

$$A = k[x, y]$$

$$f = \underbrace{x^2y}_{f_1} + \underbrace{xy^2}_{f_2} + y^2$$

$$f = (xy-1, y^2-1)$$

$$x^2y > xy^2 \text{ b.c. } (2,1) - (1,2) = (1,-1)$$

$$xy^2 > y^2 \text{ b.c. } 3 > 2.$$

$$x > y \text{ b.c. } (1,0) - (0,1) = (1,-1)$$

$$> = >_{\text{grelex}}$$

$$f_1 = xy - 1 \quad \text{⊗} \quad x^2y + xy^2 + y^2$$

$$f_2 = (y^2 - 1)$$

$$\begin{array}{r} x^2y - x \\ \hline \end{array}$$

$$\begin{array}{r} xy^2 + y^2 + x \\ \hline \end{array}$$

$$\begin{array}{r} xy^2 - y \\ \hline \end{array}$$

$$\begin{array}{r} y^2 + x + y \\ \hline \end{array}$$

$$\begin{array}{r} y^2 - 1 \\ \hline \end{array}$$

Remainder

$$\text{⊗} \quad + y + 1$$

$$\longrightarrow x$$

$$\begin{array}{r} y + 1 \\ \hline \end{array}$$

$$\longrightarrow y$$

$$\begin{array}{r} 1 \\ \hline \end{array}$$

$$\longrightarrow 1$$

$$0$$

$$\text{Conclusion: } f = (x+y)f_1 + 1 \cdot f_2 + \text{⊗} + \text{⊗} + 1$$

Note: Depends on both order of division and monomial order!