LOFAR software for beam correction and circular conversion software

T. D. Carozzi Onsala Space Observatory, Sweden

24 Nov 2012 (Created 27 Jan 2011)

1   Release news

- 24 Nov 2014, version 1.7: Reorganized package and prepared for release on github.

- 12 Feb 2013, version 1.6: Input MS file is not required to have autocorrelations. The "antennaJones.py" module now can compute Jones matrices (without resorting to an MS file). It knows about 2 different antenna element models: the default ideal electric dipole, and the Hamaker model based on code from ASTRON.

- 15 May 2012, version 1.5, First release

2   Background motivation

The Long-baseline group (LBG) noted the issue of differential Faraday rotation degradation of LOFAR observations. As this affects long-baselines most, it was decided that the LBG should try to find a solution. The short term solution that would allow further meaningful processing of LOFAR data was suggested: to convert the data output, which is currently in terms of linear polarized components, into circularly polarized components. It was also proposed that a polarimetric correction for the LOFAR beam be applied to the standard data output concurrently to the conversion to circular.

3   Mathematical background: Jones matrix for LOFAR

LOFAR receives electric fields radiated from celestial sources. We will use the IAU coordinate system at some epoch (normally J2000) for these impinging polarized electric fields, which in the linear (noncircular) basis can be written

$$\mathbf{e}_{\mathrm{radec}} = \begin{pmatrix} e_\alpha \\ e_\delta \end{pmatrix} \tag{3.1}$$

where right ascension is α and declination is δ coordinates of the equatorial system. This field is can be seen as the input to the LOFAR measurement equation. The IAU also support a circular basis which is related to the linear basis via

$$\mathbf{e}_{\text{radec}}^{\circ} = \begin{pmatrix} e_r \\ e_l \end{pmatrix} = \mathbf{C}\mathbf{e}_{\text{radec}} \tag{3.2}$$

where r, l denote the right and left handed components respectively.

$$\mathbf{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & +\mathrm{i} \\ 1 & -\mathrm{i} \end{pmatrix} \tag{3.3}$$

The output of the measurement equation is the induced voltages out of LOFAR receivers which we denote

$$\mathbf{V}_{XY} = \begin{pmatrix} V_X \\ V_Y \end{pmatrix} \tag{3.4}$$

where are the X and Y are channels associated with the LOFAR polarized antennas along the corresponding local cartesian horizontal directions of the antenna coordinate system, with Z along the vertical towards zenith.

At the time when a certain measurement is made the apparent direction of the incoming field will be different from the position given at the standard epoch due to the precession of the equinoxes. This results in an apparent rotation of the electric field given by the 2x2 rotation matrix

$$\mathbf{R}_{\text{polzprec}} \tag{3.5}$$

so

$$\mathbf{e}_{\text{radec,app}} = \mathbf{R}_{\text{polzprec}}\mathbf{e}_{\text{radec}} \tag{3.6}$$

In between in the input to LOFAR and the output from LOFAR we assume a linear matrix relation. The (potentially) complex 2x2 matrix that connects the two via a matrix multiplication is known as the Jones matrix.

3.1 Electric dipole model Jones

In this section, let us model the LOFAR response as an ideal short electric dipole (ied). To derive the Jones matrix in this case we make use of the fact that the voltage induced in an ied is proportional to the electric field along the direction of the dipole, in other words it picks up the cartesian component of the E-field along which it is extended. The electric field from some direction, however, is given in IAU system which is a spherical coordinate system. Therefore we proceed by converting the spherical components into a cartesian system.

3.2 Equatorial to ITRF

The cartesian system used by LOFAR is the cartesian ITRF. The transformation from the equatorial to ITRF is

$$\mathbf{e}_{\text{ITRF}}^{(3)} = \mathbf{T}_{\text{sph2cart}}\left(l = l_{\text{ITRF}}, m = m_{\text{ITRF}}, n = n_{\text{ITRF}}\right)\mathbf{e}_{\text{radec}} \tag{3.7}$$

with

$$\mathbf{e}_{\mathrm{ITRF}}^{(3)} = e_x\hat{\mathbf{x}} + e_y\hat{\mathbf{y}} + e_z\hat{\mathbf{z}} = (e_x, e_y, e_z)^{\mathsf{T}} \tag{3.8}$$

the $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ basis being the ITRF basis, and

$$\mathbf{T}_{\mathrm{sph2crt}}(l,m,n) = \frac{1}{\sqrt{l^2+m^2}}\begin{pmatrix} m & ln \\ -l & mn \\ 0 & -l^2-m^2 \end{pmatrix} \tag{3.9}$$

and where

$$\begin{aligned} \mathbf{s}_{\mathrm{ITRF}} &= l_{\mathrm{ITRF}}\hat{\mathbf{x}} + m_{\mathrm{ITRF}}\hat{\mathbf{y}} + n_{\mathrm{ITRF}}\hat{\mathbf{z}} \\ &= (l_{\mathrm{ITRF}}, m_{\mathrm{ITRF}}, n_{\mathrm{ITRF}})^{\mathsf{T}} \end{aligned} \tag{3.10}$$

Eq. ((3.9)) is equivalent to the $\mathbf{T}'(\Theta = \pi/2)$ in eq. (B6) in Carozzi & Woan (2009).

Now we have converted the electric field from a 2D spherical system to 3D cartesian system.

3.3   ITRF to station coordinates

Each LOFAR station has a local cartesian coordinate system called the *station coordinate system* (stn). M. Brentjens document "Station coordinate system" (2009), defines a basis system, $\hat{\mathbf{p}}, \hat{\mathbf{q}}, \hat{\mathbf{r}}$. Relative to the ITRF, stn can found by using the rotation matrix of a station that is provided in the COORDINATE_SYSTEM column of the LOFAR_ANTENNA_FIELD table in the measurement set (MS) file of the data. Let denote this matrix

$$\mathbf{M}_{\mathrm{stn2ITRF}} = \begin{pmatrix} \hat{p}_x & \hat{q}_x & \hat{r}_x \\ \hat{p}_y & \hat{q}_y & \hat{r}_y \\ \hat{p}_z & \hat{q}_z & \hat{r}_z \end{pmatrix} \tag{3.11}$$

and its inverse as

$$\mathbf{M}_{\mathrm{ITRF2stn}} = \mathbf{M}_{\mathrm{stn2ITRF}}^{\mathsf{T}} \tag{3.12}$$

With this given matrix we can transform from the ITRF system into station coordinates, so the 3D cartesian vector representation of the incoming electric field in the stn coordinate system can be expressed

$$\mathbf{e}_{\mathrm{stn}}^{(3)} = \mathbf{M}_{\mathrm{ITRF2stn}}\mathbf{e}_{\mathrm{ITRF}}^{(3)} = \mathbf{M}_{\mathrm{ITRF2stn}}\mathbf{T}_{\mathrm{sph2crt}}(\mathbf{s} = \mathbf{s}_{\mathrm{ITRF}})\mathbf{e}_{\mathrm{radec}} \tag{3.13}$$

3.4   Station coordinates to Antenna coordinates

The stn coordinate system is not aligned with the dipole directions. Let us call the coordinate system which is aligned with the dipoles the *antenna* coordinate system (XY). For all LOFAR stations, the dipoles are rotated 45° from the station coordinate

system. From Fig 1 in Brentjens (2009), the precise rotation between the station (stn) and antenna (XY) coordinates is given by the rotation matrix

$$\mathbf{M}_{\text{stn2XY}} = \frac{1}{\sqrt{2}} \begin{pmatrix} +1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \tag{3.14}$$

3.5 Conversion of electric field to voltages through ied

Now with the electric field expressed in a cartesian coordinate system aligned with the LOFAR dipoles, we can apply the ideal electric dipoles (ied) model. As mentioned above, the response is parallel to the dipole vector. LOFAR has two nominally orthogonal dipoles so the response can be modeled by the 2×3 matrix

$$\mathbf{L}_{\text{ied}} = L \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{3.15}$$

Thus the open-circuit voltage excited by the incident electric field is

$$\mathbf{V} = \mathbf{L}_{\text{ied}} \mathbf{e}_{\text{stn}}^{(3)} \tag{3.16}$$

3.6 Compound Jones matrix

The compound measurement equation for LOFAR modeled as dipoles can therefore be written as

$$\mathbf{V} = \mathbf{L}_{\text{ied}} \mathbf{M}_{\text{stn2XY}} \mathbf{M}_{\text{ITRF2stn}} \mathbf{T}_{\text{sph2crt}} \left(\mathbf{s} = \mathbf{s}_{\text{ITRF}} \right) \mathbf{R}_{\text{polzprec}} \mathbf{e}_{\text{radec}} \tag{3.17}$$

for the source polarization given in a linear basis at a standard epoch. For the circular polarization basis, the formula is

$$\mathbf{V} = \mathbf{L}_{\text{ied}} \mathbf{M}_{\text{stn2XY}} \mathbf{M}_{\text{ITRF2stn}} \mathbf{T}_{\text{sph2crt}} \left(\mathbf{s} = \mathbf{s}_{\text{ITRF}} \right) \mathbf{R}_{\text{polzprec}} \mathbf{C}^{\text{H}} \mathbf{C} \mathbf{e}_{\text{radec}}$$
$$= \mathbf{L}_{\text{ied}} \mathbf{M}_{\text{stn2XY}} \mathbf{M}_{\text{ITRF2stn}} \mathbf{T}_{\text{sph2crt}} \left(\mathbf{s} = \mathbf{s}_{\text{ITRF}} \right) \mathbf{R}_{\text{polzprec}} \mathbf{C}^{\text{H}} \mathbf{e}_{\text{radec}}^{\circ} \tag{3.18}$$

If the electric field is to be expressed in the apparent equatorial coordinate system the measurement equations are

$$\mathbf{V} = \mathbf{L}_{\text{ied}} \mathbf{M}_{\text{stn2XY}} \mathbf{M}_{\text{ITRF2stn}} \mathbf{T}_{\text{sph2crt}} \left(\mathbf{s} = \mathbf{s}_{\text{ITRF}} \right) \mathbf{e}_{\text{radec,app}} \tag{3.19}$$

and

$$\mathbf{V} = \mathbf{L}_{\text{ied}} \mathbf{M}_{\text{stn2XY}} \mathbf{M}_{\text{ITRF2stn}} \mathbf{T}_{\text{sph2crt}} \left(\mathbf{s} = \mathbf{s}_{\text{ITRF}} \right) \mathbf{C}^{\text{H}} \mathbf{e}_{\text{radec,app}} \tag{3.20}$$

Now if we want to correct the measured voltages so that they represent the electric field then we must invert the Jones matrix in these compound equations. So

$$\mathbf{e}_{\text{radec}} = \left(\mathbf{L}_{\text{ied}} \mathbf{M}_{\text{stn2XY}} \mathbf{M}_{\text{ITRF2stn}} \mathbf{T}_{\text{sph2crt}} \left(\mathbf{s} = \mathbf{s}_{\text{ITRF}} \right) \mathbf{R}_{\text{polzprec}} \right)^{-1} \mathbf{V} \tag{3.21}$$

and analogously for the other representations.

It useful to notice that the electric field in the circular basis can be obtained from the electric field in the linear basis by factoring out the **C** matrix as in equation (3.2). Likewise, the polarization in the epochal equatorial coordinate system can be computed from the apparent equatorial system by applying the polarization precession matrix $\mathbf{R}_{\text{polzprec}}$. Therefore it is minimally sufficient to compute the inverse of the Jones matrix to the linear apparent equatorial system, and then optionally apply the precession matrix and/or the circular basis matrix to obtain all the systems of interest.

The final data output of LOFAR is correlations of the complex voltages $\mathbf{V}^{(p)}, \mathbf{V}^{(q)}$ from pairs of stations p, q i.e. the visibilities $\mathscr{V}^{(p,q)} = \mathbf{V}^{(p)} \otimes \left(\mathbf{V}^{(q)} \right)^*$. So to apply these corrections to the MS data one must compute the Jones matrix for each pair of stations and apply the inverse taking care to conjugate one the stations appropriately.

4 Description of the `mscorpol` Software

4.1 Summary

The mathematics presented in the previous section to correct the measurement set (MS) data for the LOFAR antenna response has been implemented in a `python` script developed by the author called `mscorpol`, which is meant to stand for "measurement set correct for polarization response". The Jones matrix that it currently corrects for is an ideal electric dipole with effective length set to 1 [1] for all frequencies, with the look direction set to the phase reference direction given in the MS. This type of correction can seen as a purely geometric correction of the projection of the electric field done for the center of the source field. The main advantage of this correction is that it is simple to understand conceptually and it requires no auxiliary parameters beyond what is provided by the MS file. More complex models would require tables of coefficients to describe the frequency dependence and the directional dependence of the antennas.

The software also has support for computing Jones matrices for observations without an MS file, e.g. beamformed data (although it can not update this data, only compute the corresponding Jones matrices). The relevant command is `antennaJones`

- Version: 1.6

- Dependencies:

  - `python 2.6.5`
  - `numpy`
  - `pyrap`
  - `dipoleJones`

- optional:

---

[1] The effective length for a short dipole is equal to physical length of the dipole but because the absolute gain is anyways difficult to determine we can simply use 1 meter.

- – `parseAntennaField`
- – `HamakerJones`
    - ∗ Depends on: `ElementResponseMain` (a stdin/stdout version of the `lofar_element_response` ASTRON code)

- • Assumptions:

  - – Input MS file is nominal LOFAR (linearly polarized) data output
  - – The DATA column in the MS file is taken as input and results are written in the same DATA column. Thus original data is overwritten.
  - – Default setting is such that visibility data is $DATA(ANTENNA1, ANTENNA2) = V(ANTENNA1)^* \times V(ANTENNA2)$, where V() is the raw voltage of the corresponding antenna (can be flipped with internal flag).
  - – The MS file is regular: i.e. for each unique value in TIME there is a value in DATA for all baselines (ANTENNA1 and ANTENNA2 combinations)
  - – The time period of an MS file is not that long, so for the epoch precession it is sufficient to compute it at the start time.

The optional `parseAntennaField` module allows to work without MS files.

4.2 Examples

4.2.1 mscorpol

Normally the script is executed on an MS file to produce dipole corrected data in a circular basis. The shell command:

```
$ mscorpol.py -f L2011_25115_SB102.MS
```

produces corrected data in the DATA column and outputs it in the DATA column.

If one wishes to correct for the dipole project but keep the result in linear polarization basis (IAU standard frame), then use

```
$ mscorpol.py -l -f L2011_25115_SB102.MS
```

where the -l flag represents output in the linear basis.

To inspect the Jones matrices used in the correction use

```
$ mscorpol.py -j -f L2011_25115_SB102.MS
```

Note that this flag does not update the data. The MS file can either be original LOFAR data or data that already has been processed by `mscorpol`.

4.2.2 antennaJones

If you want to compute Jones matrices without necessarily having an MS file, you can use the `antennaJones` script. To run it from the shell:

```
$ antennaJones.py Hamaker SE607 '2012-04-01 01:02:03' 60 1 0.1 0.2 60E6
```

This computes Jones matrices based on the Hamaker (ASTRON) model of the LOFAR element antennas for station SE607, start time '2012-04-01 01:02:03', for a duration of 60 s in 1 s time steps for the direction 0.1 rad RA and 0.2 rad DEC at 60 MHz. Or it can be run based on observation information from a LOFAR parset file, e.g.

```
$ antennaJones.py dipole -o L29053
```

where L29052 is the LOFAR observation id.

4.3 Algorithm

The algorithm used by mscorpol version 1 is as folows. Iterate over ANTENNA1 in MS file. For current antenna number, compute the Jones matrix as per Section 2 for all the unique times. Apply the conjugated inverted Jones matrices to the selected DATA, and apply the nonconjugated Jones matrices to DATA for which ANTENNA2 matches current antenna number.