CS 161 Intro. To Artificial Intelligence

Week 3, Discussion 1B

Qian Long

Today’s Topics

- Informed Search
  - Greedy Search
  - A* Search
    - Optimality
    - Heuristic Function
- Constraint Satisfaction Problem (CSP)
  - Formulation of CSPs
  - Backtrack Search
  - Techniques for improving CSP solution

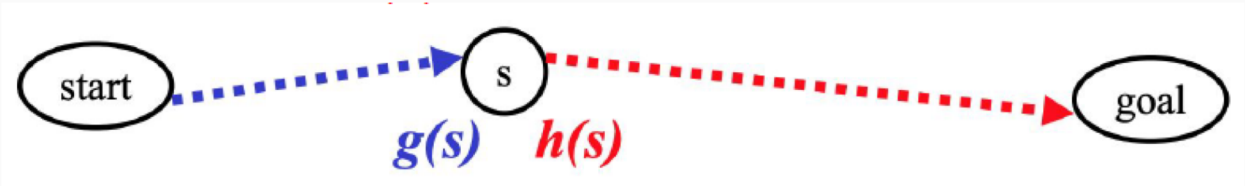
Informed Search

- Leverage problem-specific knowledge
- Also known as heuristic search

Informed Search

- Uninformed Search
  - Have access only to the problem definition
  - Knows the **actual path cost g(s)** from start to a node s in the fringe

- Informed Search
  - Have access to **heuristic function** as well as problem definition
  - The additional **h(s)** is the heuristic of **the cost from s to goal**



Informed Search

Q: Is uniform-cost search informed search?

A: **No!**

Reasons:

- It only looks backwards, it has no ability to predict future costs
- It doesn't use any domain knowledge

Best-first search

Best-first search:

- The general approach for informed search
- Choose the "best" (the most desirable) node to expand

How to determine which node is the best?

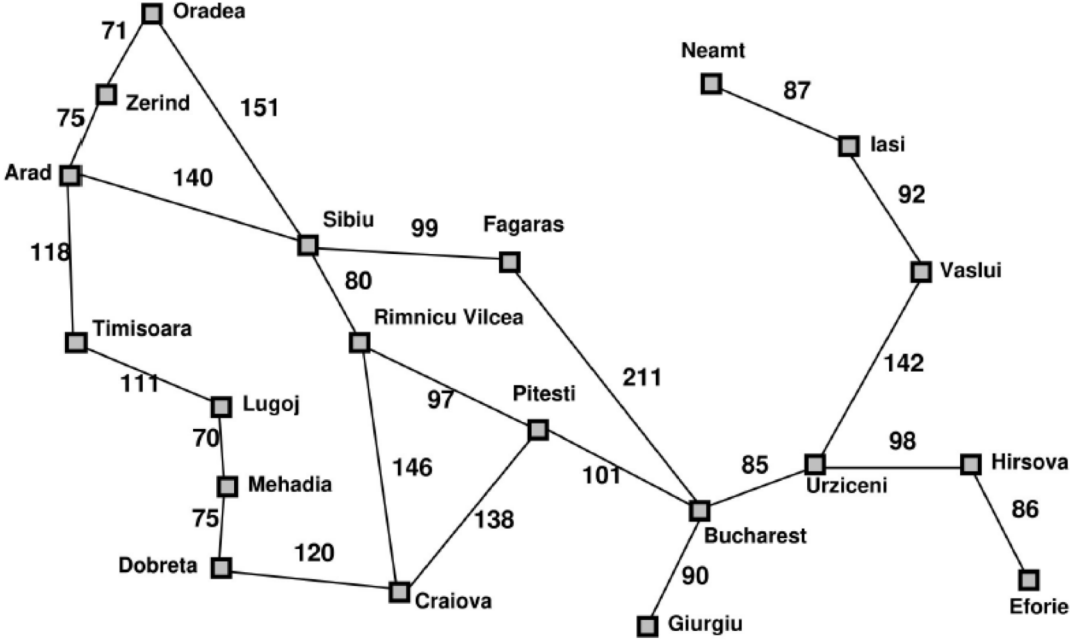
- **Evaluation function $f(n)$** (a cost estimation for node n) -- smallest $f(n)$ is usually the “best” node
- n is a node in search tree, not a state in state graph
- Often involves a **heuristic function $h(n)$**
- Fringe is a queue sorted in decreasing order of desirability

Special Cases:

- Greedy best first search
- A* search

Greedy Best-first Search

- $f(n) = h(n)$ **(heuristic)** = estimate cost from node n to the closest goal
  - E.g., h_{SLD} = straight-line distance from n to Bucharest



Straight-line distance to Bucharest	
Arad	366
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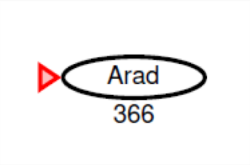
Greedy Best-first Search

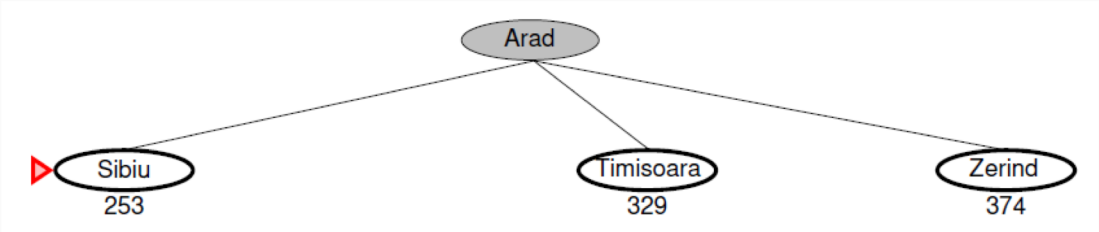
Greedy best-first search expands the node that **appears** to be closest to goal

$f(n) = h(n)$ = straight-line distance from n to Bucharest

Initial state:



After expanding Arad:



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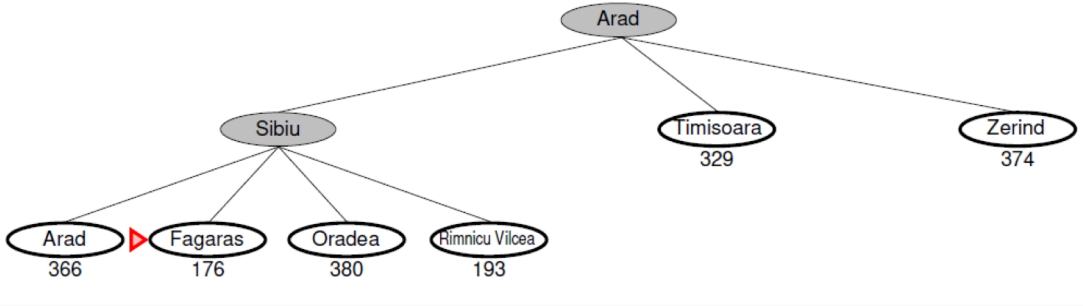
Greedy Best-first Search

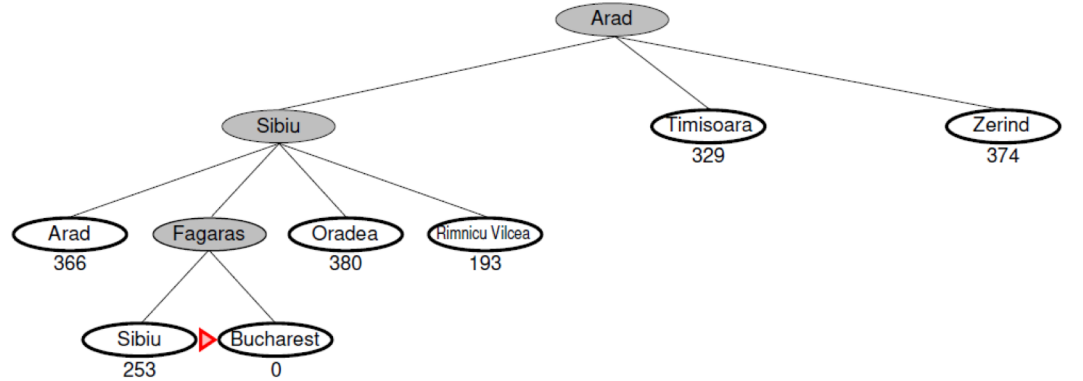
Greedy best-first search expands the node that **appears** to be closest to goal

$f(n) = h(n)$ = straight-line distance from n to Bucharest

After expanding Sibiu



After expanding Fagaras:



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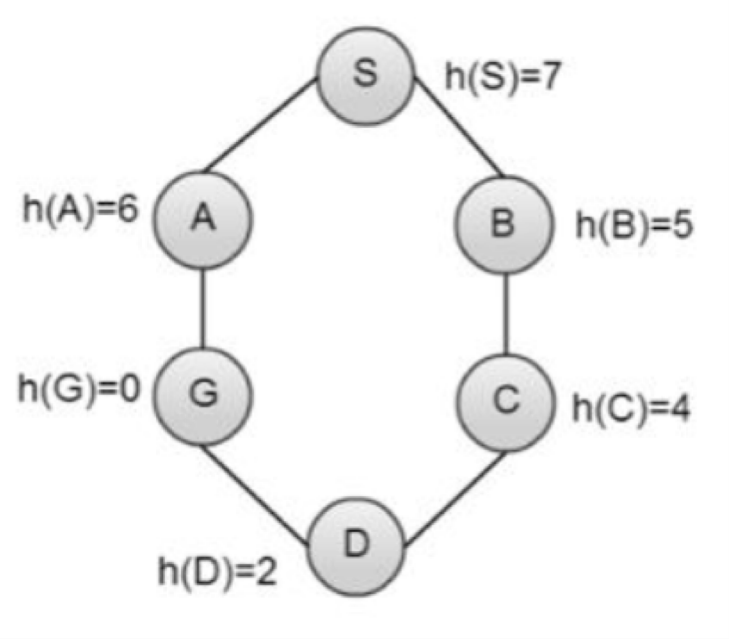
Greedy Best-first Search - Example

From S to G using Greedy best-first search:



Greedy Best-first Search - Example

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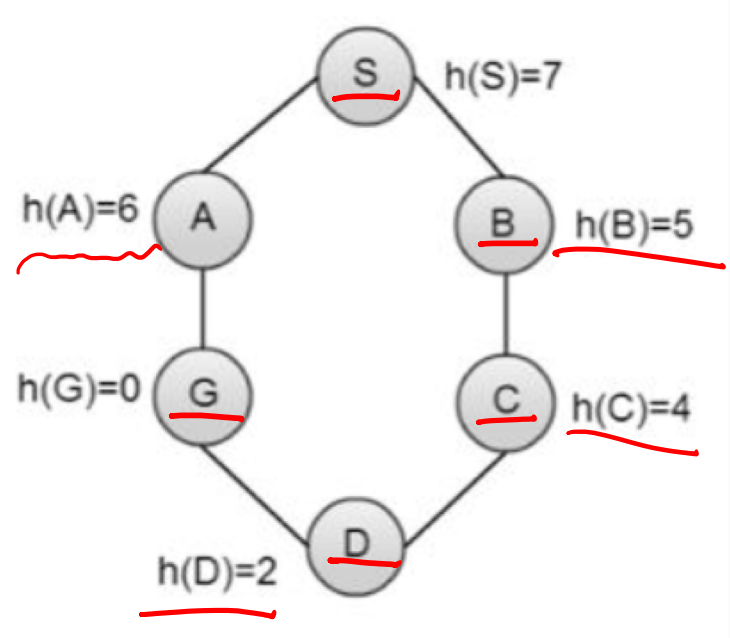
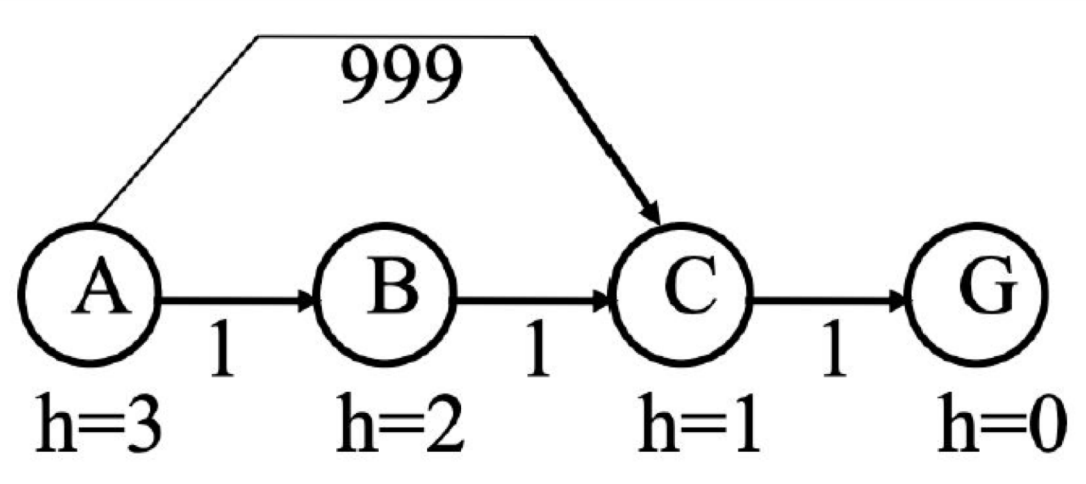


S → B → C → D → G

Greedy Best-first Search - Example

From A to G using Greedy best-first search:



Greedy Best-first Search - Example

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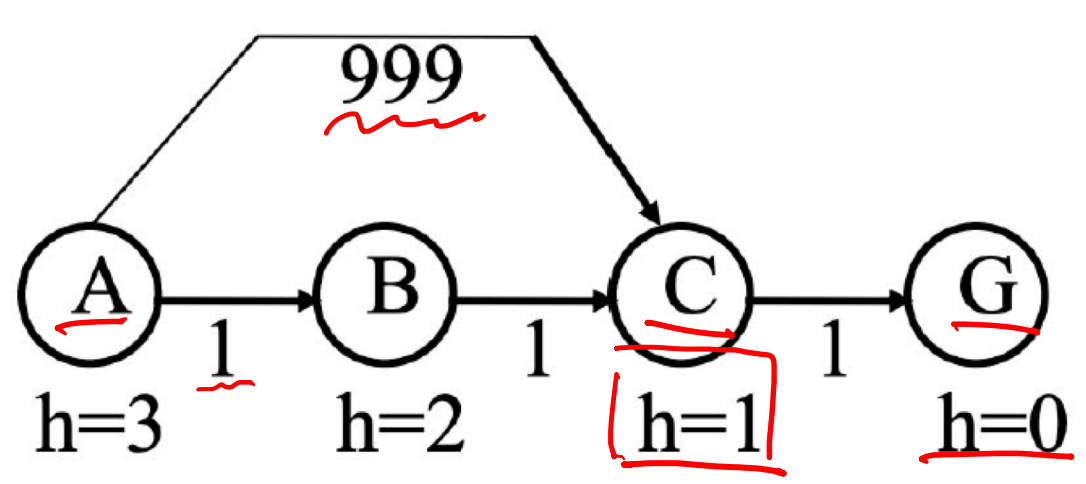


A → C → G

Problem:
Actual cost is too high!
→ Use A* search

A* Search

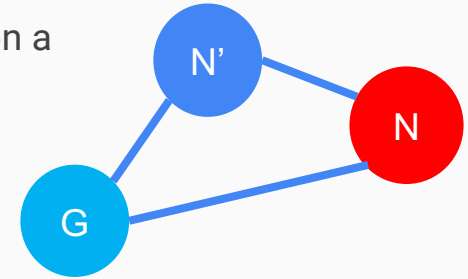
- Avoid expanding paths that are already expensive
- Goal test at expansion
- $f(n) = g(n) + h(n)$
  - $g(n)$: actual path cost so far to reach n
  - $h(n)$: estimated cost from n to goal
  - $f(n)$: estimated total cost of path through n to goal

A* Search

Theorem: A* search is optimal

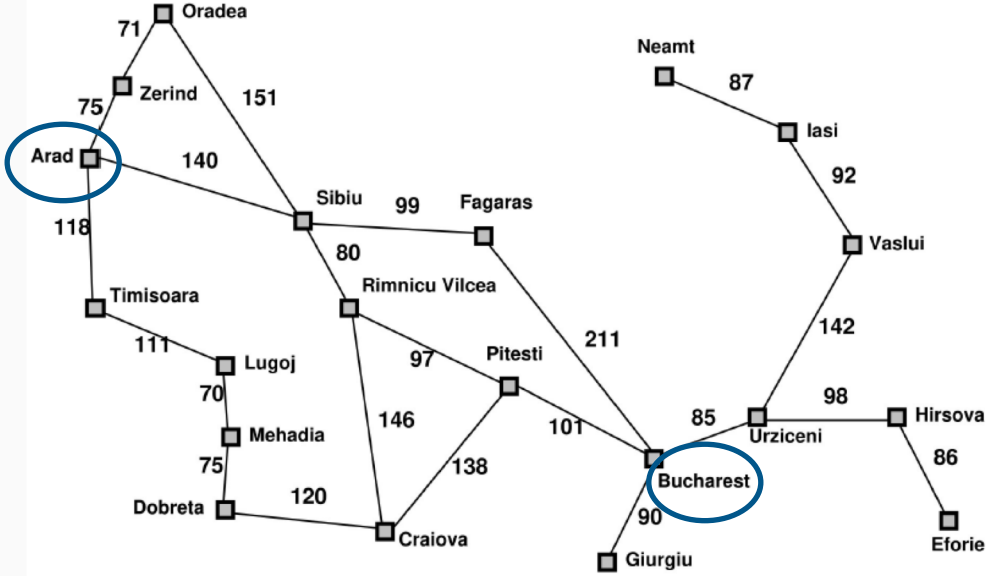
The properties of **heuristic h(n)** for A*: (they are the conditions for A*'s optimality)

- **Admissibility**
  - $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost from node n to goal state (also need h(n)>=0)
  - Admissible heuristic cost should **never overestimate** the actual cost of a node
- **Consistency (Monotonicity)**
  - $h(n) \leq c(n, a, n') + h(n')$
    - n' is a successor of n
    - $c(n, a, n')$ is the cost of path from n to n' by choosing action a
    - $f(n)$ is non-decreasing along any path
    - Alike to "triangle inequality"



A* Search

- Example: from Arad to Bucharest
- $f(n) = g(n) + h(n)$
  - $h(n) = h_{SLD}$ = straight-line distance from n to Bucharest



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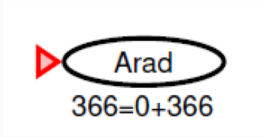
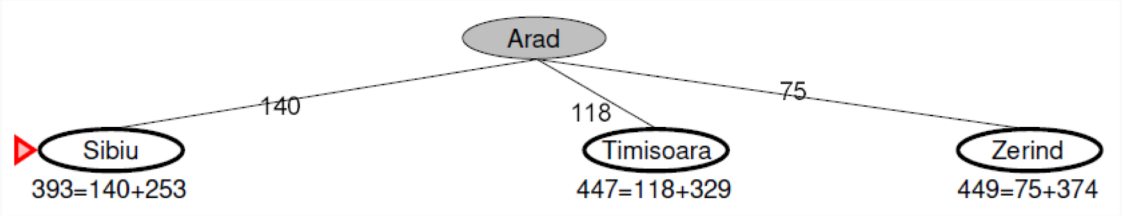
A* Search

- $f(n) = g(n) + h(n)$
  - $h(n) = h_{SLD}$ = straight-line distance from n to Bucharest

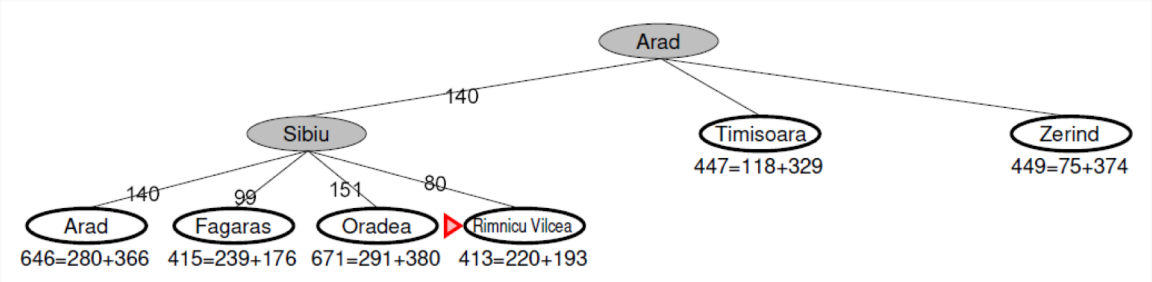
Initial state:



After expanding Arad:



After expanding Sibiu:



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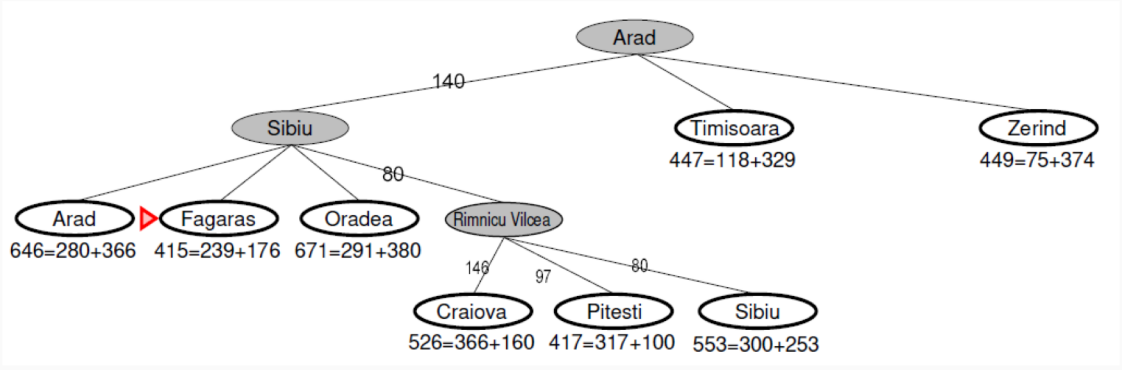
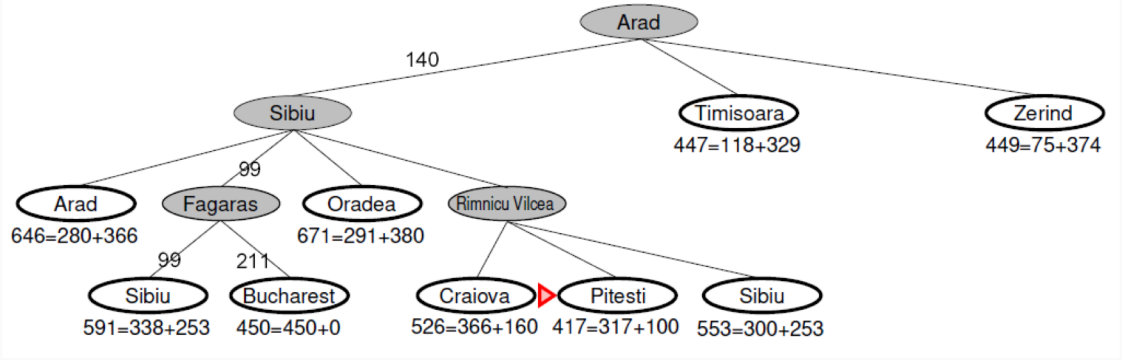
A* Search

- $f(n) = g(n) + h(n)$
  - $h(n) = h_{SLD}$ = straight-line distance from n to Bucharest

After expanding Rimnicu Vilcea:



After expanding Fagars:



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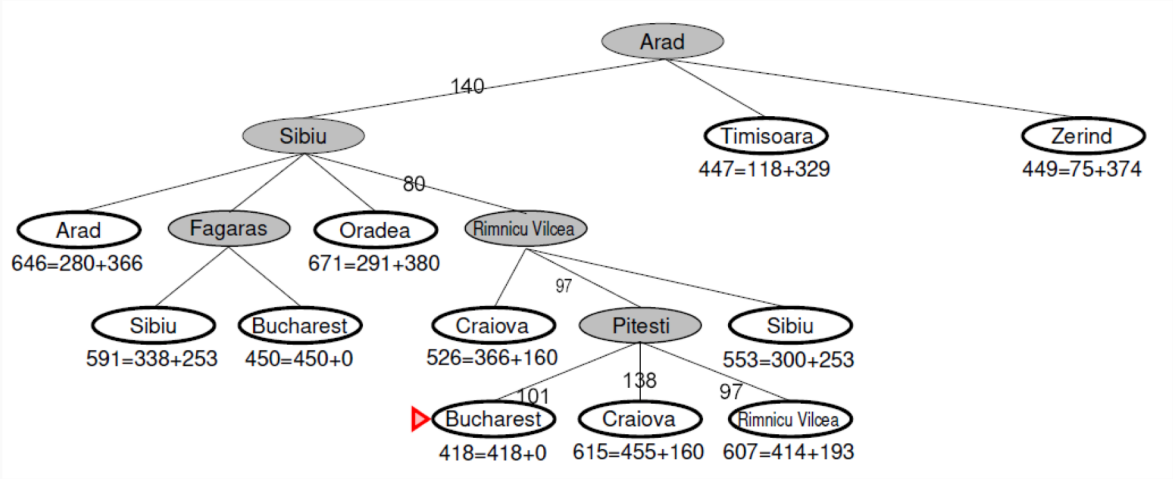
A* Search

- $f(n) = g(n) + h(n)$
  - $h(n) = h_{SLD}$ = straight-line distance from n to Bucharest

After expanding Pitesti:



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A* Search - Optimality

- Admissibility
  - $h(n) \leq h^*(n)$
    - $h^*(n)$: true cost from node n to goal state
  - Theorem: If *h(n)* is admissible, A* is optimal
- Consistency (Monotonicity)
  - $h(n) \leq cost(n, n') + h(n')$ (n' is successor of n)
    - $f(n)$ is non-decreasing along any path
- Consistency implies admissibility

A* Search - Optimality

- Consistency is stronger than admissible (**consistency implies admissibility**)
- If h(n) is consistent, the values of f(n) along any path are nondecreasing
  - It's because: f(n') = g(n') + **h(n')** = g(n) + **c(n,a,n') + h(n')** >= g(n) + **h(n)** = f(n)
  - Where c(n, a, n') is the cost of path from n to n' (a successor of n) by choosing action a

Heuristic Function

How to figure out good heuristics?

- A good heuristic should be admissible but as close to the true cost as possible
  - **Admissible** heuristic cost should **never overestimate** the actual cost of a node
  - I.e. it must be “optimistic” → we never overlook a node that is actually good

- Admissible heuristics can be derived from the exact solution cost of a **relaxed version** of the problem
  - Relaxed problems are problems with fewer restrictions on the actions (but with same initial state and goals)

CSP - Formulation

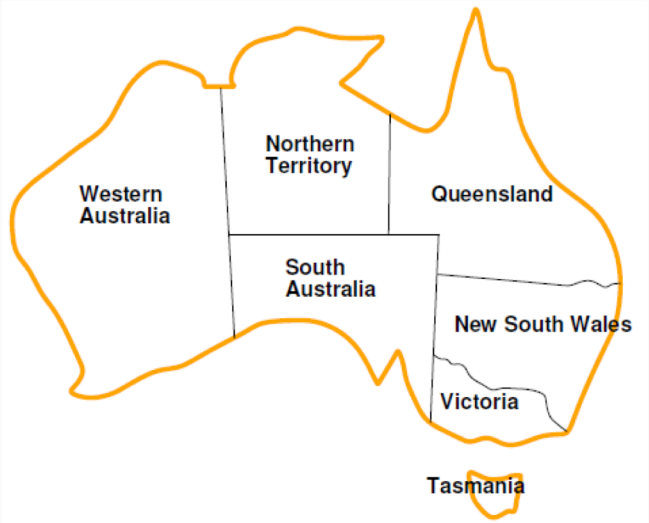
Components of Constraint Satisfaction Problem (CSP):

X is a set of variables, $\{X_1, \ldots, X_n\}$.
D is a set of domains, $\{D_1, \ldots, D_n\}$, one for each variable.
C is a set of constraints that specify allowable combinations of values.

- Each domain Di consists of a set of allowable values {v1,…,vk} for the corresponding Xi
- A **state** in CSP: an assignment of values to some or all variables
  - Partial assignment: assign values to only some of the variables
  - Complete assignment: every variable is assigned (otherwise partial assignment)
  - Consistent/Legal assignment: an assignment that does not violate any constraints
- A **solution** in CSP: a consistent, complete assignment

CSP – Formulation Example

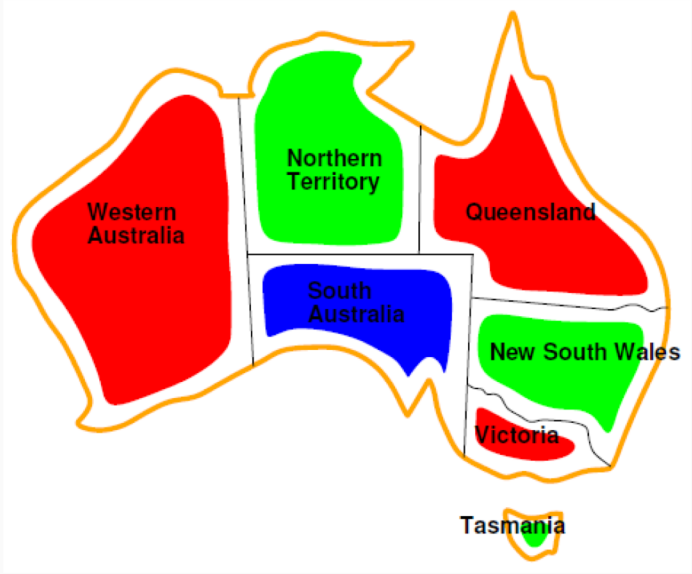
Map Coloring:



- **Variables**: WA, NT, Q, NSW, V , SA, T
- **Domains**: Di = {red, green, blue}
- **Constraints**: adjacent regions must have different colors
  - E.g. WA ≠ NT (if the language allows this),
  - E.g. (WA,NT) ∈ {(red, green), (red, blue), (green, red), (green, blue), . . .}

CSP – Formulation Example

Map Coloring:



- Solutions are assignments satisfying all constraints,
  - e.g., {WA=red, NT =green, Q=red, NSW =green, V =red, SA=blue, T =green}

Backtrack Search

Backtracking search: the basic uninformed algorithm for CSPs

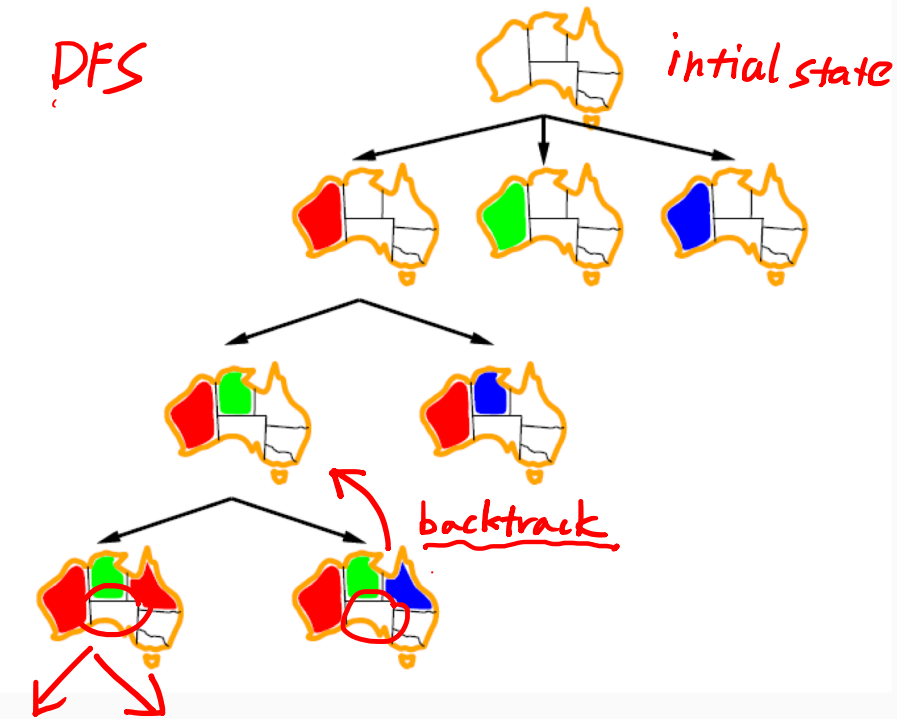
- It's basically a depth-first search for CSPs with single-variable assignments:
  - Chooses values for one variable at a time and **backtracks when a variable has no legal values left to assign**

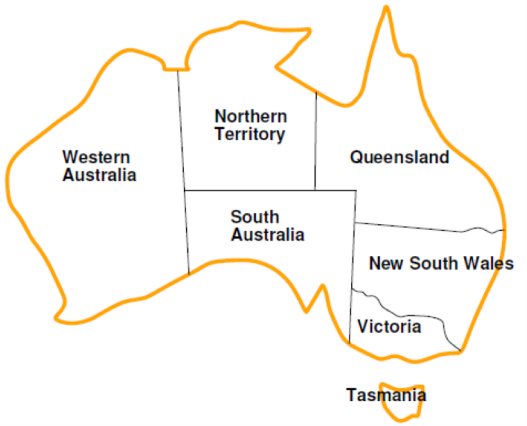
```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add {var = value} to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

Backtrack Search - Example

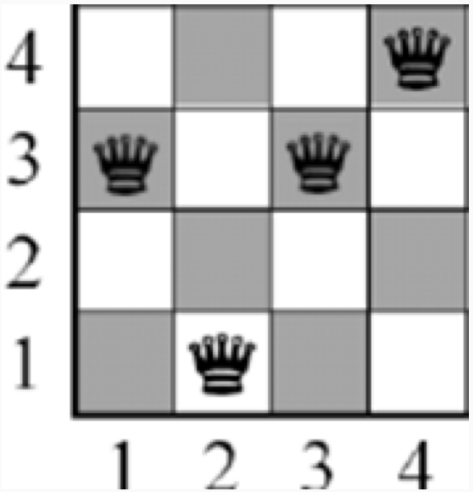
- When to backtrack? → when a variable has no legal values left to assign





Backtrack Search - Example

- 4-Queens Puzzle (assume each queen in each column) as a CSP:
  - Variables: Q1, Q2, Q3, Q4 -- row indices of each queen
  - Domains Di = {1, 2, 3, 4}
  - Constraints:
    - Qi ≠ Qj (cannot be in same row)
    - |Qi − Qj | ≠ |i − j| (or same diagonal)
- Backtracking search:
  - (Q1, Q2, Q3, Q4):
  - (1,X,X,X) → (1, 3,X,X) → No legal assign for Q3, backtracking
  - (1, 4,X,X) → (1, 4, 2,X) → No legal assign for Q4, backtracking
  - (1, 4, 3, X) → Not a legal assign for Q3, backtracking
  - (2,X,X,X) → (2, 4,X,X) → (2, 4, 1,X) → (2, 4, 1, 3), Bingo!



Improving Backtracking Efficiency

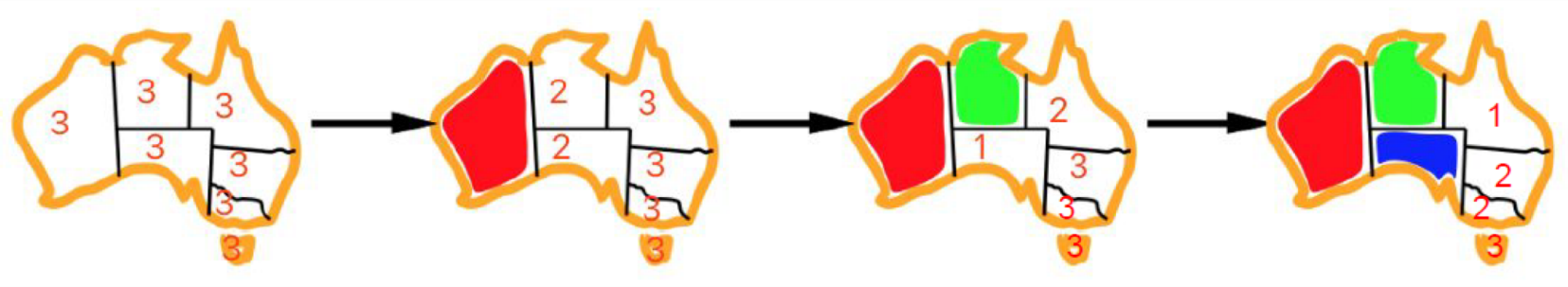
3 techniques (heuristics) to improve efficiency:

- How to select unassigned **variable**?
  - Most constrained variable / Minimum remaining values (MRV)
  - Most constraining variable / Degree heuristic

- In what order should we assign **values** to each variable?
  - Least constraining value

Improving Backtracking Efficiency

- How to select unassigned **variable**?
  - Most constrained variable / Minimum remaining values (MRV):
    - choose the **variable** with the fewest legal values
    - If no legal values left, fail immediately

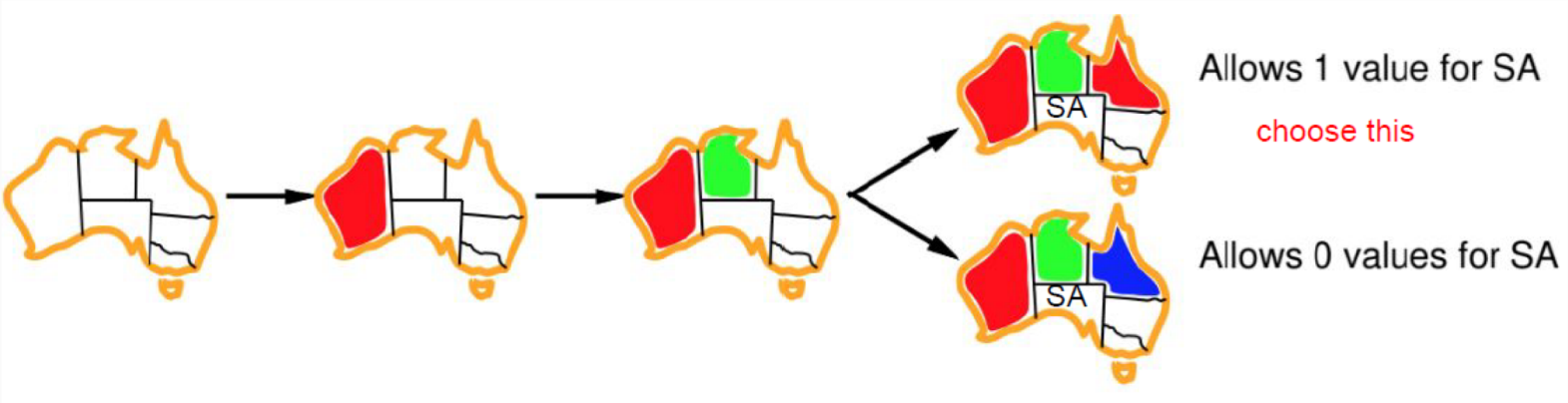


Improving Backtracking Efficiency

- How to select unassigned **variable**?
  - Most constrained variable / Minimum remaining values (MRV)
  - Most constraining variable / Degree heuristic:
    - Choose the **variable** with the most constraints on remaining variables
    - Attempt to reduce branching factor on future choice
    - **Useful as a tie-breaker**

Improving Backtracking Efficiency

- In what order should we assign **values** to each variable?
  - Least constraining value:
    - Choose the **value** that rules out the fewest values in the remaining variables
    - Leave the maximum flexibility for subsequent variable assignments



Questions?

- My slides take the following materials as references:
  - Xiao Zeng’s slide

Thank you!