$A = k[x,y]$

$f = x^2y + xy^2 + y^2$

$F = (\overbrace{xy-1}^{f_1}, \overbrace{y^2-1}^{f_2})$

$> \; = \; >_{grevlex}$

$x^2y > xy^2$ b.c. $(2,1)-(1,2) = (1,-1)$

$xy^2 > y^2$ b.c. $3 > 2$.

$x > y$ b.c. $(1,0)-(0,1) = (1,-1)$

$f_1 = xy - 1$

$f_2 = y^2 - 1$

x: $\quad x^2y + xy^2 + y^2$

$\quad x^2y - x$

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y: $\quad xy^2 + y^2 + x$

$\quad xy^2 - y$

1: $\quad y^2 + x + y$

$\quad y^2 - 1$

Can't divide! $\quad x + y + 1 \longrightarrow x$

$\quad y + 1 \longrightarrow y$

$\quad 1 \longrightarrow 1$

$\quad 0$

Remainder

Conclusion: $f = (x+y)f_1 + 1 \cdot f_2 + x + y + 1$

Note: Depends on both order of division and monomial order!