

Bicycle Wheel  
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Identification  
and Optimal  
Truing  
Algorithm

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# Bicycle Wheel System Identification and Optimal Truing Algorithm

Aaron Hunter

October 7, 2019

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# Introduction

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- Wheel ‘truizing’ is the process of adjusting spoke tension to minimize lateral and radial variations
- Commercial robotic bicycle wheel truing machines use a heuristic truing method.
- This method mimics the actions a human might take to perform the task.
- This method can be inefficient or ineffective requiring human intervention

The work that follows demonstrates an optimal approach to wheel truing using system identification techniques and feedback control to achieve wheel alignment.

# The Bicycle Wheel

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- The bicycle wheel is a structure consisting of a rim, a hub, and spokes connecting the hub to the rim
- The spokes are under tension to provide the stiffness to wheel structure
- The rim is under compression
- The wheel can be considered a system of springs in parallel and series combinations
- Spoke tension must be high enough to support the load (bicycle and rider) but not so high that the rim warps laterally
- Spokes patterns vary from radial to nearly tangential

# Bicycle Wheel Geometry

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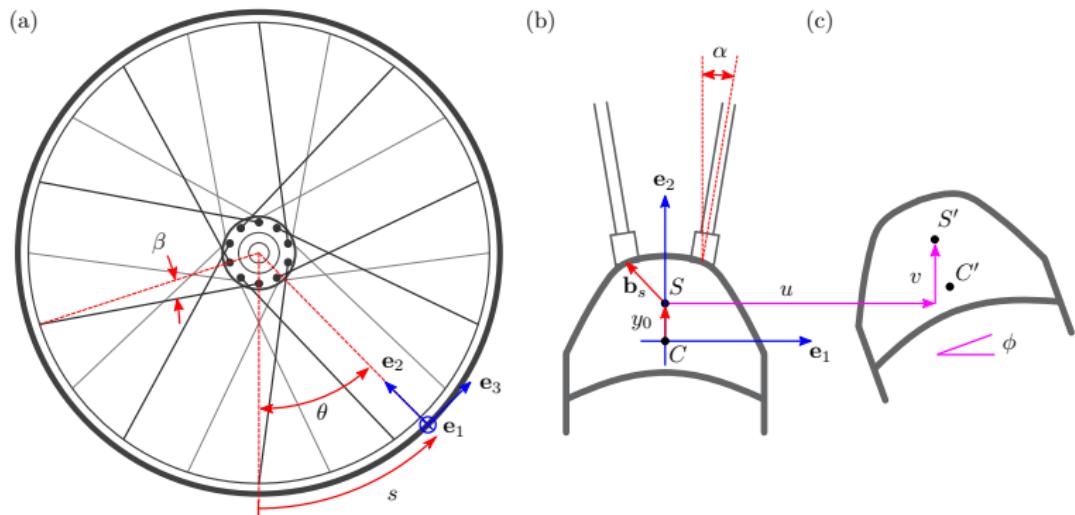


Image credit: Ford, Matthew, *Reinventing the Wheel: Stress Analysis, Stability, and Optimization of the Bicycle Wheel*, PhD. Dissertation Northwestern University, December 2018.

# Wheel Truing

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- Spoke tension is adjusted by changing the effective length of the spoke via a threaded nipple seated in the rim
- Spoke tension consists of lateral, radial, and tangential components at the rim
- Spoke tension is adjusted such that rim is ‘true’ in both lateral and radial dimensions, and desired mean tension is achieved
- Conventional truing algorithm:
  - 1 Adjust mean tension
  - 2 Minimize lateral variations
  - 3 Minimize radial variations
  - 4 Repeat until all desired specifications are met

# Apparatus

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References

- Centrimaster Comfort Wheel Truing Stand
- WheelFanatyk Digital Tension Gauge
- Canon EOS M, prime 22mm lens
- Wheel: Stans ZTR Alpha Rim, DT Swiss Competition spokes, White Industries MI5 hub



# System Identification Methodology

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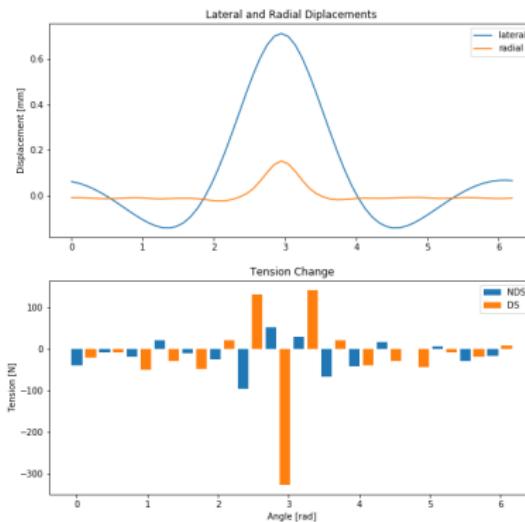
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Identify the lateral, radial and tension changes ('gain curves') induced by a unit spoke adjustment for each spoke



Theoretical gain curves derived for a generic wheel using <https://github.com/dashdotrobot/bike-wheel-calc>

# Measurements Using Computer Vision

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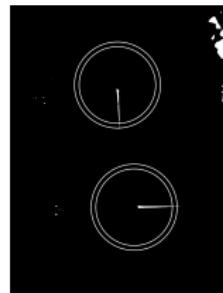
(a)



(b)



(c)



(d)

Algorithm to interpret gauge displacements:

- ① Reference image (a)
- ② Measurement (or zero value) image (b)
- ③ Subtract measurement from reference (c)
- ④ Binary threshold and mask (d)
- ⑤ Calculate angle of centroid from gauge center
- ⑥ Calculate displacement measurement from angle

# Spoke Tension Measurements



WheelFanatyk spoke tension meter. Image credit: <https://www.wheelfanatyk.com>

- Digital spoke tension measurements
- Data collected through USB to PC
- Meter collects displacement of spoke by calibrated spring
- Reference measurement accounts for variation of spoke thickness
- Tension values interpolated from calibration table

# Measurement Data Vector

Measurement data collected for a wheel under test:

$$\mathbf{U} = \begin{bmatrix} u(\theta = 1\frac{2\pi}{2n_s}) \\ u(\theta = 2\frac{2\pi}{2n_s}) \\ \vdots \\ u(\theta = 2n_s\frac{2\pi}{2n_s}) \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} v(\theta = 1\frac{2\pi}{2n_s}) \\ v(\theta = 2\frac{2\pi}{2n_s}) \\ \vdots \\ v(\theta = 2n_s\frac{2\pi}{2n_s}) \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} t(\theta = 1\frac{2\pi}{n_s}) \\ t(\theta = 2\frac{2\pi}{n_s}) \\ \vdots \\ t(\theta = n_s\frac{2\pi}{n_s}) \end{bmatrix}$$
$$\mathbf{Y} = \begin{bmatrix} \mathbf{U} \\ \mathbf{V} \\ \mathbf{T} \end{bmatrix}$$

- $n_s$  = number of spokes
- $\theta$  = rim measurement location where  $\theta = 0$  is taken to be the valve hole
- $u(\theta)$  = lateral measurement at  $\theta$
- $v(\theta)$  = radial measurement at  $\theta$
- $t(\theta)$  = spoke tension measurement at  $\theta$

# Gain Matrices

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Let  $\mathbf{u}_s = u_s(\theta)$  be the lateral displacement vector (gain curve) for every discrete  $\theta$  around the rim induced by turning spoke  $s$  by one rotation. The matrix of all  $\mathbf{u}_s$  gain curves is defined to be the lateral 'gain' matrix  $\Phi_u$ . The radial and tension gain matrices are similarly defined.

$$\Phi_u = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \dots \quad \mathbf{u}_{n_s}]$$

$$\Phi_v = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \dots \quad \mathbf{v}_{n_s}]$$

$$\Phi_t = [\mathbf{t}_1 \quad \mathbf{t}_2 \quad \dots \quad \mathbf{t}_{n_s}]$$

# Measurement Prediction

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Let  $\mathbf{d}$  be a vector of spoke rotations. Let  $\mathbf{Y}_b$  be the baseline wheel measurements, that is, the state of the wheel prior to the application of  $\mathbf{d}$ . The predicted lateral, radial, and tension measurements,  $\hat{\mathbf{Y}}$ , after applying  $\mathbf{d}$  to the spokes is given by:

$$\Phi = \begin{bmatrix} \Phi_u \\ \Phi_v \\ \Phi_t \end{bmatrix}$$

$$\hat{\mathbf{Y}} = \mathbf{Y}_b + \Phi\mathbf{d}$$

# Multi-objective Least Squares

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Predict the set of spoke rotations,  $\hat{\mathbf{d}}$ , that yield given set of measurements,  $\mathbf{Y}$  given the weighting factors  $\mu_v$ ,  $\mu_t$  and desired state  $\mathbf{Y}_d = [\mathbf{u}_d, \mathbf{v}_d, \mathbf{T}_d]^T$ :

$$\tilde{\Phi} = \begin{bmatrix} \Phi_u \\ \Phi_v \sqrt{\mu_v} \\ \Phi_t \sqrt{\mu_t} \end{bmatrix} \Delta \tilde{\mathbf{Y}} = \begin{bmatrix} \mathbf{u} - \mathbf{u}_d \\ (\mathbf{v} - \mathbf{v}_d) \sqrt{\mu_v} \\ (\mathbf{T} - \mathbf{T}_d) \sqrt{\mu_t} \end{bmatrix}$$
$$\hat{\mathbf{d}} = \tilde{\Phi}^\dagger \Delta \tilde{\mathbf{Y}}$$

Where  $\tilde{\Phi}^\dagger$  is the pseudo-inverse of  $\tilde{\Phi}$ . The weighting factors represent the tradeoff between the lateral, radial, and tension variables and are found through evaluation of the wheel specification and exhaustive simulation.

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- $\hat{\mathbf{d}}$  is the best fit of spoke rotations that result in the measured state relative to some desired state
- To achieve the desired state apply  $-\hat{\mathbf{d}}$  to the system
- Spoke turns are difficult to apply accurately. Instead predict the state of the system after each spoke adjustment
- Adjust the spoke until the *lateral* displacement at the spoke location matches prediction
- After all spokes adjusted, measure state of the system
- Demonstrated graphically in the next slides...

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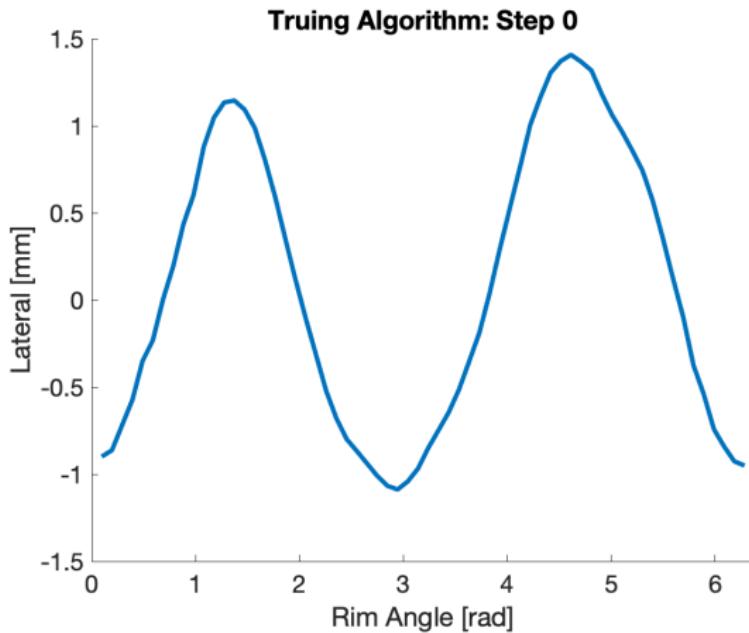
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Thick curve is predicted profile, diamond is lateral target

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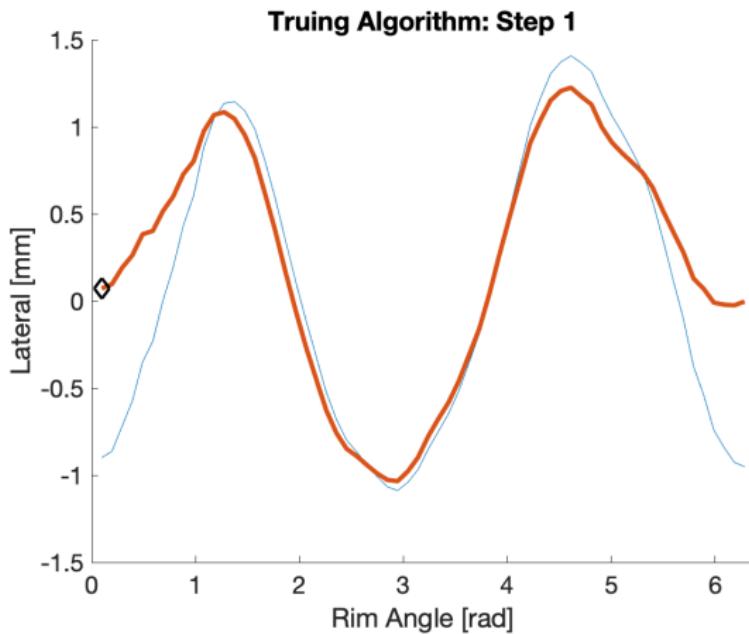
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Thick curve is predicted profile, diamond is lateral target

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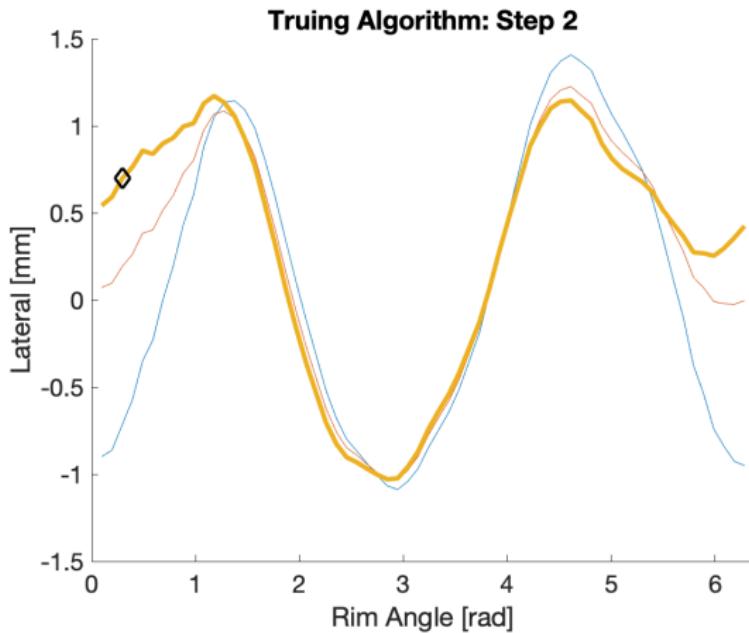
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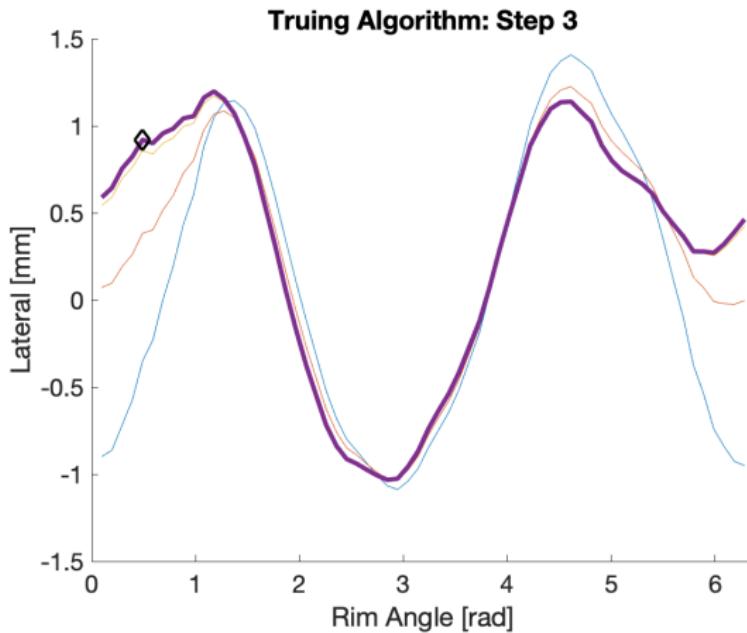
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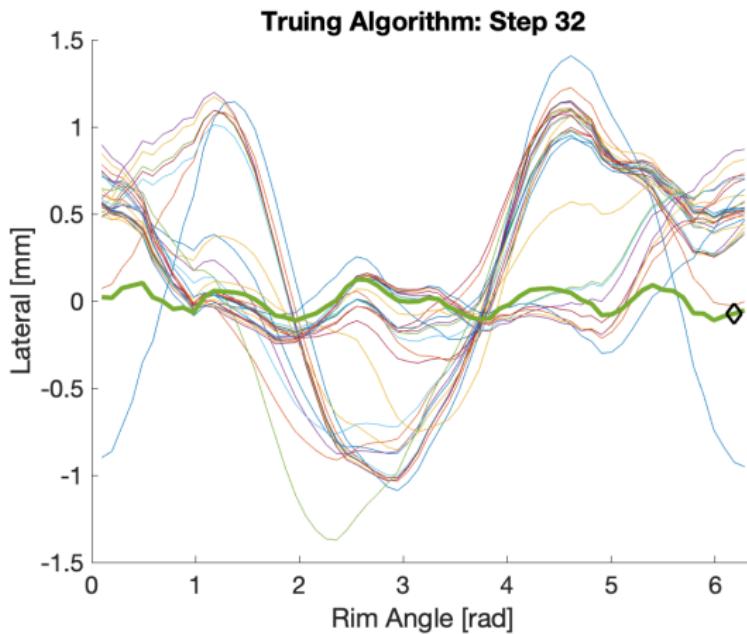
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Thick curve is predicted profile, diamond is lateral target

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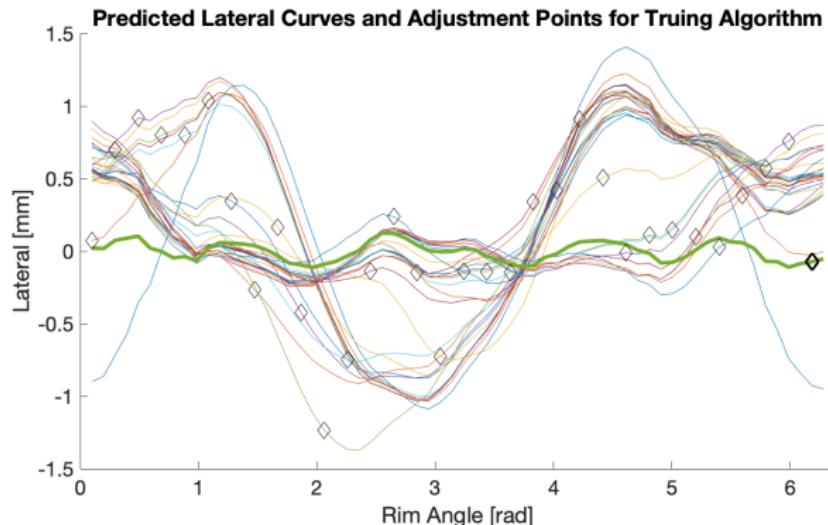
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- Predicted lateral profiles *after* each spoke adjustment
- Diamonds represent lateral targets
- Green represents final (trued) profile

# Computer Vision Validation

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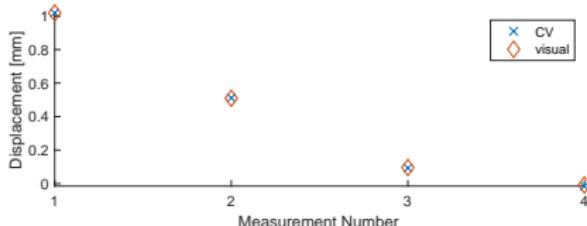
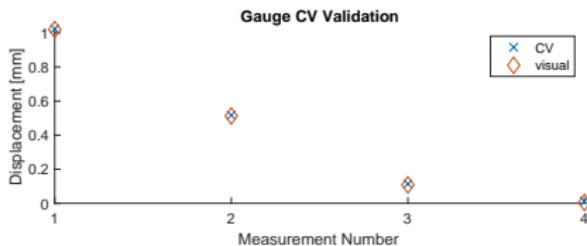
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- Four dual gauge readings were recorded
- Gauges were set to 1mm, 0.5mm, 0.1mm, and 0mm
- Visual analysis and CV algorithm results compared
- Visual analysis resolution is 0.0135mm
- Results agree to  $\pm 0.007\text{mm}$

# Tension Gain Curves

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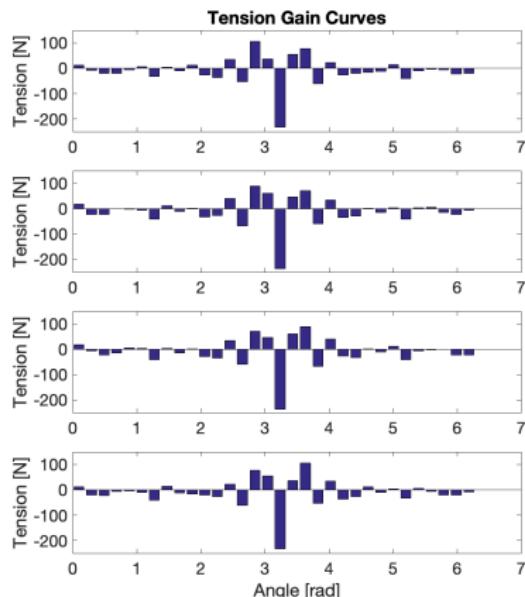
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- 32 tension curves were collected
- Four distinct patterns identified:
  - Non-drive side leading
  - Drive side leading
  - Non-drive side trailing
  - Drive side trailing
- Tension meter model biased (experiments 4,5)  
 $\implies$  final model changed to average of all patterns



# Lateral and Radial Gain Curves

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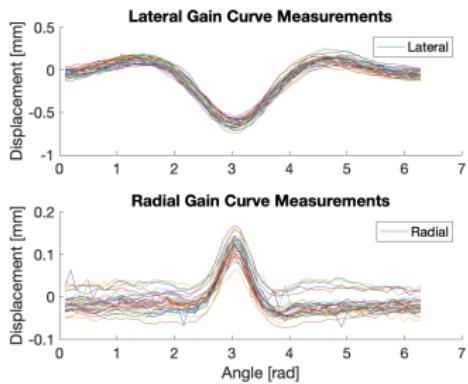
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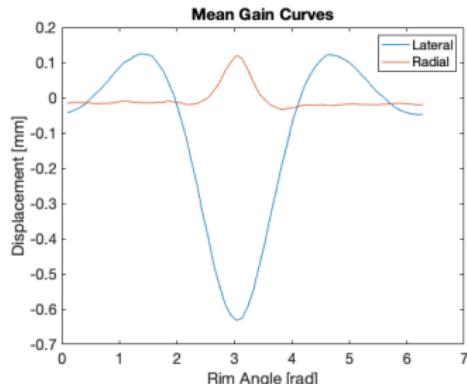
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(a)



(b)

- 32 lateral and radial curves measured (a)
- Mean gain curves used for model (b)
- Curves normalized to same rim angle and side for clarity

# Truing Algorithm Simulation

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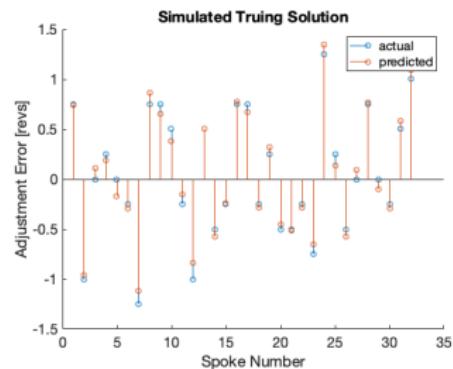
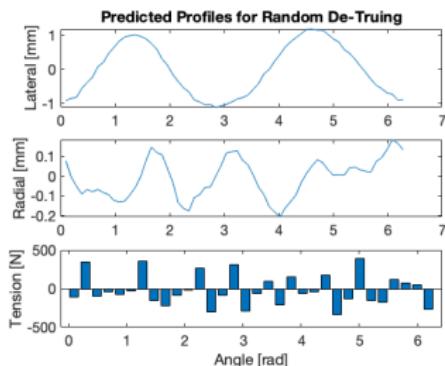
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- Random spoke displacement vector,  $\mathbf{d}$ , generated
- Noise added to simulated profile
- Spoke displacement vector,  $\hat{\mathbf{d}}$ , predicted
- Weighting factors adjusted for performance

# Simulation Error

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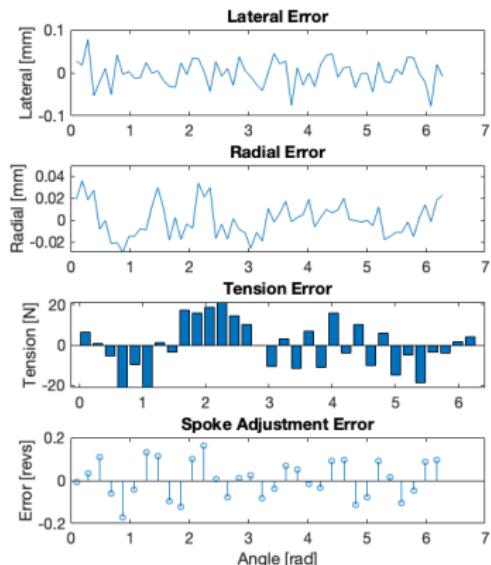
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- Lateral displacement is more sensitive than radial with same spec
- Units comparison of tension to lateral  
 $\implies \mu_t < \frac{0.6}{300}$
- Weighting factors yielding satisfactory performance:
  - $\mu_v = 0.5$
  - $\mu_t = 1.0e - 5$



# Experiment 1: De-true Test Wheel

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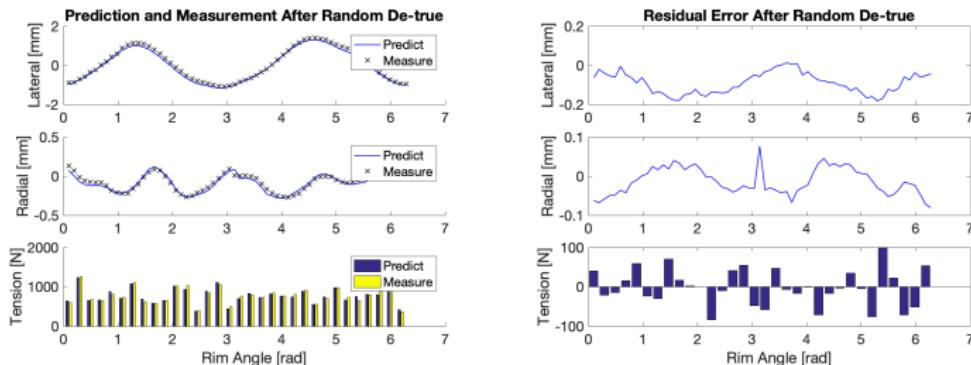
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- Simulation spoke vector  $\mathbf{d}$  applied to manually-trued test wheel
- Spokes adjusted using lateral feedback
- The model predicts the experimental results well; some residual structure is evident

# Experiment 2: True Test Wheel

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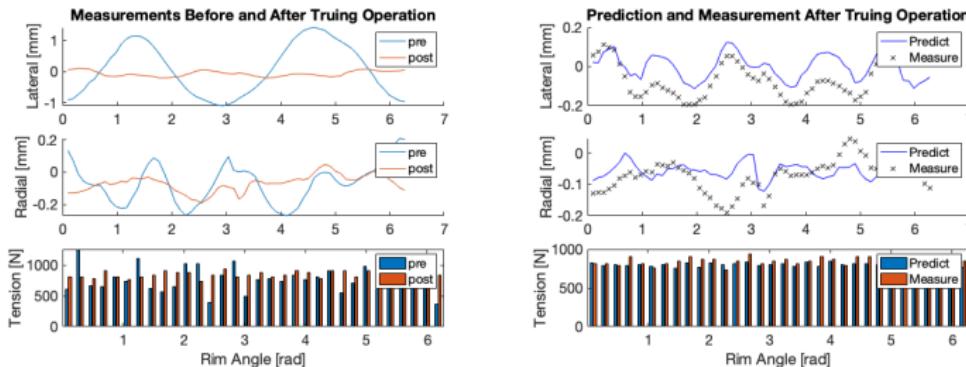
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Wheel measurements ( $\mu \pm \sigma$ ):

Parameter	Initial	Final
Lateral [mm]	$0.078 \pm 0.820$	$-0.062 \pm 0.091$
Radial [mm]	$-0.067 \pm 0.122$	$-0.070 \pm 0.052$
Tension [N]	$776 \pm 205$	$849 \pm 48$

Single iteration effectively trues a wheel

# Experiment 3: Iterate Truing Algorithm

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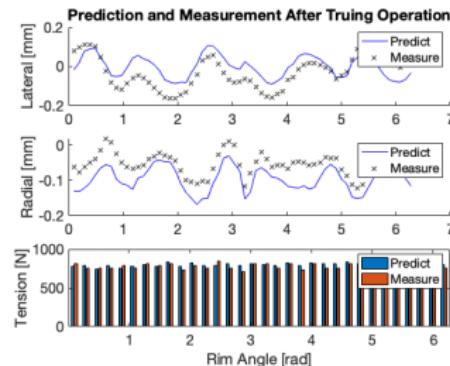
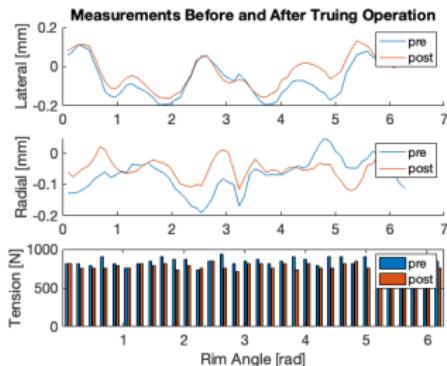
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Wheel measurements ( $\mu \pm \sigma$ ):

Parameter	Initial	Final
Lateral [mm]	$-0.062 \pm 0.091$	$-0.029 \pm 0.083$
Radial [mm]	$-0.070 \pm 0.052$	$-0.052 \pm 0.032$
Tension [N]	$849 \pm 48$	$781 \pm 33$

Small but significant improvement after second iteration

# Experiment 4: Target Tension=1000N

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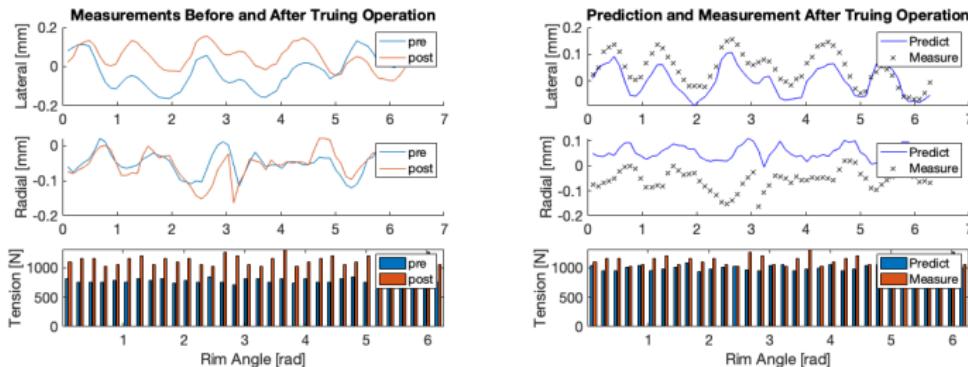
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Wheel measurements ( $\mu \pm \sigma$ ):

Parameter	Initial	Final
Lateral [mm]	$-0.029 \pm 0.083$	$0.045 \pm 0.063$
Radial [mm]	$-0.052 \pm 0.032$	$-0.056 \pm 0.039$
Tension [N]	$781 \pm 33$	$1126 \pm 76$

Tension exceeds target and non-uniform

# Experiment 5: Second Iteration

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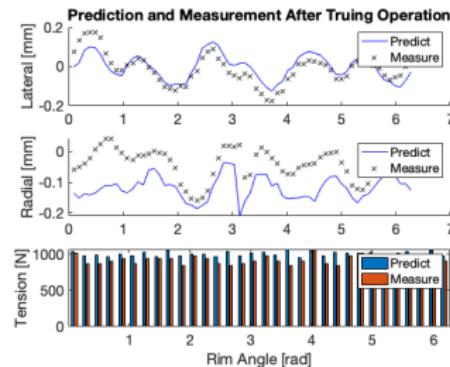
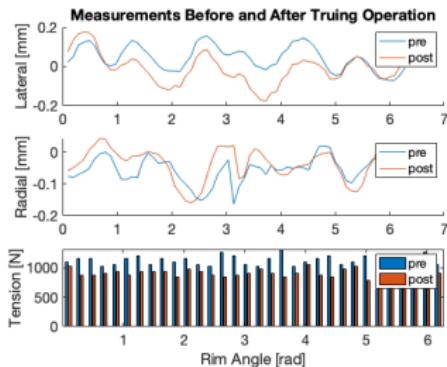
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Wheel measurements ( $\mu \pm \sigma$ ):

Parameter	Initial	Final
Lateral [mm]	$0.045 \pm 0.063$	$-0.012 \pm 0.079$
Radial [mm]	$-0.056 \pm 0.039$	$-0.037 \pm 0.050$
Tension [N]	$1126 \pm 76$	$914 \pm 63$

Tension below target tension and non-uniform!

# Experiment 4 & 5: Analysis

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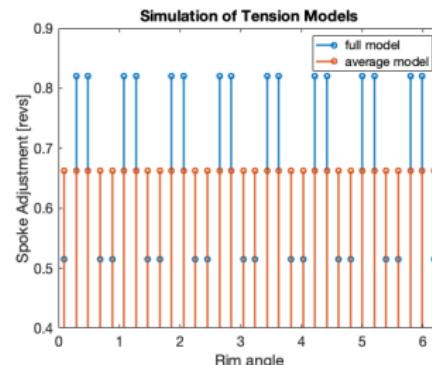
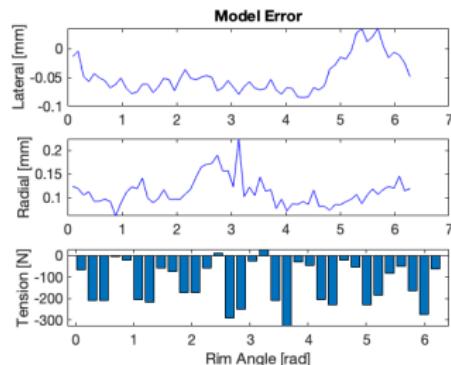
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- Model error demonstrates periodic bias in tension
- A constant change in spoke displacement should result in constant tension change
- Simulation of a 200N tension change on perfectly true wheel highlights the model error

Change tension model to average

# Experiment 6: Average Tension Model

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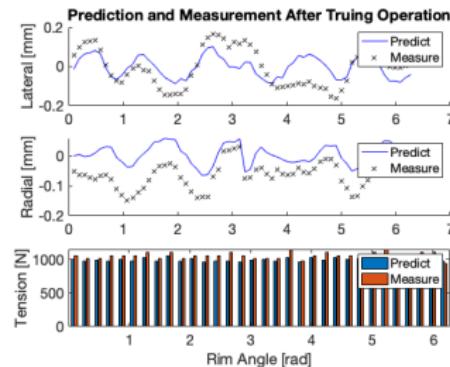
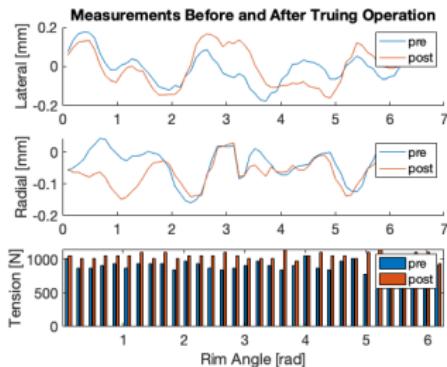
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Wheel measurements ( $\mu \pm \sigma$ ):

Parameter	Initial	Final
Lateral [mm]	$-0.012 \pm 0.079$	$-0.004 \pm 0.098$
Radial [mm]	$-0.037 \pm 0.050$	$-0.064 \pm 0.042$
Tension [N]	$914 \pm 63$	$1056 \pm 48$

Symmetric tension model improves tension uniformity

# Conclusions

- System identification techniques used to develop linear model of a wheel subject to spoke tension inputs
- Computer vision techniques provide finer resolution from analog gauges than visual estimations
- Computer vision vastly reduces time for data collection
- The model can be used to predict:
  - Lateral displacement
  - Radial displacement
  - Tension changes
- Multi-object least squares approximations of these parameters used to find optimal spoke displacements for wheel truing
- Wheel truing algorithm minimizes errors in tension adjustment using lateral feedback

# Conclusions

- Truing performance limited by spoke tension adjustment resolution:
  - Spoke twist during adjustment
  - Friction at nipple/spoke thread interface
- Truing performance also limited by tension measurement accuracy and resolution
  - Small effects due to spoke patterns not captured by model

# Future Work

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- Comparison with theoretical models
- Extension of model to include tension offset (side to side)
- Improved resolution/accuracy tensiometer
- Mechatronic implementation

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