ± 0.032	-0.056 ± 0.039
Tension	781 ± 33	1126 ± 76

Tension exceeds target and non-uniform

Experiment 5: Second Iteration



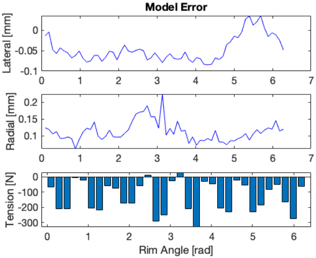


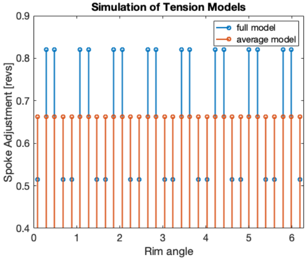
Wheel measurements ($\mu \pm \sigma$):

Parameter	Inital	Final
Lateral	0.045 ± 0.063	-0.012 ± 0.079
Radial	-0.056 ± 0.039	-0.037 ± 0.050
Tension	1126 ± 76	914 ± 63

Tension below target tension and non-uniform!

Experiment 4 & 5: Analysis

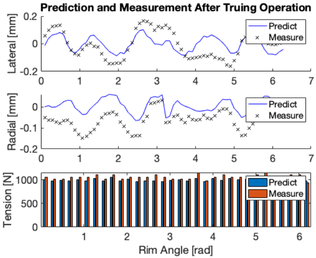




- Model error demonstrates periodic bias in tension
- A constant change in spoke displacement should result in constant tension change
- Simulation of a 200N tension change on perfectly true wheel highlights the model error

Average tension model more accurate

Experiment 6: Symmetric Tension Model



Wheel measurements ($\mu \pm \sigma$):

Parameter	Inital	Final
Lateral	-0.012 ± 0.079	-0.004 ± 0.098
Radial	-0.037 ± 0.050	-0.064 ± 0.042
Tension	914 ± 63	1056 ± 48

Symmetric tension model improves tension uniformity

Conclusions

References