COM SCI 161A HW 5 Solution

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Problem 1. Equivalent sentences

Solution 1a. $P \implies \neg Q$, $Q \implies \neg P$ are equivalent
We can convert implies into following sentences: $\neg P \vee \neg Q$, $\neg Q \vee \neg P$. Building truth tables with these gives,

Worlds	P	Q	$\neg P \vee \neg Q$	$\neg Q \vee \neg P$
w_1	0	0	1	1
w_2	0	1	1	1
w_3	1	0	1	1
w_4	1	1	0	0

$M[\neg P \vee \neg Q] = \{w_1,w_2,w_3\}$ and $M[\neg Q \vee \neg P] = \{w_1,w_2,w_3\}$. Since the models for both the sentences are same, the sentences are equivalent.

Solution 1b. $P \iff \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$ are equivalent
We can convert equivalence into following: $((\neg P \vee \neg Q) \wedge (Q \vee P))$. Building truth tables gives,

Worlds	P	Q	$((\neg P \vee \neg Q) \wedge (Q \vee P))$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
w_1	0	0	0	0
w_2	0	1	1	1
w_3	1	0	1	1
w_4	1	1	0	0

$M[((\neg P \vee \neg Q) \wedge (Q \vee P))] = \{w_2,w_3\}$ and $M[((P \wedge \neg Q) \vee (\neg P \wedge Q))] = \{w_2,w_3\}$. Since the models for both the sentences are same, the sentences are equivalent.

Problem 2. Valid, unsatisfiable or neither

Solution 2a. $(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$ is neither valid nor unsatisfiable,
Removing implies gives us following;

$$(\neg Smoke \vee Fire) \implies (Smoke \vee \neg Fire)$$

$$\neg(\neg Smoke \vee Fire) \vee (Smoke \vee \neg Fire)$$

$$(Smoke \wedge \neg Fire) \vee (Smoke \vee \neg Fire)$$

Worlds	Smoke	Fire	$(Smoke \wedge \neg Fire) \vee (Smoke \vee \neg Fire)$
w_1	0	0	1
w_2	0	1	0
w_3	1	0	1
w_4	1	1	1

Sentence is satisfiable in world w_1, w_3 and w_4 but is not valid as it doesn't satisfy all the worlds.

Solution 2b. $(Smoke \implies Fire) \implies ((Smoke \vee Heat) \implies Fire)$ is neither valid nor unsatisfiable,
Removing implies gives us following;

$$(\neg Smoke \vee Fire) \implies (\neg(Smoke \vee Heat) \vee Fire)$$

$$\neg(\neg Smoke \vee Fire) \vee ((\neg Smoke \wedge \neg Heat) \vee Fire)$$

$$(Smoke \wedge \neg Fire) \vee ((\neg Smoke \wedge \neg Heat) \vee Fire)$$

Worlds	Smoke	Fire	Heat	$(Smoke \wedge \neg Fire) \vee ((\neg Smoke \wedge \neg Heat) \vee Fire)$
w_1	0	0	0	1
w_2	0	0	1	0
w_3	0	1	0	1
w_4	0	1	1	1
w_5	1	0	0	1
w_6	1	0	1	1
w_7	1	1	0	1
w_8	1	1	1	1

Sentence is satisfiable in all worlds except w_2 hence it is not valid.

Solution 2c. $((Smoke \wedge Heat) \implies Fire) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire))$ is valid,
Removing implies and equivalence gives us following;

$$(\neg(Smoke \wedge Heat) \vee Fire) \iff ((\neg Smoke \vee Fire) \vee (\neg Heat \vee Fire))$$

$$(\neg S \vee \neg H \vee F) \iff (\neg S \vee F \vee \neg H))$$

$$((\neg S \vee \neg H \vee F) \implies (\neg S \vee F \vee \neg H)) \wedge ((\neg S \vee F \vee \neg H) \implies (\neg S \vee \neg H \vee F))$$

$$(\neg(\neg S \vee \neg H \vee F) \vee (\neg S \vee F \vee \neg H)) \wedge (\neg(\neg S \vee F \vee \neg H) \vee (\neg S \vee \neg H \vee F))$$

$$((S \wedge H \wedge \neg F) \vee (\neg S \vee F \vee \neg H)) \wedge ((S \wedge \neg F \wedge H) \vee (\neg S \vee \neg H \vee F))$$

Worlds	Smoke	Fire	Heat	$((S \wedge H \wedge \neg F) \vee (\neg S \vee F \vee \neg H)) \wedge ((S \wedge \neg F \wedge H) \vee (\neg S \vee \neg H \vee F))$
w_1	0	0	0	1
w_2	0	0	1	1
w_3	0	1	0	1
w_4	0	1	1	1
w_5	1	0	0	1
w_6	1	0	1	1
w_7	1	1	0	1
w_8	1	1	1	1

Sentence is satisfiable in all worlds hence it is valid.

Problem 3. Unicorn problem

Solution 3a. Knowledge Base
We consider following definition of symbols for the unicorn: A : Mythical, B : Mortal, C : Mammal, D : Horned, E : Magical. Then our Knowledge base consists of following:

1. $A \implies \neg B$
2. $\neg A \implies (B \wedge C)$
3. $(\neg B \vee C) \implies D$
4. $D \implies E$

Solution 3b. CNF
Using KB to convert to CNF form by removing implies and simplifying not and or,

$$(A \implies \neg B) \wedge (\neg A \implies (B \wedge C)) \wedge ((\neg B \vee C) \implies D) \wedge (D \implies E)$$

$$(\neg A \vee \neg B) \wedge (A \vee (B \wedge C)) \wedge (\neg(\neg B \vee C) \vee D) \wedge (\neg D \vee E)$$
$$(\neg A \vee \neg B) \wedge ((A \vee B) \wedge (A \vee C)) \wedge ((B \wedge \neg C) \vee D) \wedge (\neg D \vee E)$$
$$(\neg A \vee \neg B) \wedge (A \vee B) \wedge (A \vee C) \wedge (B \vee D) \wedge (\neg C \vee D) \wedge (\neg D \vee E)$$

Solution 3c i. Unicorn is not Mythical
We represent KB in CNF form as Δ and query as α, then we do resolution of $\Delta \wedge \neg\alpha$ to check for unsatisfiability,

1. $(\neg A \vee \neg B)$
2. $(A \vee B)$
3. $(A \vee C)$
4. $(B \vee D)$
5. $(\neg C \vee D)$
6. $(\neg D \vee E)$
7. $\neg A$ $\quad$ $(\neg\alpha)$
8. B $\quad$ (2,7)
9. C $\quad$ (3,7)
10. D $\quad$ (5,9)
11. E $\quad$ (6,10)

We see that no more resolution can be done and it is satisfiable, hence unicorn is not mythical.

Solution 3c ii. Unicorn is Magical
We define Δ similar to previous,

1. $(\neg A \vee \neg B)$

2. $(A \vee B)$
3. $(A \vee C)$
4. $(B \vee D)$
5. $(\neg C \vee D)$
6. $(\neg D \vee E)$
7. $\neg E$ $\quad (\neg\alpha)$
8. $\neg D$ $\quad$ (6,7)
9. $\neg C$ $\quad$ (5,8)
10. B $\quad$ (4,8)
11. $\neg A$ $\quad$ (1,10)
12. C $\quad$ (3,11)
13. $Contradiction$ $\quad$ (9,12)

We see that resolution leads to unsatisfiability hence, unicorn is magical.

Solution 3c iii. Unicorn is Horned
We define Δ similar to previous,

1. $(\neg A \vee \neg B)$
2. $(A \vee B)$
3. $(A \vee C)$
4. $(B \vee D)$
5. $(\neg C \vee D)$
6. $(\neg D \vee E)$
7. $\neg D$ $\quad (\neg\alpha)$
8. B $\quad$ (4,7)
9. $\neg C$ $\quad$ (5,7)
10. A $\quad$ (3,9)
11. $\neg B$ $\quad$ (1,10)
12. $Contradiction$ $\quad$ (8,11)

We see that resolution leads to unsatisfiability hence, unicorn is horned.

Problem 4. NNF

Solution 4a. Figure 1 is decomposable, not deterministic and not smooth
Figure 1 is decomposable because all the AND gates inputs do not share variables.
Figure 1 is not deterministic, because the world with {A=1, B=0, C=1, D=0} shows that the inputs to top OR gate is not mutually exclusive. This is the only assignment that is common for top OR gate.
Figure 1 is not smooth because the second OR gate in third row has input $\{C, (\neg C \wedge \neg D)\}$ i.e. do not share variable.

Solution 4b. Figure 2 is decomposable, not deterministic and smooth
Figure 2 is decomposable because all the AND gates inputs do not share variables.
Figure 2 is not deterministic, because the first OR gate in third row has same inputs, i.e. not mutually exclusive.
Figure 2 is smooth because the all OR gates share variables.

Problem 5. Weighted Model Count

Solution 5a. $(\neg A \wedge B) \vee (\neg B \wedge A)$

Worlds	A	B	$(\neg A \wedge B) \vee (\neg B \wedge A)$
w_1	0	0	0
w_2	0	1	1
w_3	1	0	1
w_4	1	1	0

$$\text{Weighted Model Count} = \omega(w_2) + \omega(w_3)$$

$$\text{Weighted Model Count} = \omega(\neg A)\omega(B) + \omega(\neg B)\omega(A)$$

$$\text{Weighted Model Count} = 0.9 * 0.3 + 0.7 * 0.1 = 0.34$$

Solution 5b. Figure 3
The count on the root would be $\omega(\neg A)\omega(B) + \omega(\neg B)\omega(A) = 0.34$ which is **equal** to the Weighted Model count from previous question.

Solution 5c. Figure 4
The Weighted Model count for Figure 4 is 0.5. See below figure for computation,

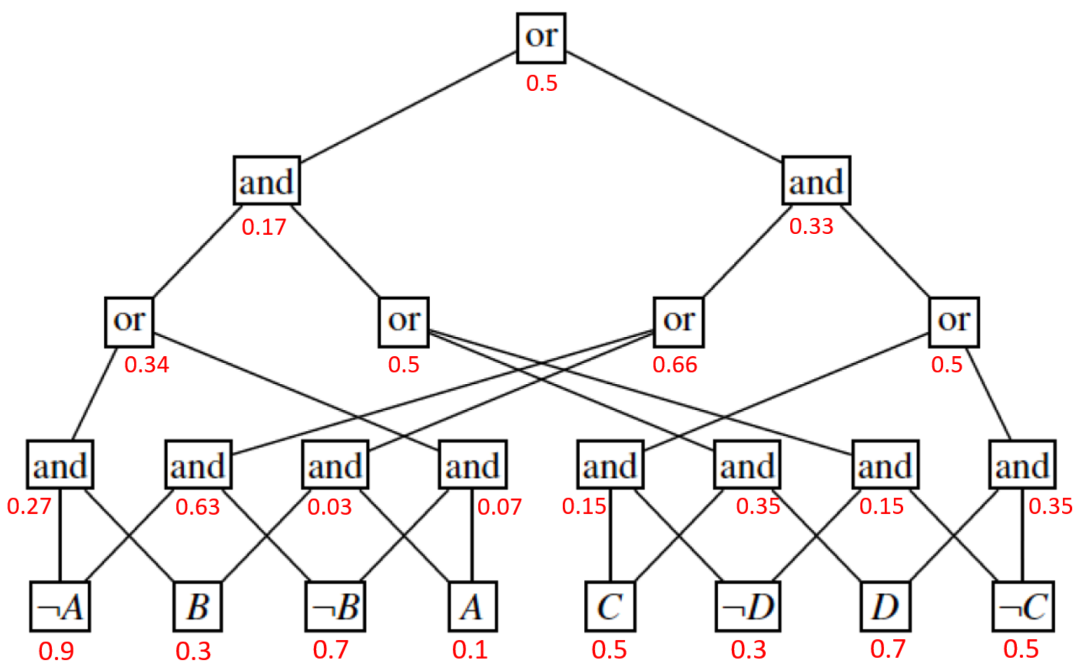

Figure 4