CS 161 Intro. To Artificial Intelligence

Week 10, Discussion 1B

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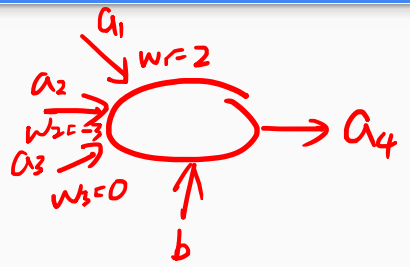
Today’s Topics

- **Neural Network (NN)**
  - Neurons
  - Feedforward NN
  - Neurons with Step Activation Functions
  - NN as a Function
  - Training NN
  - Strength & Limitation of NN
- Final Review

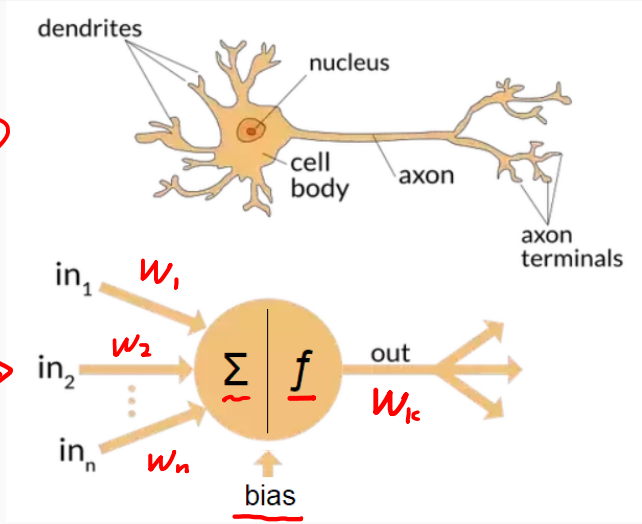
Neurons

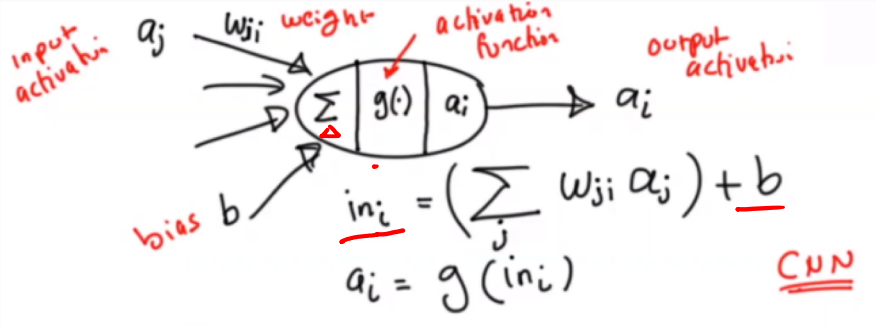
Important Terms:

- Activations
  - input
  - output
- Weights
- Bias
- Activation functions – $g(x)$
  - Binary step functions
    - E.g. Step, Sign
  - Linear functions
  - Non-linear functions
    - E.g. Sigmoid, ReLU



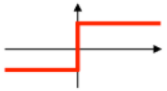
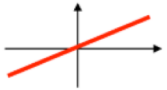
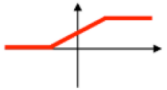
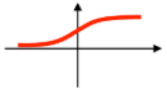
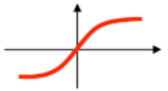
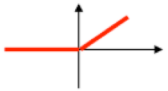
$$a_4 = g(2a_1 - 3a_2 + b)$$





Activation Functions

- Activation functions
  - Binary step functions
    - E.g. Step, Sign
  - Linear functions
    - Same as linear regression
  - Non-linear functions
    - E.g. Sigmoid, ReLU

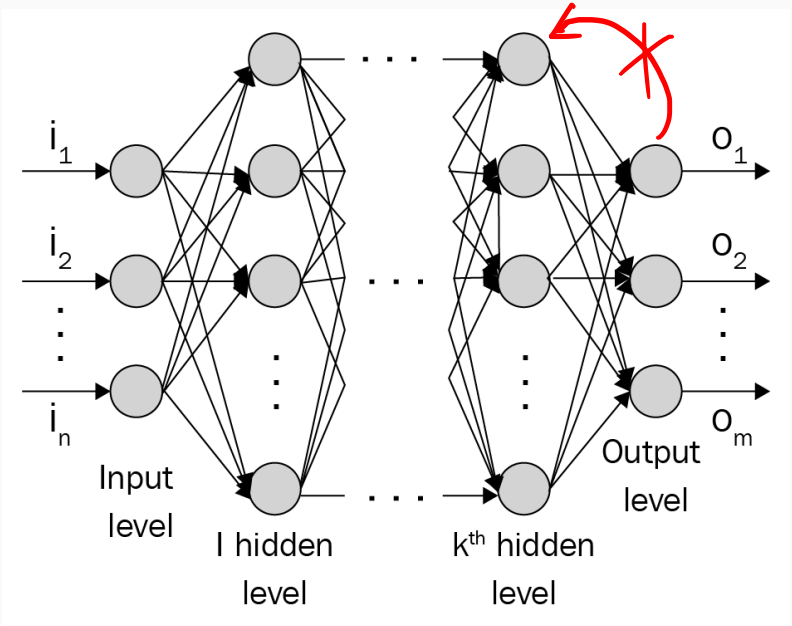
Activation function	Equation	Example	1D Graph
Unit step (Heaviside)	$\phi(z) = 1$ if $z \geq t$ $\phi(z) = 0$ if $z < t$	Perceptron variant	
Sign (Signum)	$\phi(z) = 1$ if $z \geq 0$ $\phi(z) = -1$ if $z < 0$	Perceptron variant	
Linear	$\phi(z) = z$	Adaline, linear regression	
Piece-wise linear	$\phi(z) = \begin{cases} 1, & z \geq \frac{1}{2}, \\ z + \frac{1}{2}, & -\frac{1}{2} < z < \frac{1}{2}, \\ 0, & z \leq -\frac{1}{2}, \end{cases}$	Support vector machine	
Logistic (sigmoid)	$\phi(z) = \frac{1}{1 + e^{-z}}$	Logistic regression, Multi-layer NN	
Hyperbolic tangent	$\phi(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$	Multi-layer Neural Networks	
Rectifier, ReLU (Rectified Linear Unit)	$\phi(z) = max(0, z)$	Multi-layer Neural Networks	

Source of image: http://rasbt.github.io/mlxtend/user_guide/general_concepts/activation-functions/

Feedforward NN

A NN is also a universal function approximator

- A simple NN can represent a wide variety of functions when given appropriate parameters
- **Feedforward NN**: a NN that connections between the nodes do not form a cycle. Recurrent NNs are not feedforward

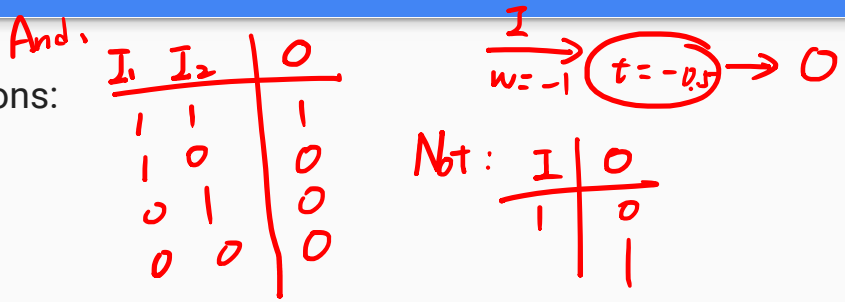


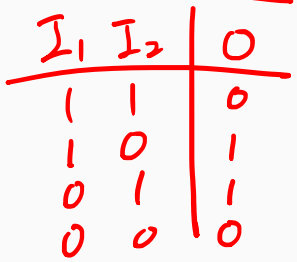
Source of image: https://subscription.packtpub.com/book/big_data_and_business_intelligence/9781789346640/1/ch01lvl1sec12/architecture-of-a-simple-neural-network

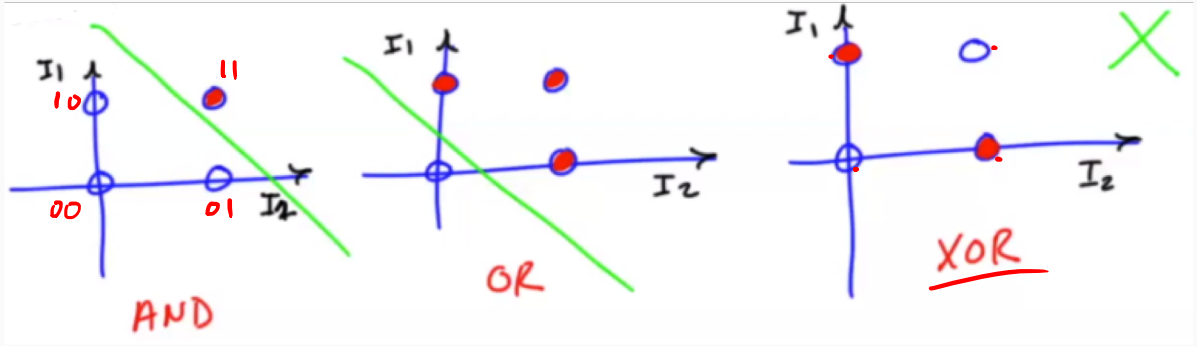
Neurons with Step Activation Functions

Example Functions:

- And
- Or
- Not
- Limitation:
  - A single neuron can only represent linearly separable function
  - E.g. XOR won't work

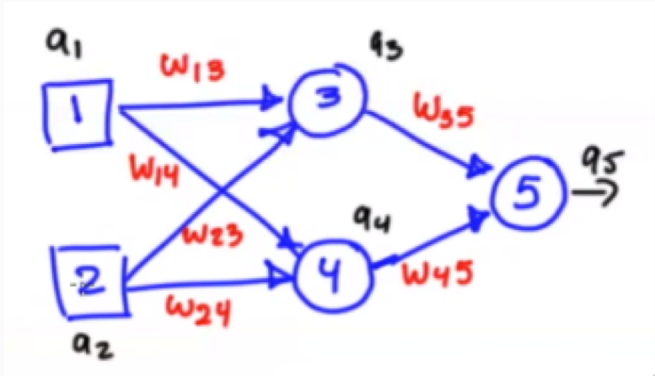






NN as a Function

$$a_5 = g(w_{35}a_3 + a_4 w_{45})$$
$$= g(w_{35}\, g(w_{13}a_1 + w_{23}a_2) + w_{45}\, g(w_{14}a_1 + w_{24}a_2))$$

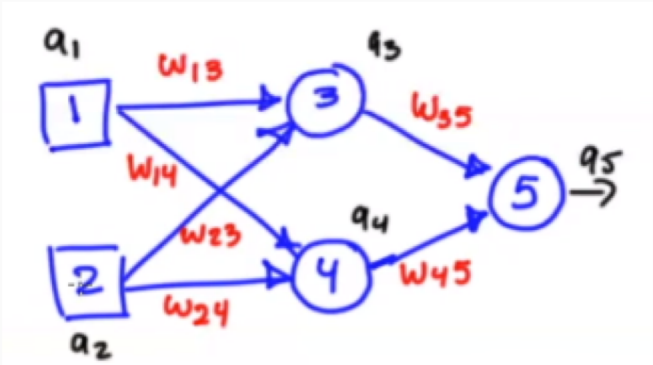


- $a_5 = f(a_1, a_2, w_{13}, w_{14}, \ldots, w_{45})$
- If we are given the dataset, then given each input (data case), output activation is a function of weights
  - $\boldsymbol{a_5 = f(w_{13}, w_{14}, \ldots, w_{45})}$

	a_1	a_2	a_5
I_1			O_1
I_2			O_2
⋮			⋮
I_n			O_n

Training NNs

$$a_5 = g(w_{35} a_3 + a_4 w_{45})$$
$$= g(w_{35}\, g(w_{13} a_1 + w_{23} a_2) + w_{45}\, g(w_{14} a_1 + w_{24} a_2))$$



Loss function is used to find **optimal weights**:

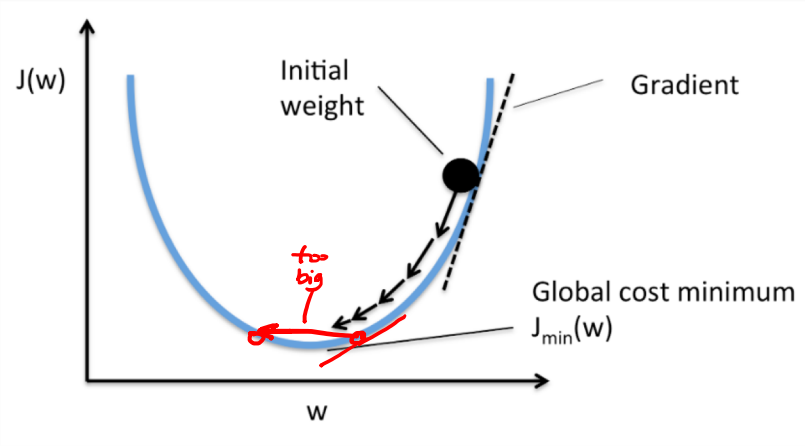
- Cross Entropy (CE)
- Mean Square Error (MSE):
  - MSE = $\frac{1}{n}\sum_{i=1}^{n}(NN(I_i) - O_i)^2$
  - $NN(I_i)$ is $\mathbf{f_i}(\boldsymbol{w_1}, \boldsymbol{w_2}, \ldots, \boldsymbol{w_k})$

	a_1	a_2	a_5
I_1			O_1
I_2			O_2
⋮	⋮		⋮
I_n			O_n

Training NNs

Loss function is what we want to optimize:

- Mean Square Error (MSE):
  - MSE = $\frac{1}{n}\sum_{i=1}^{n}(NN(I_i) - O_i)^2$
  - $NN(I_i)$ is $\mathbf{f_i}(\boldsymbol{w_1}, \boldsymbol{w_2}, \ldots, \boldsymbol{w_k})$

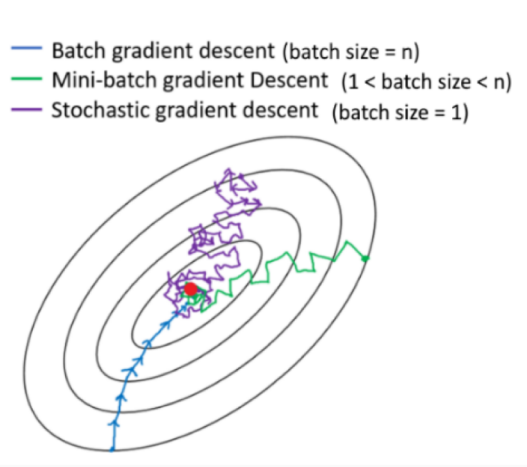


Gradient Descent (GD) is often used to find weights that optimize loss functions

- Partial derivatives: $(\frac{\delta f}{\delta w_1}, \frac{\delta f}{\delta w_2}, \ldots, \frac{\delta f}{\delta w_k})$ → this vector is called **gradient**
- If step size is too big, we may miss the optimal value
- GD has many variations (e.g. Adam optimizer), can have different step sizes, etc.
- Calculation proceeds backwards through the network → **backpropagation**

Source of image: http://rasbt.github.io/mlxtend/user_guide/general_concepts/gradient-optimization/

Training NNs



Other important concepts about training NNs:

- Performance metric: the **accuracy** obtained from test or validation data
- Train / validation / test data:
  - Test data: should never be used or seen in the training step
  - Validation data: normally part of the training data, may be used for cross-validation
- Epoch: one iteration of gradient descent(s)
  - Classical GD goes through all training examples to compute MSE = $\frac{1}{n}\sum_{i=1}^{n}(NN(I_i) - O_i)^2$
  - **Stochastic GD (SGD)** goes through single examples or batches to compute MSE → more efficient
- Batch (mini-batch): a subset of the training data → no need to compute MSE on entire dataset
  - Often divided randomly
  - Batch size is a hyper-parameter (e.g. 32, 64, 128)
- Stopping criteria: can based on # of epochs, loss and performance metrics (using validation data)

Source of image: https://datascience.stackexchange.com/questions/52884/possible-for-batch-size-of-neural-network-to-be-too-small

Strength & Limitation of NN

Strength:

- Universal function estimator – very expressive
- Conceptually easy to build
- Allow us to do things that couldn't do before
    - E.g. CNN for image analysis

Limitation:

- Requires huge amount of data
- Not robust
- Results are not easy to explain → needs explainable AI

Today’s Topics

- Neural Network (NN)
  - Neurons
  - Feedforward NN
  - NN as a Function
  - Training NN
  - Strength & Limitation of NN
- **Review of Final**

Final Review – Content before Midterm

LISP

- quote or ‘ : everything under it is kept symbolic
- nil or (): empty list
- car: first element of the list, cdr: the rest of the list (always a list)
- (**cons** arg1 arg2): reverse of car+cdr
- (**list** arg1 … argn): construct a list ‘(arg1 … argn)
- (**append** ‘(l11 l12 …) ‘(l21 l22 …) … ‘(ln1 ln2 …)): ‘(l11 l12 … l21 l22 … ln1 ln2 ...)
- predicates: atom, listp, null, equal
- (cond (cond1 value1) (cond2 value2) … (condn valuen))
- (let ((var1 value1) … (varn valuem)) (expression))
    - let: parallel assignment, let*: sequential assignment

Final Review – Content before Midterm

LISP

- (defun functionName (arg1 … argn) (expression))
- Calling a function: (functionName arg1 … argn)
- General form for a lisp recursion function

```
(defun functionName (arg1 … argn)
        (cond
                (baseCase someValue)
                (Case1 someValue/recursiveCall)
                …
                (t someValue/recursiveCall)
        )
)
```

Final Review – Content before Midterm

SEARCH

- Search Problem Formulation
  - Initial state, State space, Actions, Transition model, Goal Test
  - 8 queens - complete formulation and incremental formulation
- State space and search tree
- Solution
  - A path from initial state to goal state
- Node **generation** and **expansion**
- **Properties** of search strategies
  - Completeness, Optimality, Time complexity, Space complexity

Final Review – Content before Midterm

Uninformed Search:

- **Breadth-first search**: expands the shallowest nodes first
    - Complete, optimal for unit step costs, exponential space complexity.
- **Uniform-cost search**: expands the node with lowest path cost
    - Complete, optimal
- **Depth-first search**: expands the deepest unexpanded node first.
    - Neither complete nor optimal, but has linear space complexity.
- **Depth-limited search:** adds a depth bound to DFS
- **Iterative deepening search**: calls depth-first search with increasing depth limits until a goal is found.
    - Complete, optimal for unit step costs, time complexity comparable to breadth-first search, linear space complexity.

Final Review – Content before Midterm

Informed Search:

- Informed search methods have access to heuristic function h(n)
  - Evaluate cost from node n to goal
  - **Admissible**, consistent
- **Greedy search** expands nodes with minimal h(n)
  - Not always optimal but efficient
- **A-star search** expands nodes with minimal g(n) + h(n)
  - Complete and optimal
  - Tree-search version when h is admissible
  - Graph-search version when h is consistent

Final Review – Content before Midterm

Constraint Satisfaction:

- Constraint satisfaction problem formulation
    - Variables, Domains, Constraints
- **Backtracking DFS**
    - Fail and backtrack when a consistent assignment is not possible
- **Heuristics**: increase the efficiency of backtracking DFS
    - Variable selection
        - Most constraint variable / Minimum Remaining Values heuristic
        - Most constraining variable / Degree heuristic
    - Value selection
        - Least constraining value

Final Review – Content before Midterm

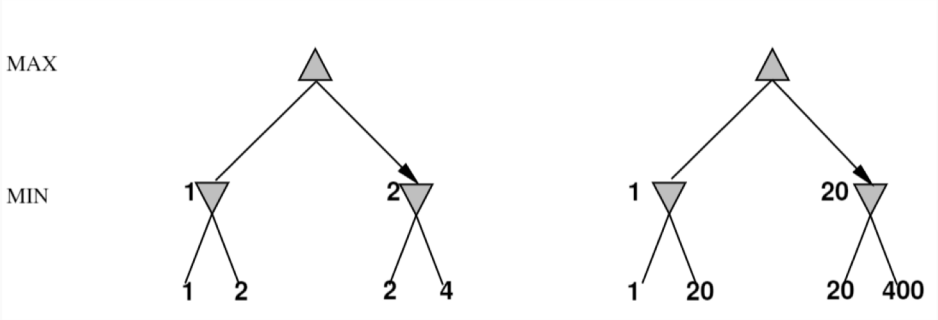
Constraint Satisfaction:

- **Constraint propagation**
  - Arc consistency
  - Forward checking (variable-level arc consistency)
- **Problem Structure**
  - Tree-structured CSP

Final Review – Content before Midterm

Game Playing: Basics

- **Minimax**:
  - a utility value for all goal states (leaves)
  - max player: value is max of its children
  - min player: value is min of its children
  - value of the root: the value of the game outcome



Alpha-beta pruning

- Motivation: skip branches that won't matter to improve efficiency

Final Review – Content before Midterm

Propositional Logic:

- Syntax and semantics
- Important terms: model, satisfiability, validity, entailment, etc.
- **Syntactic forms**: CNF, DNF, Horn clauses, NNF, DNNF
    - All but horn clauses are universal. DNF, horn and DNNF are tractable
- **Propositional Inference**:
    - Inference rules
    - Method 1: Proof by enumeration - model checking: E.g. Using a truth table
    - Method 2: Proof by refutation (resolution):
        - Step 1: Convert KB to CNF
        - Step 2: Keep applying inference rules until an empty clause appear

            → Showing $\Delta \wedge \neg\alpha$ is unsatisfiable!
    - Method 3 (SAT solver) and 4 (NNF circuits).

List of Topics

Lisp and search strategies

1. Evaluate a simple LISP expression or function, or choose a sentence to complete it.
2. Understand differences among search algorithms and determine completeness, optimality, time, and space complexity for any of them.
3. Understand backtracking DFS and heuristics (variable order, value order, etc.) in constraint satisfaction problems.
4. MINIMAX and α-β pruning.

List of Topics

Propositional logic (PL) and first-order logic (FOL):

5. The concepts in PL and FOL, e.g. satisfiability, validity, entailment, consistency.
6. Translate English to FOL sentences, or the other way around.
7. Convert a propositional or first-order logic sentence to CNF. Perform Skolemization.
8. Apply resolution or other inference rules to PL/FOL sentences. Completeness and soundness of inference rules.
9. Find unifiers for two FOL sentences.
10. Decide whether a propositional or first-order sentence entails another sentence.

Reasoning over uncertainty:

11. Independence, conditional independence. Bayes rule.
12. Given background information, compute probabilities for events. → use Bayesian rule
13. Compute probability for PL sentences given possible worlds.

List of Topics

Bayesian Network:

14. Model a problem as a Bayesian network.
15. Identify Markovian assumptions encoded by a Bayesian network (its semantics). Give joint probability using the chain rule.
16. Utilize d-separation to identify independence.

    → 3 types of valves, relationship between d-separation and independence

Machine Learning and Neural Network:

17. Concepts about Machine learning.
18. Definition for Entropy. Choose splitting attributes for a decision tree.
19. Concepts about Neural Network. Given input and NN structure, predict output.

Questions?

My slides take the following materials as references:

- Prof. Darwiche's lecture video

Good luck with the final.

Thank you!