Bicycle Wheel System Identification and Optimal Truing Algorithm

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July 6, 2020

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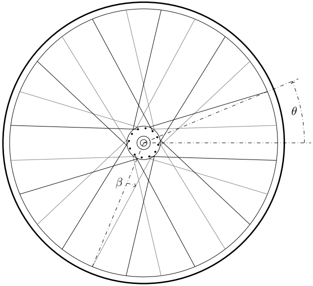
Introduction

- Wheel 'truing' is the process of adjusting spoke tension to minimize lateral, radial, and tension variations
- Commercial robotic bicycle wheel truing machines employ an open loop algorithm that operates iteratively
- This method may not achieve wheel specifications requiring human intervention

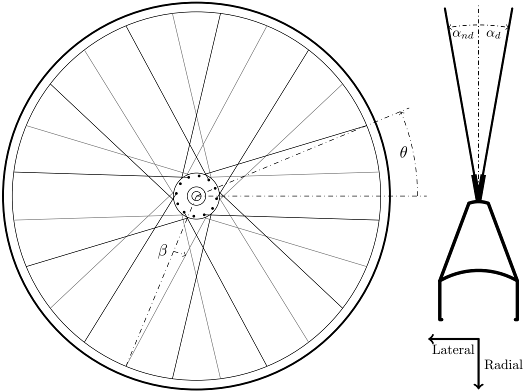
The work that follows demonstrates an optimal approach to wheel truing using system identification techniques and feedback control to achieve wheel alignment.

The Bicycle Wheel



- The bicycle wheel is a structure consisting of a rim, a hub, and spokes connecting the hub to the rim
- The spokes are under tension to provide the lateral stiffness and radial strength
- Spoke tension must be high enough to support the load but not so high that the rim buckles laterally
- Spokes patterns vary from radial to nearly tangential

Bicycle Wheel Geometry



- θ is the rim angle of the spoke
- β is the spoking angle (tangential vs radial)
- α_d and α_{nd} are the drive side and non-drive side lateral angles

Wheel Truing

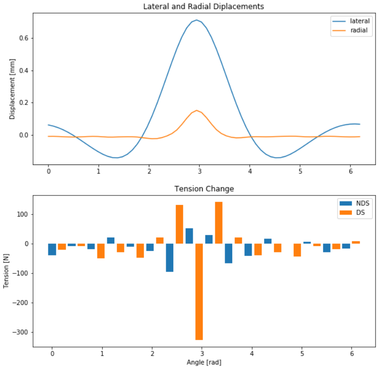
- Spoke tension is adjusted by tightening or loosening the spoke via a threaded nipple seated in the rim
- Spoke tension consists of lateral, radial, and tangential components at the rim
- Spoke tension is adjusted such that rim is ‘true’ in both lateral and radial dimensions, and desired mean tension is achieved
- Conventional truing algorithm:
  1. Adjust mean tension
  2. Minimize lateral variations
  3. Minimize radial variations
  4. Repeat until all desired specifications are met

Apparatus

- Centrimaster Comfort Wheel Truing Stand
- WheelFanatyk Digital Tension Gauge
- Canon EOS M, prime 22mm lens
- Wheel: Stans ZTR Alpha Rim, DT Swiss Competition spokes, White Industries MI5 hub

System Identification Methodology

Identify the lateral, radial and tension changes ('gain curves') induced by a unit spoke adjustment for each spoke



Theoretical gain curves derived for a generic wheel using https://github.com/dashdotrobot/bike-wheel-calc

Measurements Using Computer Vision

(a) (b) (c) (d)

Algorithm to interpret gauge displacements:

1. Reference image (a)
2. Measurement (or zero value) image (b)
3. Subtract measurement from reference (c)
4. Binary threshold and mask (d)
5. Calculate angle of centroid from gauge center
6. Calculate displacement measurement from angle

Measurement Data Vector

Measurement data collected for a wheel under test:

$$\mathbf{U} = \begin{bmatrix} u(\theta = 1\frac{2\pi}{2n_s}) \\ u(\theta = 2\frac{2\pi}{2n_s}) \\ \vdots \\ u(\theta = 2n_s\frac{2\pi}{2n_s}) \end{bmatrix} \mathbf{V} = \begin{bmatrix} v(\theta = 1\frac{2\pi}{2n_s}) \\ v(\theta = 2\frac{2\pi}{2n_s}) \\ \vdots \\ v(\theta = 2n_s\frac{2\pi}{2n_s}) \end{bmatrix} \mathbf{T} = \begin{bmatrix} t(\theta = 1\frac{2\pi}{n_s}) \\ t(\theta = 2\frac{2\pi}{n_s}) \\ \vdots \\ t(\theta = n_s\frac{2\pi}{n_s}) \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{U} \\ \mathbf{V} \\ \mathbf{T} - \bar{T} \end{bmatrix}$$

- n_s = number of spokes
- $\theta =$ rim angle measurement location where $\theta = 0$ is taken to be the valve hole
- $u(\theta), v(\theta), t(\theta) =$ lateral, radial, and tension measurements taken at rim angle θ
- $\bar{T} =$ mean spoke tension

Gain Matrices

Let $\mathbf{u_s}=u_s(\theta)$ be the lateral displacement vector (gain curve) for every discrete θ around the rim induced by turning spoke s by one rotation. The matrix of all $\mathbf{u_s}$ gain curves is defined to be the lateral 'gain' matrix Φ_u. The radial and tension gain matrices are similarly defined.

$$\Phi_u = \begin{bmatrix} \mathbf{u_1} & \mathbf{u_2} & \dots & \mathbf{u_{n_s}} \end{bmatrix}$$
$$\Phi_v = \begin{bmatrix} \mathbf{v_1} & \mathbf{v_2} & \dots & \mathbf{v_{n_s}} \end{bmatrix}$$
$$\Phi_t = \begin{bmatrix} \mathbf{t_1} & \mathbf{t_2} & \dots & \mathbf{t_{n_s}} \end{bmatrix}$$

Measurement Prediction

Let $\mathbf{d}$ be a vector of spoke rotations. Let $\mathbf{Y_b}$ be the baseline wheel measurements, that is, the state of the wheel prior to the application of $\mathbf{d}$. The predicted lateral, radial, and tension measurements, $\hat{\mathbf{Y}}$, after applying $\mathbf{d}$ to the spokes is given by:

$$\Phi = \begin{bmatrix} \Phi_u \\ \Phi_v \\ \Phi_t \end{bmatrix}$$

$$\hat{\mathbf{Y}} = \mathbf{Y_b} + \Phi \mathbf{d}$$

Multi-objective Least Squares

Compute the optimal vector of spoke rotations, $\mathbf{d}_{ls}$, from the set of measurements, $\{\mathbf{u}, \mathbf{v}, \mathbf{T}\}$ given the weighting factors μ_v and μ_t:

$$\tilde{\Phi} = \begin{bmatrix} \Phi_u \\ \Phi_v \sqrt{\mu_v} \\ \Phi_t \sqrt{\mu_t} \end{bmatrix} \quad \tilde{\mathbf{Y}} = \begin{bmatrix} \mathbf{u} \\ \mathbf{v}\sqrt{\mu_v} \\ (\mathbf{T} - \bar{T})\sqrt{\mu_t} \end{bmatrix}$$

$$\mathbf{d}_{ls} = \tilde{\Phi}^{\dagger} \tilde{\mathbf{Y}}$$

Where $\tilde{\Phi}^{\dagger}$ is the pseudo-inverse of $\tilde{\Phi}$. The weighting factors represent the tradeoff between the lateral, radial, and tension variables and are found through evaluation of the wheel specification, as well as the noise and accuracy of the measurements.

Tension Targeting

- Targeting a different tension, T_d, can be accomplished by substituting T_d for $\bar{T}$ into $\mathbf{Y}$
- However:
  - T_d is weighted when calculating $\mathbf{d}$–not desireable
  - Non-ideality in tension gain curves is amplified when T_d is significantly different than $\bar{T}$
- Note: *ensemble average tension changes do not affect lateral and radial displacements.*
- Equivalently, a constant added to $\mathbf{d}$ does not effect lateral and radial displacements
- Therefore average tension changes $(T_d - \bar{T})$ are separable from total tension changes $(\Phi_t \mathbf{d})$ and predicted with higher confidence

Tension Targeting

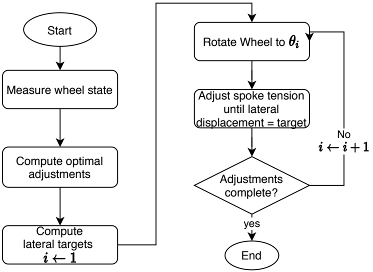
A better approach is to subtract the mean adjustment value from $\mathbf{d}_{ls}$ and to calculate a constant factor, d_{cm} that induces the desired change in mean tension:

$$\begin{aligned} \mathbf{d} &= -(\mathbf{d}_{ls} - \bar{d}_{ls}) \\ d_{cm} &= (T_d - \bar{T})/c \end{aligned}$$

where c is the proportionality constant between mean tension and mean adjustment. The predicted tension vector, $\hat{\mathbf{T}}$, becomes:

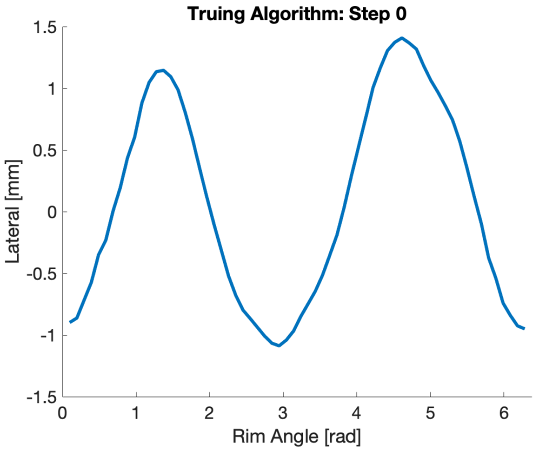
$$\begin{aligned} \hat{\mathbf{T}} &= \bar{T} + \Phi_t \mathbf{d} + d_{cm} c \\ &= T_d + \Phi_t \mathbf{d} \end{aligned}$$

Truing Algorithm



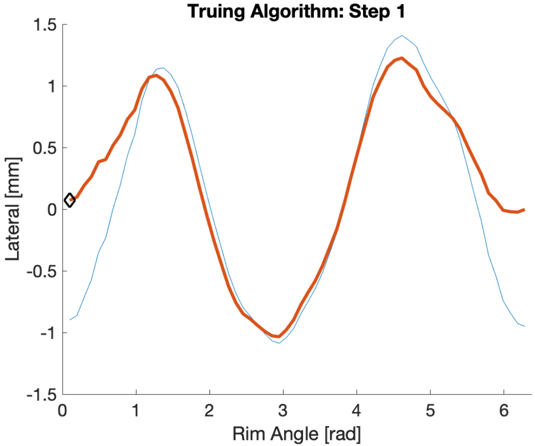
- Spoke adjustment vector: current $\rightarrow$ desired state
- Difficult to apply precisely (twist, friction)
- Use model to predict state after adjustment!

Truing Algorithm



Thick curve is predicted profile, diamond is lateral target

Truing Algorithm



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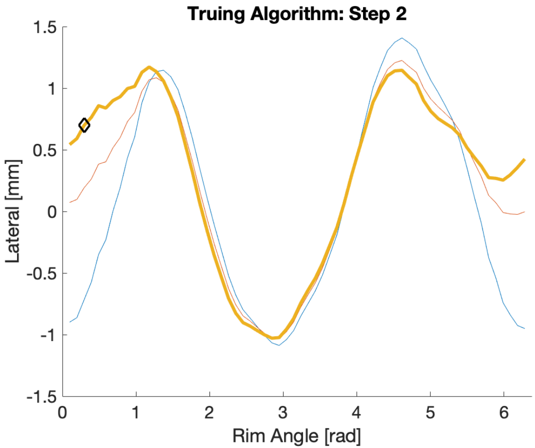
Truing Algorithm



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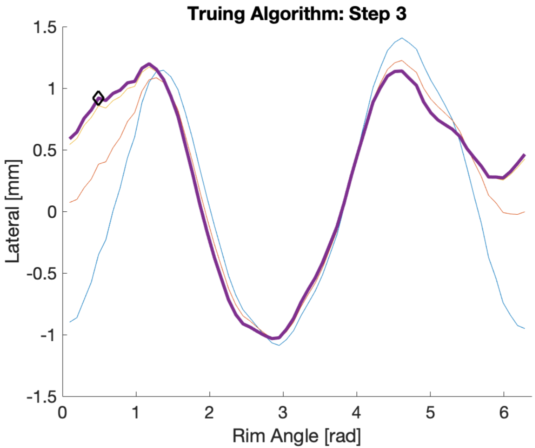
Truing Algorithm



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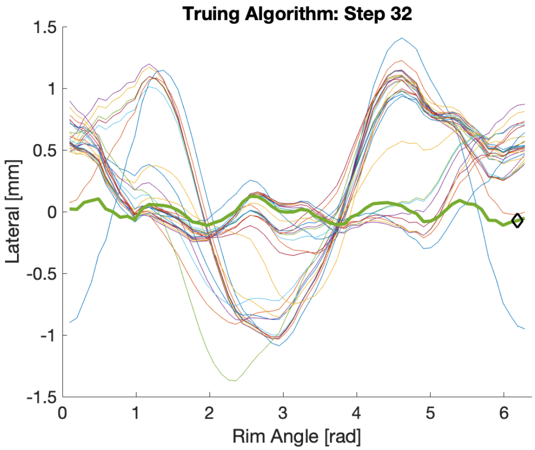
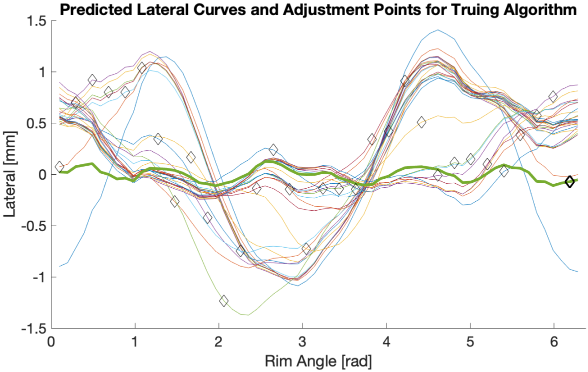
Truing Algorithm



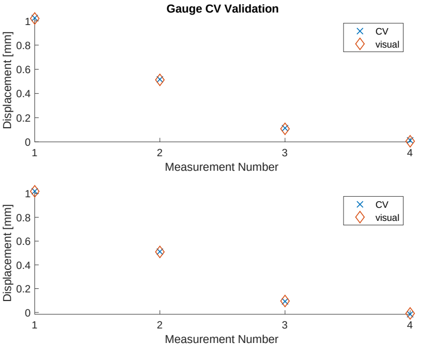
Thick curve is predicted profile, diamond is lateral target

Truing Algorithm



- Predicted lateral profiles *after* each spoke adjustment
- Diamonds represent lateral targets
- Green represents final (trued) profile

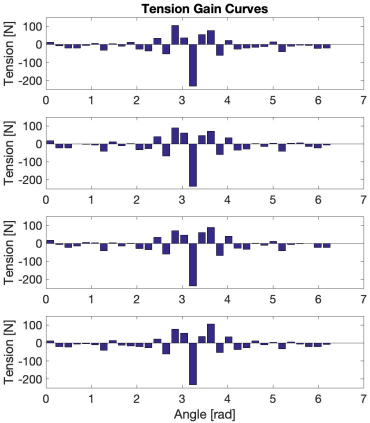
Computer Vision Validation



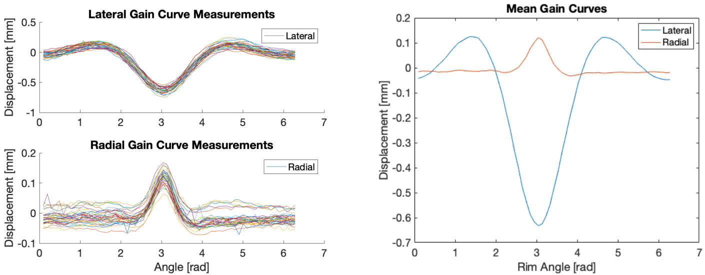
- Four dual gauge readings were recorded
- Gauges were set to 1mm, 0.5mm, 0.1mm, and 0mm
- Visual analysis and CV algorithm results compared
- Visual analysis resolution is 0.0135mm
- Results agree to $\pm$ 0.007mm

Tension Gain Curves

- 32 tension curves were collected
- Four distinct patterns identified:
  - Non-drive side leading
  - Drive side leading
  - Non-drive side trailing
  - Drive side trailing



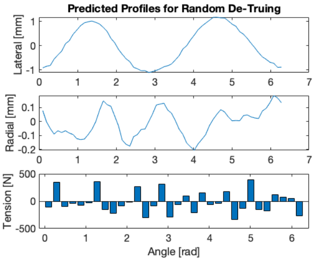
Lateral and Radial Gain Curves

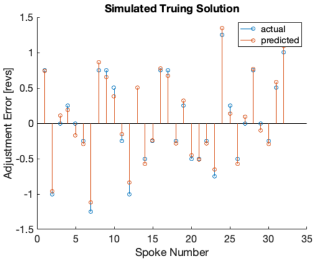


(a) (b)

- 32 lateral and radial curves measured (a)
- Mean gain curves used for model (b)
- Curves normalized to same rim angle and side for clarity

Truing Algorithm Simulation

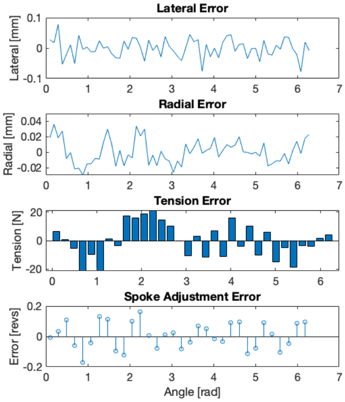




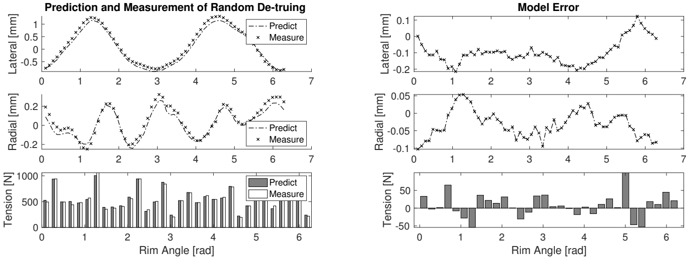
- Random spoke displacement vector, $\mathbf{d}$, generated
- Noise added to simulated profile
- Spoke displacement vector, $\hat{\mathbf{d}}$, predicted
- Weighting factors adjusted for performance

Weighting Factors

- Lateral displacement is more sensitive than radial with same spec
- Units comparison of tension to lateral $\implies \mu_t < \frac{0.6}{300}$
- Weighting factors yielding satisfactory performance:
  - $\mu_v = 0.5$
  - $\mu_t = 1.0e - 5$

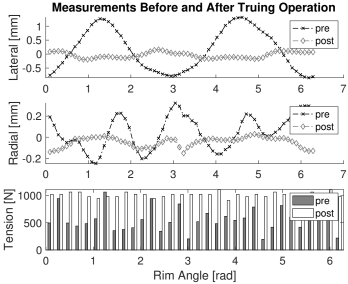


Experiment 1: De-true Test Wheel



- Simulation spoke vector $\mathbf{d}$ applied to manually-trued test wheel
- Spokes adjusted using lateral feedback
- The model predicts the experimental results well

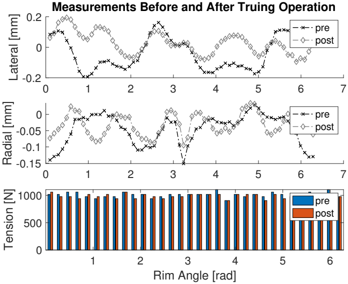
Experiment 2: True Test Wheel to 1000N



Parameter	Inital ($\mu \pm \sigma$)	Final ($\mu \pm \sigma$)
Lateral [mm]	0.160 ± 0.736	-0.037 ± 0.107
Radial [mm]	0.050 ± 0.158	-0.046 ± 0.047
Tension [N]	556 ± 211	1009 ± 45

Single iteration effectively trues a wheel

Experiment 3: Iterate Truing Algorithm



Wheel measurements ($\mu \pm \sigma$):

Parameter	Inital	Final
Lateral [mm]	-0.037 ± 0.107	0.028 ± 0.073
Radial [mm]	-0.046 ± 0.047	-0.031 ± 0.034
Tension [N]	1009 ± 45	989 ± 39

Small but significant improvement after second iteration

Conclusions

- System identification techniques used to develop linear model of a wheel subject to spoke tension inputs
- Computer vision techniques provide finer resolution from analog gauges than visual estimations
- Computer vision vastly reduces time for data collection
- The model can be used to predict:
  - Lateral displacement
  - Radial displacement
  - Tension changes
- Multi-objective least squares approximations of these parameters used to find optimal spoke displacements for wheel truing
- Wheel truing algorithm minimizes errors in tension adjustment using lateral feedback

Conclusions

- Truing performance limited by spoke tension adjustment resolution:
  - Spoke twist during adjustment
  - Friction at nipple/spoke thread interface
- Truing performance also limited by tension measurement accuracy and resolution
  - Small effects due to spoke patterns not captured by model

Future Work

- Comparison with theoretical models
- Extension of model to include tension offset (side to side)
- Improved resolution/accuracy tensiometer
- Mechatronic implementation

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Spoke Tension Measurements

WheelFanatyk spoke tension meter. Image credit: https://www.wheelfanatyk.com

- Digital spoke tension measurements
- Data collected through USB to PC
- Meter collects displacement of spoke by calibrated spring
- Reference measurement accounts for variation of spoke thickness
- Tension values interpolated from calibration table