COM SCI 161A HW 9 Solution

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Problem 1. Decision Tree

Solution 1. For first step, we compare $ENT(D|A)$, and $ENT(D|B)$ and $ENT(D|C)$

$$ENT(D|A) = P(a)ENT(D|a) + P(\bar{a})ENT(D|\bar{a})$$

$$ENT(D|A) = \frac{11}{22}(-\frac{7}{11}\log\frac{7}{11} - \frac{4}{11}\log\frac{4}{11}) + \frac{11}{22}(-\frac{3}{11}\log\frac{3}{11} - \frac{8}{11}\log\frac{8}{11})$$
$$ENT(D|A) = 0.5 * 0.9457 + 0.5 * 0.8453 = 0.8955$$

$$ENT(D|B) = P(b)ENT(D|b) + P(\bar{b})ENT(D|\bar{b})$$
$$ENT(D|B) = \frac{14}{22}(-\frac{8}{14}\log\frac{8}{14} - \frac{6}{14}\log\frac{6}{14}) + \frac{8}{22}(-\frac{2}{8}\log\frac{2}{8} - \frac{6}{8}\log\frac{6}{8})$$
$$ENT(D|B) = \frac{14}{22} * 0.9852 + \frac{8}{22} * 0.8113 = 0.922$$

$$ENT(D|C) = P(c)ENT(D|c) + P(\bar{c})ENT(D|\bar{c})$$
$$ENT(D|C) = \frac{7}{22}(-\frac{4}{7}\log\frac{4}{7} - \frac{3}{7}\log\frac{3}{7}) + \frac{15}{22}(-\frac{6}{15}\log\frac{6}{15} - \frac{9}{15}\log\frac{9}{15})$$
$$ENT(D|C) = \frac{7}{22} * 0.9852 + \frac{15}{22} * 0.9709 = 0.9755$$

We see that entropy is minimum for $ENT(D|A)$, thus we choose attribute A to split first. Now conditioned on $A = a$, we compare $ENT(D|B, a)$ and $ENT(D|C, a)$

$$ENT(D|B, a) = P(b|a)ENT(D|b, a) + P(\bar{b}|a)ENT(D|\bar{b}, a)$$

$$ENT(D|B, a) = \frac{7}{11}(-\frac{7}{7}\log\frac{7}{7} - \frac{0}{7}\log\frac{0}{7}) + \frac{4}{11}(-\frac{0}{4}\log\frac{0}{4} - \frac{4}{4}\log\frac{4}{4})$$
$$ENT(D|B, a) = 0$$

$$ENT(D|C, a) = P(c|a)ENT(D|c, a) + P(\bar{c}|a)ENT(D|\bar{c}, a)$$
$$ENT(D|C, a) = \frac{4}{11}(-\frac{1}{4}\log\frac{1}{4} - \frac{3}{4}\log\frac{3}{4}) + \frac{7}{11}(-\frac{6}{7}\log\frac{6}{7} - \frac{1}{7}\log\frac{1}{7})$$

$$ENT(D|C,a) = \frac{4}{11} * 0.8113 + \frac{7}{11} * 0.5917 = 0.6715$$

We see that entropy is minimum for $ENT(D|B,a)$, thus conditioned on $A = a$, we choose B to split next. Since the entropy is 0 now we dont need to split any further for this branch.

Now conditioned on $A = \bar{a}$, we compare $ENT(D|B,\bar{a})$ and $ENT(D|C,\bar{a})$

$$ENT(D|B,\bar{a}) = P(b|\bar{a})ENT(D|b,\bar{a}) + P(\bar{b}|\bar{a})ENT(D|\bar{b},\bar{a})$$

$$ENT(D|B,\bar{a}) = \frac{7}{11}(-\frac{1}{7}\log\frac{1}{7} - \frac{6}{7}\log\frac{6}{7}) + \frac{4}{11}(-\frac{2}{4}\log\frac{2}{4} - \frac{2}{4}\log\frac{2}{4})$$

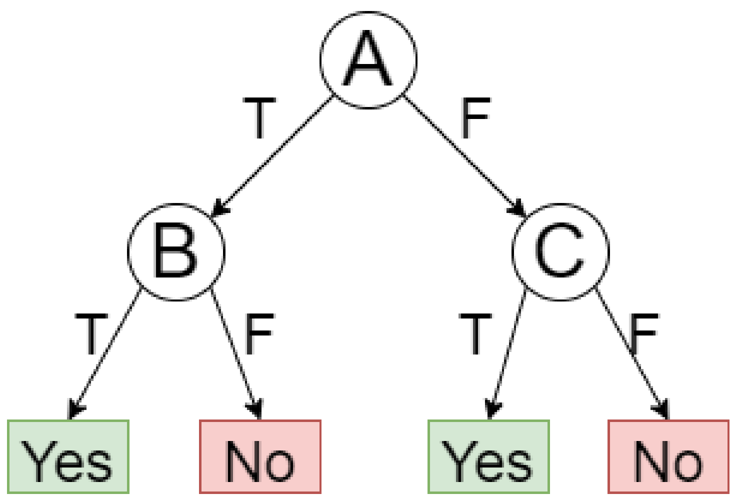
$$ENT(D|B,\bar{a}) = \frac{7}{11} * 0.5917 + \frac{4}{11} * 1 = 0.7402$$

$$ENT(D|C,\bar{a}) = P(c|\bar{a})ENT(D|c,\bar{a}) + P(\bar{c}|\bar{a})ENT(D|\bar{c},\bar{a})$$

$$ENT(D|C,\bar{a}) = \frac{3}{11}(-\frac{3}{3}\log\frac{3}{3} - \frac{0}{3}\log\frac{0}{3}) + \frac{8}{11}(-\frac{0}{8}\log\frac{0}{8} - \frac{8}{8}\log\frac{8}{8})$$

$$ENT(D|C,\bar{a}) = 0$$

We see that entropy is minimum for $ENT(D|C,\bar{a})$, thus conditioned on $A = \bar{a}$, we choose C to split next. Since the entropy is 0 now we dont need to split any further for this branch. The final decision tree is as below:



Problem 2. Neural Network

Solution 2a.

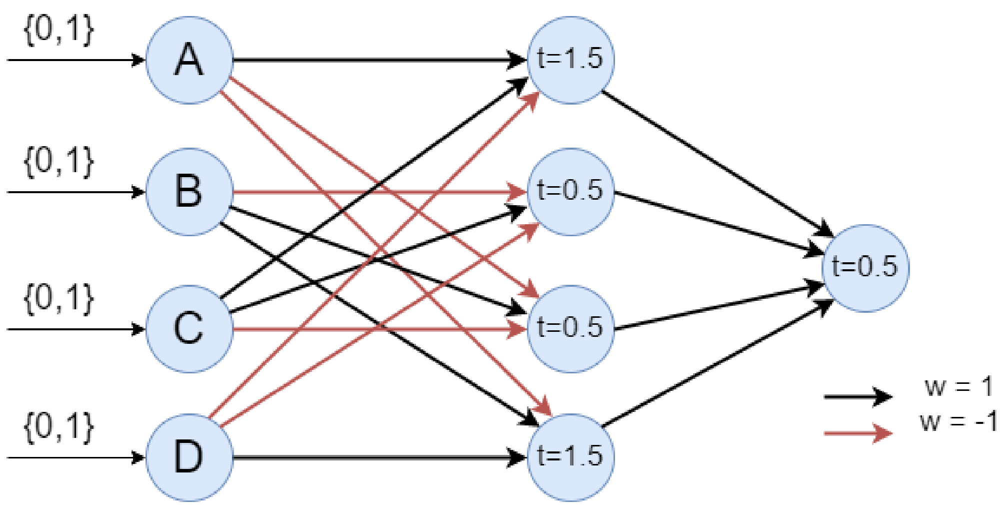
$$(A \vee \neg B) \oplus (\neg C \vee D) = ((A \vee \neg B) \wedge \neg(\neg C \vee D)) \vee (\neg(A \vee \neg B) \wedge (\neg C \vee D))$$

$$(A \vee \neg B) \oplus (\neg C \vee D) = ((A \vee \neg B) \wedge (C \wedge \neg D)) \vee ((\neg A \wedge B) \wedge (\neg C \vee D))$$

$$(A \vee \neg B) \oplus (\neg C \vee D) = (A \wedge C \wedge \neg D) \vee (\neg B \wedge C \wedge \neg D) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge D)$$

The neural network for this is as follows:

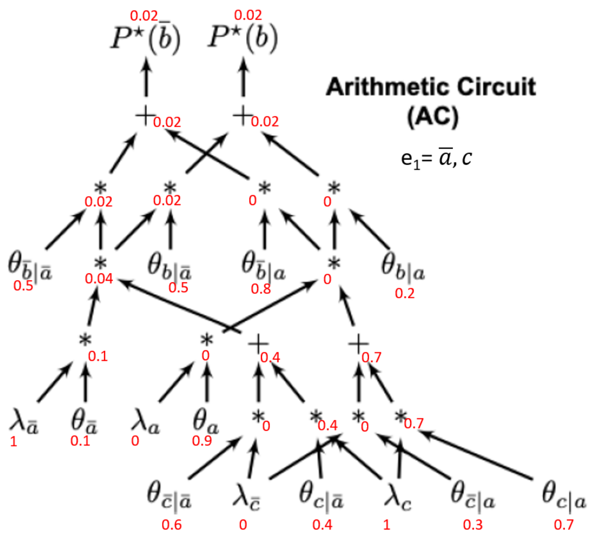
Red arrow represents weight being -1 and black arrows represent weight being 1. All



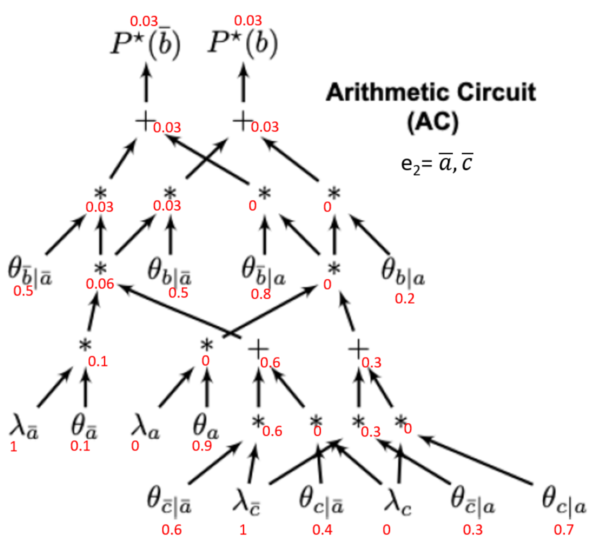
the activations are threshold activations with respective thresholds. The first layer neurons performs as AND gate for the three input connections. Last layer neuron performs as OR gate for four input connections.

Problem 3. Arithmetic Circuit

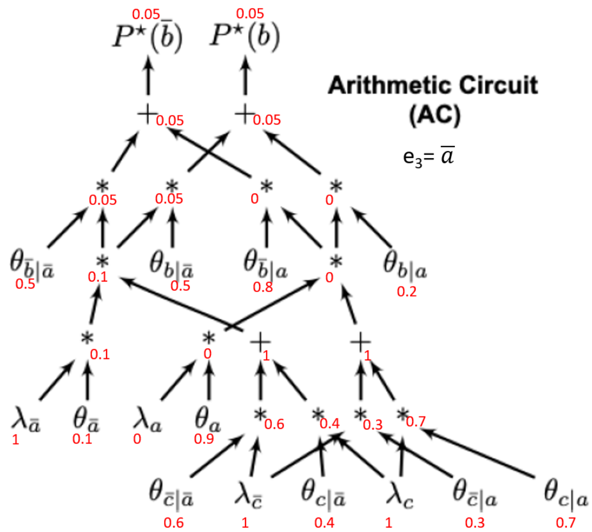
Solution 3a)i). For evidence $e_1 = \bar{a}, c$, we put $\lambda_a = 0, \lambda_{\bar{a}} = 1, \lambda_c = 1, \lambda_{\bar{c}} = 0$ and compute the circuit as below and get $P^*(\bar{b}) = 0.02$ and $P^*(b) = 0.02$.



Solution 3a)ii). For evidence $e_2 = \bar{a}, \bar{c}$, we put $\lambda_a = 0, \lambda_{\bar{a}} = 1, \lambda_c = 0, \lambda_{\bar{c}} = 1$ and compute the circuit as below and get $P^*(\bar{b}) = 0.03$ and $P^*(b) = 0.03$.



Solution 3a)iii). For evidence $e_3 = \bar{a}$, we put $\lambda_a = 0, \lambda_{\bar{a}} = 1, \lambda_c = 1, \lambda_{\bar{c}} = 1$ and compute the circuit as below and get $P^*(\bar{b}) = 0.05$ and $P^*(b) = 0.05$.



Solution 3b). The two circuit outputs represents the joint probability distribution over b and the respective evidence. $P^*(b) = P(b, e_i)$ and $P^*(\bar{b}) = P(\bar{b}, e_i)$ where e_i is the respective evidence.

Solution 3c).

$$Pr(\bar{b}|e_1) = \frac{Pr(\bar{b}, e_1)}{P(e_1)}$$

$$Pr(\bar{b}|e_1) = \frac{Pr(\bar{b}, \bar{a}, c)}{P(\bar{a}, c)}$$

$$Pr(\bar{b}|e_1) = \frac{Pr(\bar{b}, \bar{a}, c)}{P(\bar{a})P(c|\bar{a})}$$

$$Pr(\bar{b}|e_1) = \frac{0.02}{0.1 * 0.4} = 0.5$$

$$Pr(\bar{b}|e_2) = \frac{Pr(\bar{b}, e_2)}{P(e_2)}$$

$$Pr(\bar{b}|e_2) = \frac{Pr(\bar{b}, \bar{a}, \bar{c})}{P(\bar{a}, \bar{c})}$$

$$Pr(\bar{b}|e_2) = \frac{Pr(\bar{b}, \bar{a}, \bar{c})}{P(\bar{a})P(\bar{c}|\bar{a})}$$

$$Pr(\bar{b}|e_2) = \frac{0.03}{0.1 * 0.6} = 0.5$$

$$Pr(\bar{b}|e_3) = \frac{Pr(\bar{b}, e_3)}{P(e_3)}$$

$$Pr(\bar{b}|e_3) = \frac{Pr(\bar{b}, \bar{a})}{P(\bar{a})}$$

$$Pr(\bar{b}|e_3) = \frac{0.05}{0.1} = 0.5$$